EFFICIENT TOPOLOGY DESIGN METHODS FOR NEXT GENERATION ACCESS NETWORKS

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LIST OF ACRONYMS

AETH	Active Ethernet: a current AON network technology
AON	Active Optical Network
BCA	Branch Contracting Algorithm: fast heuristic algorithm for topology design of Passive Optical Networks
CAPEX	Capital expenditure : cost of deploying the network
CFL	Capacitated Facility Location problem
CO	Central Office
CPMP	Capacitated P-Median Problem
DSL	Digital Subscriber Line network
DU	Distribution Unit
FTTx	Fiber to the X, collective term for fully or partly optical access networks (FTTH, FTTB, FTTC architectures), sometimes used as a synonym for NGA
GIS	Geographic Information System: the system to store geographical data
GPON	Gigabit Passive Optical Network: a current PON network technology
INCA	Iterative Neighbor Contracting Algorithm: fast heuristic algorithm for topology design of Active Optical Networks
MIP	Mixed Integer Programming
NGA	Next Generation Access network
NTD	NGA Topology Design (NTD) problem Term for the addressed general mathematical problem, defined in section 2.
OPEX	Operating expense: cost of operating the network
P2P	Point-to-Point Network
PON	Passive Optical Network
QP	Quadratic Programming
SA	Simulated Annealing
SACD	Stepwise Allocation of Critical DUs: fast heuristic algorithm for topology design of Digital Subscriber Line networks
SU	Subscriber Unit
VDSL	Very high bit-rate Digital Subscriber Line: a current DSL network technology

1. Introduction

Telecommunications research and development is motivated by the continuous evolution of service needs and requirements. New technologies are emerging, new principles are evolving, and the networks are facing new and new challenges. Beyond the primary challenge of increasing bandwidth needs, future (internet) services have several additional requirements, e.g. for latency or reliability of the communication network.

Evolution of core and access networks is strongly connected: core networks have to serve the traffic arriving from access networks, or in contrary: even if enormous bandwidth is available in the core network, customers without the necessary high speed access do not benefit from it. After the glorious last decade of optical transmission in core networks, time has come for fiber communications in the access networks. The technical and economic challenges were preventing the exploitation of optical transmission in the access network until the recent years. Optical network technologies based on simple, cheap equipment, such as passive optical networks were the fundamental enablers of the development. The term "Next Generation Access" (NGA) network refers to the fully or partly optical access networks, fulfilling the above mentioned requirements of future (internet) services.

1.1. Next Generation Access Networks (NGA)

Access networks of the (near) future have to face a set of service requirements that enforces substantial changes in the network technology: complete or partial replacement of the copper networks with optical fiber. In the short term, 100 Mbit/s bandwidth requirements have to be fulfilled on a per customer basis, and for the 2020 time horizon, the European goal is to keep up with capacity growth of at least 1 Gb/s in the wireless, and 10 Gb/s in the wired access. Offered new services, e.g. HD VoD (High Definition Video on Demand), VoIP (Voice over IP), videoconferencing and high speed internet access require high bandwidth and low delay simultaneously [1]-[2].

The depth of fiber installation in the network makes distinction between Fiber to the Home (FTTH) or Fiber to the Building (FTTB) architectures, i.e. the complete replacement of the copper network with optical fiber (with or without respect to the in-building cabling); and Fiber to the Cabinet (FTTC) or Fiber to the Neighborhood (FTTN) architectures, i.e. partial replacement of the copper network with optical fiber [3]. Considering the access network structure, a differentiation can be made between point-to-point and point-multipoint systems. In the latter case, several aggregation/distribution nodes exist in the cable plant, and the demand points are connected to the Central Office (CO) or Point of Presence (PoP) through these. In the case of point-to-point networks, the demand points are connected directly to their respective Central Office via a dedicated optical fiber. Point-to-point is also referred to as home run, while point-to-point as star architecture in several publications.

Even though numerous competing NGA network technologies exist, a classification can be made based on their architecture principles:

Passive optical networks (PON)

Passive optical networks rely completely on optical connectivity. They have a point-to-multipoint architecture with passive devices as aggregation/distribution nodes within the cable plant. Currently existing PON technologies are e.g. APON [9], BPON [10], EPON [11] or GPON [12] and their 10G counterparts, 10G EPON [13] and XGPON [14], while WDM PON systems are just showing up [4]-[7].

Active optical networks (AON)

Active optical networks are similar to their passive counterparts, with the significant difference of using active aggregation/distribution equipment in the cable plant. Active Ethernet networks are a significant current example of the AON architecture [15].

Digital subscriber line networks (DSL)

The complete replacement of copper network by optical fiber clearly has some technical advantages. However, the huge investment needs may be prohibitive. The partial replacement, leading to Fiber to the Cabinet (FTTC) or Fiber to the Neighborhood (FTTN) architectures provide a reasonable tradeoff between investment and service level improvement simply by reusing the existing copper connectivity on the very last segment of the access networks. All of the xDSL (e.g. ADSL [18], ADSL2 [19], ADSL2+ [20], VDSL [21], VDSL2 [22]…) technologies represent the family of DSL networks: these have very similar network architecture, at least on a higher abstraction level [23].

Point-to-point fiber networks (P2P)

In contrary to the point-multipoint PON/AON network architectures, in the case of P2P networks, the demand points are connected directly to their respective Central Office (i.e. Point of Presence) via a dedicated optical fiber [16]-[17].

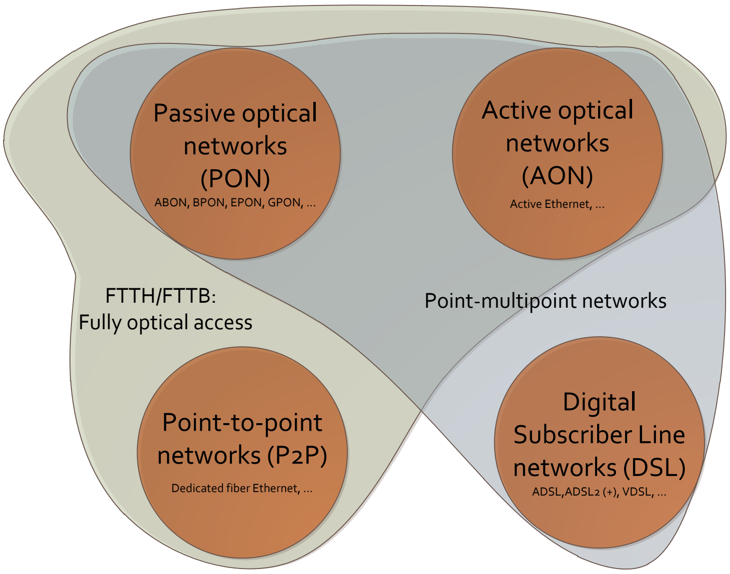


FIGURE 1 NGA TECHNOLOGY CLASSIFICATION

1.2. Network Planning

Optical networks provide a future proof platform for a wide range of services at the expense of replacing the cable plant. Unfortunately this “expense” is an enormous investment, which has to be justified by long term sustainability. Deployment costs have to be minimized, therefore optimal network planning plays crucial role regarding profitability. During optimal access network topology design, various aspects have to be considered simultaneously. Reducing cost of the network deployment (CAPEX – CAPital EXpenditure) is a natural requirement, and also the future OPEX (OPerational EXpenses) have to be considered, even though the latter is more difficult to see in advance. Coupling these with the administrative requirements of the operator and physical limitations of the technology leads to a really complex optimization problem.

The current practice for access network planning could be described as a "design guesswork" of highly experienced engineers, which is probably not an optimal use of the highly valuable human creativity: the complete network planning process is very time consuming. Therefore the work described in my study is devoted to an algorithmic approach of this problem, which on the one hand could speed up the process, and on the other hand, the mathematic interpretation allows evaluation of the network topology regarding optimality.

The methods developed and presented in this study are devoted to strategic topology design in an algorithmic way. The term "strategic topology design" stands for a high-level design, including location of the network elements, layout of the optical cable plant and a complete system design, but lacks the details of a low-level design.

We have to shortly mention here that digital maps and GIS databases (Geographic Information System) are an "enabling technology" for the algorithmic, computer aided network design, solving the most important practical difficulty. Digital maps became available in the recent years for typically any considerable access network service area, and as we will see, this geographic information serves as the primary input data of the optimization problem.

Novelty of the results presented in this study lies in the fact that we are solving a problem algorithmically by computers, which was earlier done by a human guesswork, and no scalable algorithmic solution was known that was capable to handle problem sizes of practical interest.

1.2.1. Techno-economic evaluation

As an important additional application, such a strategic topology design provides input for detailed preliminary cost estimation, and offers thorough techno-economic evaluation and comparison of the NGA technologies in focus. Techno-economic evaluation addresses the relationship between technical decisions and their economic impact. Deploying a NGA networks clearly brings technical advantages over the legacy copper network, however an investment decision must be justified by economic considerations too.

The first and foremost step of any economic evaluation is the estimation of the network deployment cost. The existence of an algorithmic network design methodology leads to a significant improvement in this field: knowing the optimized topology itself, helps to calculate the necessary expenses. Therefore availability of a topology design methodology for all viable NGA technologies supports the optimal choice among suitable network architectures.

The novelty of our approach is the combination of network design with the techno-economic approach. The "state of the art" techno-economic methodologies are using simplified geometric models for cost estimation, instead of the optimized network topology itself. As our recent results have shown, the concept of integrating network design into techno-economic evaluation is a significantly more accurate and reliable methodology [50]-[51].

1.3. Related Work

Theory of network design in itself has a long history and a massive research background [24]. The above described high economic and technical impact brought wide audience to this field, thus network design for optical access networks also triggered various efforts. However, algorithmic network design for access networks was not possible in the absence of digital maps and GIS databases for a long time, and the computational capacity also set tight restrictions. Due to the recent advances in this field, algorithmic network design became a viable opportunity. Therefore, research efforts started from various directions. At the time of writing this study there were some initial results and efforts in the literature, however all of them were constrained by the contradictory requirements of scalability and the need to avoid oversimplification.

S. U. Khan from the University of Texas (US) performed pioneer work in the field of algorithmic network design for PON networks [26] in 2005, even if his work was focusing on a Manhattan grid topology, which is a simplified approach to network design in general [27]. B. Lakic and M. Hajduczenia from Nokia-Siemens Networks Portugal have investigated the possibility to apply k-means clustering for demand points in the Euclidean space (neglecting the street system geometry), and then genetic algorithm for path generation [28]. This notable combination of clustering and genetic algorithm for PON network planning is then compared to hand-made network plans. Despite the promising results, not considering the street system at all is an example of serious oversimplification: the access network must typically follow the cable paths along streets, not crossing e.g. private buildings and properties, or rivers for example.

E. Bonsma et al. from British Telecom have investigated an Evolutionary Algorithm approach, which has scalability problems, restricting its use to small schematic sample networks of a hundred demand points [29]. Another approach for use of genetic algorithm was published in [30] by A. Kokangul and A. Ari from Turkish Telecommunication Company, that has similar scalability issues: it presents a case study with 28 demand points in a PON network. In [31] we find another genetic algorithm solution, with similar limitations on scalability: its capabilities are demonstrated in a sample network with 122 nodes. In [32] A. Haidine, R. Zhao et al. from Dresden University published another metaheuristic technique, namely particle swarm optimization for the demand point clustering (partitioning) problem of a VDSL access network with feeder fiber.

The exact optimization, such as the highly complex Mixed Integer Programming (MIP) approach was demonstrated in [33] by S. Chamberland from École Polytechnique de Montréal, which promises high quality results, but suffers from serious scalability problems: the large number of defined constraints and the dimensions of the resulting MIP matrix limits its usable range for hundreds of demand points.

Later, concurrently with our research, similar research activities have started at University of Melbourne. Initially J. Li and G. Shen published in [34] a two-level random restart iterative heuristic algorithm considering geographic constraints in 2008. The published results were promising, therefore we have implemented their RALA (Recursive Allocation and Location algorithm) for comparison, and we have found that it gives almost the same results as our methods (within 1-2%); however its time consumption is 2-3 orders of magnitude higher due to the numerous iteration steps. Later on, in [35] they presented another recursive method for "greenfield" network planning, independent from existing infrastructure, street system or geographic constraints at all: in their terminology "greenfield" stands for an infrastructureless case, minimizing connection lengths in the Euclidean space.

Clustering methods, and in particular the k-means algorithm was applied in different other publications for creating the demand point groups in point-multipoint networks. A. Agata and Y. Horiuchi from KDDI R&D Labs (Japan) have published a method using Voronoi-diagrams for splitting the service area, and then k-means for clustering, which is a highly effective solution, even if it does not solve the "bin packing" type problem of PON splitters with relatively low splitting capacity [36]: k-means was originally not designed for clustering problems with fixed size clusters.

NGA network planning has impressive literature addressing other aspects, e.g. marketing and economic considerations, or predicting future cost and demand parameters, however these do not belong closely to the scope of this study, therefore I do not present them in details. An overall view on business and economic aspects is given in [39] by FTTH Council Europe.

Techno-economic evaluation of access networks addresses the tradeoff between technical superiority and economic viability. Typically a higher investment results in higher service quality, however the relationship is far from linear. A thorough investigation of options, considering costs, service requirements and viable technologies is necessary to find the most suitable solution for the given service area. This interdisciplinary research area between telecommunication technology and economy is referred to as techno-economics. A An overall description of a techno-economic evaluation framework is given in [40] by the members of an EU research collaboration, including AGH (Kraków), KTH (Stockholm), IBBT (Gent) and BME (Budapest). A nice techno-economic overview of European telecommunication investment options is outlined in [41] by B. T. Olsen (Telenor) et al.

Every techno-economic methodology focuses on the primary question of the initial investment. Two fundamental approaches exist for estimating the cost of network deployment: in cases where sufficient statistical data exists, the typical cost per customer multiplied by the amount of customer premises gives an acceptable estimation, as the work of J. L. Riding, J. C. Ellershaw et al. [45] shows. Otherwise, the network topology has to be modeled, and the approximate values for fiber lengths and network equipment have to be summarized. Since statistical data obviously does not exist for new network technologies, the network topology modeling becomes the only viable solution in this case.

The generally used geometric models are attractive due to their simplicity: using some aggreagate descriptors of the service area (e.g. diameter, population or density), a regular triangle or square network model is built, and with basic trigonometric and geometric formula, dimensions of the resulting cable plant and network equipment list is derived. A sequence of EU research projects was developing such geometric models, e.g. TITAN, OPTIMUM, TERA or TONIC [46], and we also find recent results in this field in [47] - [48].

As our recent results and publications have proven, this very attractive simplicity of geometric models reduces reliability and accuracy of the estimations (initial results in [49], short comparison in [50], in-depth comparison in [51]). It shows a really important field for algorithmic network design: if we have the ability to create the specific network topology for the chosen service area instead of a geometric model, it will definitely lead to higher accuracy for network deployment cost estimation.

1.4. Research objectives

The problem in focus is motivated by the practical problem of network planning: a fiber access network topology is typically designed by hand, that takes a long time for highly qualified professionals of network operators. A computer-aided automatic planning process could speed up the process, and also a well-defined mathematical approach serves as a valuable evaluation and benchmark to the result of the planning process.

The initial research objective is to achieve a deep understanding of the problem, and to clarify its ***theoretic background***. A fundamental principle of the mathematic interpretation is to avoid oversimplification that hides important practical aspects of the problem. Therefore accurate and realistic network and cost models are necessary, that allow technology agnostic, general theoretic discussion at the same time. Difficulty of the formal modeling lies in the conflicting requirements of general, theoretic approach and practical applications of the proposed methodology.

The ***model*** should be able to handle different current and future NGA architectures and technologies, at the necessary abstraction level for theoretic research. Therefore a general ***formulation*** and model for the NGA Topology Design (NTD) problem will be given, followed by the definition of its ***special cases*** for Passive Optical Networks (PON), Active Optical Networks (AON), Digital Subscriber Line (DSL) and Point-to-Point (P2P) networks, according to the above presented NGA technology classification. Based on the model and the formal representation, the mathematical problem should be analyzed, from the point of view of ***complexity*** and ***approximability*** [52].

Modeling, formulation and analysis supports the efforts towards an ***optimization methodology***, which is as-fast-as-possible and as-good-as-possible at the same time. Normally methods with polynomial time complexity are accepted as fast algorithms. However, a large family of optimization problems, as the addressed NGA Topology Design (NTD) problem belongs to NP-hard problems, therefore "as-good-as-possible" in this case refers to an approximation instead of exact optimization.

A fundamental requirement is to develop methods that are scalable enough for real-world scenarios, i.e. large-scale topology design problems, with up to thousands or even tens of thousands of demand points. These large-scale problems have to be solved within reasonable time, even though *reasonable time* for the offline problem of topology design practically means it has to be solved, regardless of time. However, as we will see later, it is still a hard challenge, due to complexity reasons.

Finally, for ***evaluation*** purposes, and in order to provide a benchmark for the proposed methods and algorithms (and any future proposals), reference methods are necessary. These will be built on generally accepted, renowned concepts of optimization, e.g. quadratic/linear programming or widely known metaheuristic approaches.

In summary, goals of the research presented in this study are the following:

- modeling and formulation
- analysis of the problem to be solved
- proposing solutions, solving the optimization problem
- evaluation of the proposed solutions

1.5. Research methodology

Since the work described in this study is focusing on algorithmic topology design for NGA networks, including the theoretic background, proposed solutions and their evaluation, the initial step will be the introduction of a formal ***graph model*** and the ***optimization*** problem formulation, including its constraints, objectives, parameters and special cases.

Once the problem is formulated, a complexity and approximability analysis will be carried out, providing in-depth knowledge on the most significant components and underlying subproblems of the NGA topology design problem. Tools of ***graph theory*** and ***algorithm theory*** will be applied in the sections devoted to these issues, and ***linear reductions*** will be constructed for complexity and approximability analysis of the problems. The algorithmic analysis should provide reasonable requirements regarding scalability and accuracy of the heuristics.

The proposed methodology is using highly specialized ***heuristic*** algorithms, since the general optimization techniques did not meet the requirements for scalability and accuracy. ***Decomposition*** helps to separate the subproblems and handle the strong cross-dependence among them.

For evaluation purposes, two well-known optimization techniques will be used. For exact optimization, a ***quadratic programming*** formulation is given, which will be linearized, and with a notable transformation of the problem, dimensions of the resulting ***linear programming*** formulation were significantly reduced, in order to find lower bounds for numerical evaluation. Finally, a ***simulated annealing*** scheme will be presented, that will be used as a benchmark for the highly specialized, highly efficient heuristics.

2. THE TOPOLOGY DESIGN PROBLEM

The introductory section outlines the topic and the wider area, which is addressed by the research described in the study. This section is devoted to the formal and detailed description of the addressed, investigated and later solved problem, the NGA Topology Design (NTD) problem, as it will be called throughout the study.

The work described in the study is focusing clarifying the theoretic background of algorithmic topology design (optimization) for NGA networks, and then, based on the theoretic work, a methodology will be proposed for it that fulfills the necessary requirements for scalability and accuracy. In order to carry out mathematical analysis, the problem has to be formulated first. A well-defined model and formal representation is a prerequisite of the comprehensive complexity and approximability analysis, and also supports identification of subproblems and key points.

Therefore in the first subsection the formal model is described, that has to be as realistic as possible, representing all decisive characteristics of the NTD problem. The problem will be then formulated as a standard optimization problem, defined by the solution space, the model (variables), the objective and the constraints in the second subsection. Finally the most important special cases are defined, based on the classification of various present and future NGA technologies.

2.1. PARAMETERIZED GRAPH MODEL

Network designers are facing really diverse problems for various NGA technologies; however by finding the proper level of abstraction these problems can be represented by the same formal model. The applied network graph model is intended to represent all the significant information for the topology design process: the geographic and infrastructural data of the area where the network will be deployed, the technology specific constraints and the cost parameters. These are explained in the following paragraphs.

Typically an access network topology consists of a Central Office (CO), a set of Subscriber Units (SU), and a cable plant connecting the demand points (households/subscribers) to the central office. In a point-multipoint structure, these demand points are organized into groups, and demand points within a single group share a Distribution Unit (DU), which aggregates their traffic towards the CO. The network segment interconnecting the DUs and the CO are referred to as *feeder network* segment, while the *distribution network* connects the demand points (households) to the DUs. Schematic view of such a point-multipoint access network is depicted on Figure 2.

The service area itself, where the network is to be installed, may have many attributes that are important for various stages of the network deployment process. Focusing on the topology design itself, traces or paths along which network connections may be realized represent the most important information. Typically network links cannot be built wherever desired, they must follow existing cable paths, the street system or other infrastructure: the cable deployment and civil works (trenching, digging) is allowed in publicly owned land or on the existing infrastructure. The set of these so-called “available network links” (where cables may be deployed) serves as the ***edges*** of the graph model.

Several important ***nodes*** are to be identified in the graph, with respect to their role, e.g. demand points, location of the central office, or the set of locations where the distribution units of a point-multipoint network may be installed (e.g. cabinets with power supply, manholes, etc). The latter set will be referred to as “*available DU locations*”.

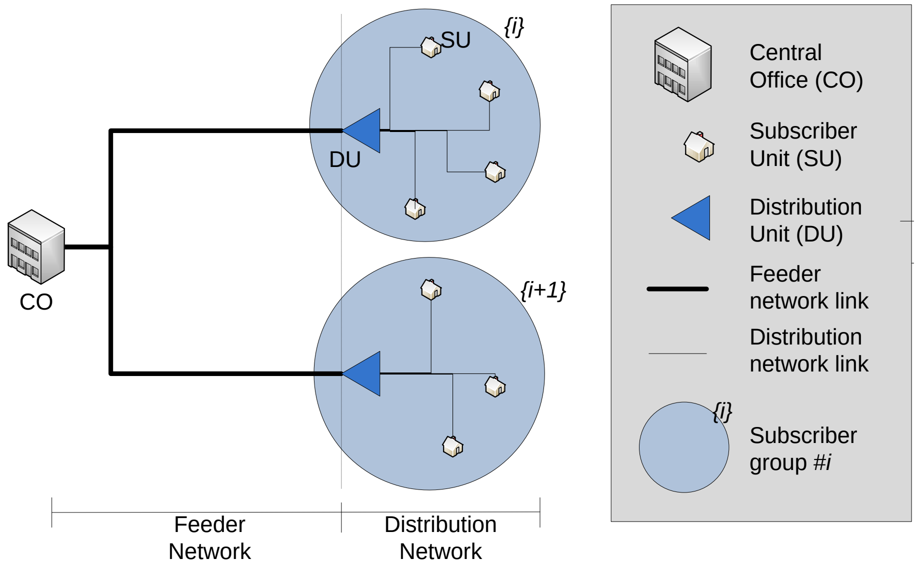


FIGURE 2 POINT-MULTIPOINT ACCESS NETWORK ARCHITECTURE

Therefore, an NGA topology design (NTD) problem is defined by the following data:

- map of the service area, traces and paths, i.e. the available network links
- demand point nodes, with their location, demand, and the drop cables connecting them to the network graph
- the Central Office (CO) location
- a set of locations, where these distribution units may be installed

These altogether define a network graph $G = (V, E)$, the set of DU locations $\Omega = \{DU_j\} \subseteq V$ and the set of demand points (potential subscribers) $S = \{s_i\} \subseteq V$. The set of graph edges represent all the "available network links", which could be used in the cable plant. The graph model of the schematic network of Figure 2 is given on Figure 3.

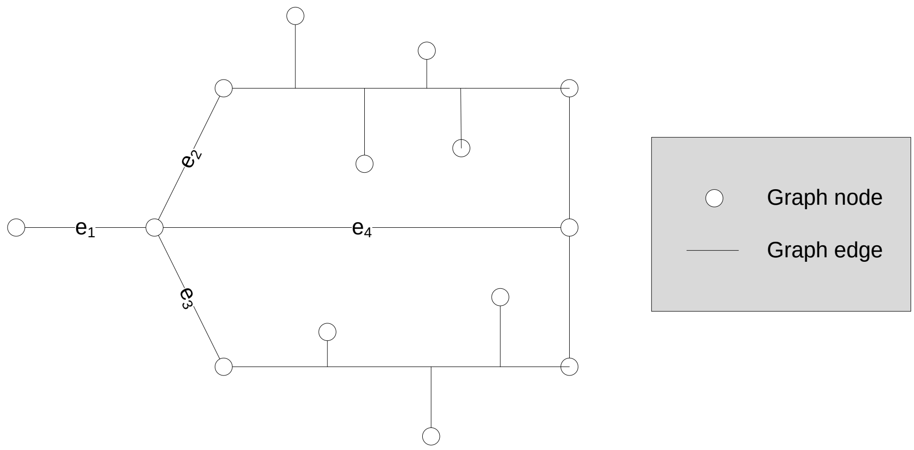


FIGURE 3 NETWORK GRAPH

A set of parameters is assigned to these network elements, e.g. length and cost of edges, location (coordinates) of demand points, or capacity of distribution unit locations. By these parameters and properties of nodes and edges, the various network elements, and the role of nodes or edges in the network graph model is identified. Figure 4 shows the parameterized network graph model, prepared for calculations, while on Figure 5, a respective network topology is given, with DU locations, demand point groups and the set of actually used network links.

Obviously the model is flexible in the sense that the optional parameters may be defined and assigned to graph elements. Such flexibility allows the use of any specific network inventory and GIS database, while the clear schematic graph structure keeps the network modeling straightforward.

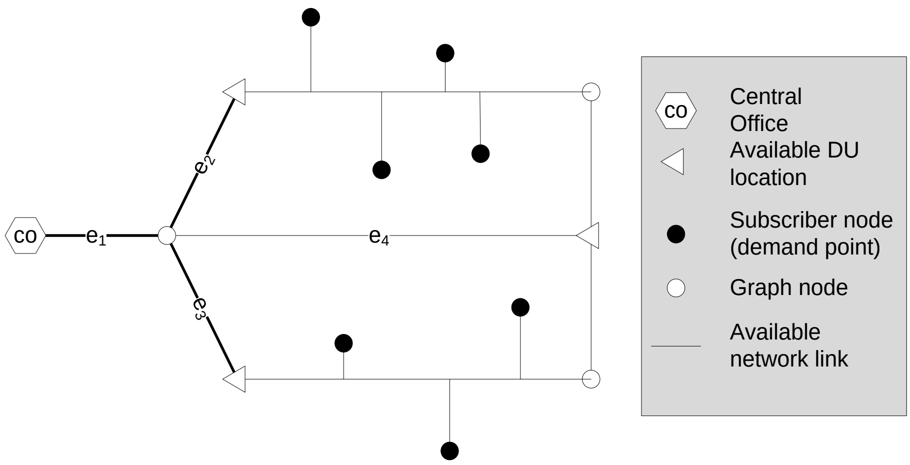


FIGURE 4 PARAMETERIZED NETWORK GRAPH MODEL

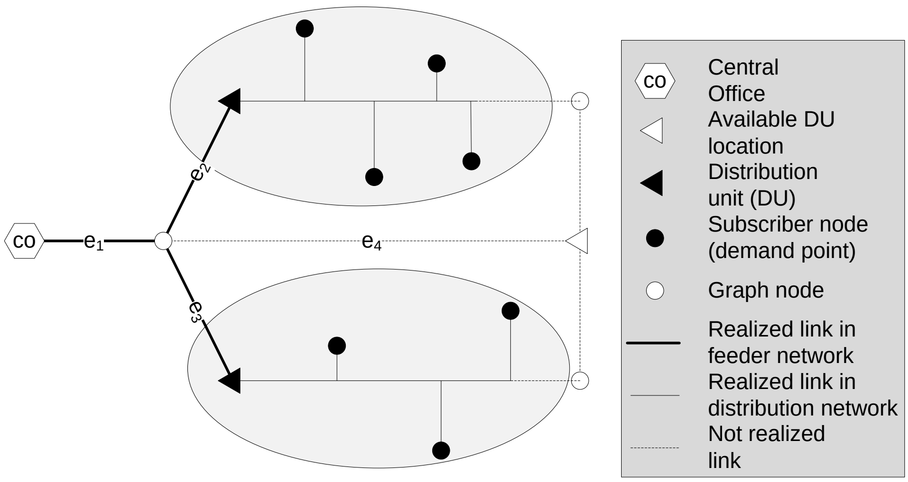


FIGURE 5 RESULTING NETWORK TOPOLOGY ON THE NETWORK GRAPH

2.2. COST FUNCTION (EXPENDITURES)

The necessary investment consists of capital expenditures (CAPEX), i.e. the cost of deploying the network; and operational expenditures (OPEX) for maintenance and operation costs [42]. Beyond this, both capital and operational expenses may be divided into topology-dependent and topology independent components. E.g. fiber costs obviously depend on the topology, while rental of the CO building does not. Table 1 and the following paragraphs give an overview of the decisive network deployment cost factors, their significance and topology dependency. Since the study is devoted to topology design and optimization, the focus will be on topology-dependent network deployment costs (CAPEX).

2.2.1. NETWORK EQUIPMENT

The *central office* (CO) serves as the interconnection between the access network and higher network segments (e.g. metro / core network). Active equipment is used in the CO that requires power supply and cooling. Their price may be significant, and partially topology-dependent: the more distribution units (Splitter, Switch or DSLAM) are deployed, the more complex central office infrastructure is necessary.

Demand points are connected to the CO via *distribution units (DUs)*. In PON networks, the demand points are served through relatively cheap passive optical splitters – while the active DUs used in AON and DSL network are expensive and require power supply that also increases the operational expenses (OPEX). The amount of necessary distribution units depends on the structure of the point-multipoint topology.

Subscriber units serve as the termination of the access network at the demand points. The contribution of these to the total cost is determined by the amount of demand points, thus independent from the network topology.

Formally, the equipment cost is composed of the central office (CO), the distribution unit (DU) and the subscriber unit (SU) costs:

$$C_{Equipment} = C_{CO} + \sum_{Subscribers} C_{SU} + \sum_{Groups} C_{DU} \qquad (1)$$

2.2.2. CABLE PLANT

The cost of the cable plant has two fundamental components: the labor cost of deploying the cables, and the raw material cost of the cables deployed. Cable deployment is typically the most significant among all cost factors, and obviously both are heavily topology-dependent. Deployment cost may be further detailed: (1) trenching and (2) installing a cable in an existing cable duct (*installation cost,* C_0) is independent from the amount of fibers to be deployed, while a smaller, additive cost represents the price of each fiber itself (*fiber cost,* C_v).

Co-existence of these components results in a stepwise cost function, as depicted on Figure 6. Difference between labor and installation costs may be overwhelming if complete trenching (digging) is necessary, but moderate, if existing cable ducts are used.

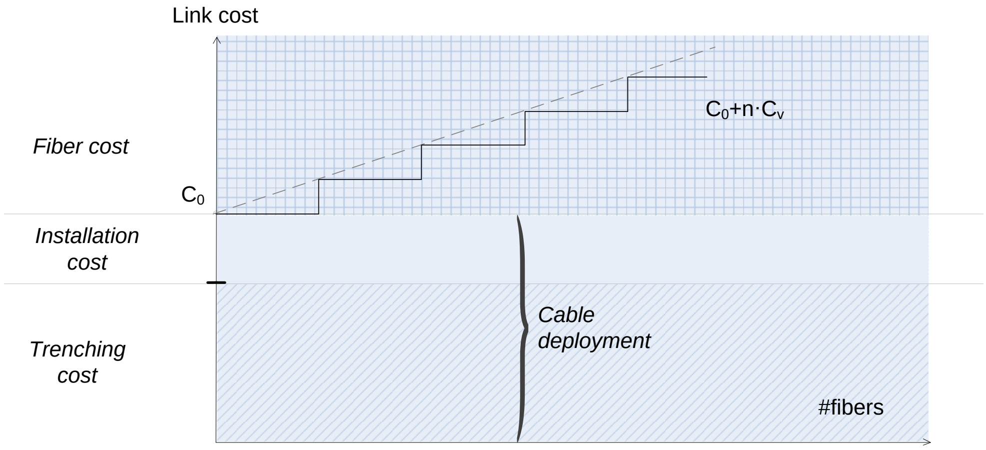


FIGURE 6 CABLE PLANT COSTS

More formally, by summing up the cost of deployment and fiber:

$$C_{cable\ plant} = C_{deployment} + C_{fiber} = \sum_{e:F(e)>0} \left(C_0(e) + \sum_{F(e)} C_v(e) \right) \tag{2}$$

Here $F(e)$ is an auxiliary function indicating the installed capacity upon link e (obviously the labor cost is to be paid only for installed network links). The trenching and installation costs are aggregated into $C_0(e)$ cable deployment cost. The fiber costs on link e are represented by $C_v(e)$.

This cost function, with possibly different values for every network link, or group of network links offers the possibility to include infrastructure information into the network model: various cable deployment costs can be assigned to various areas, subject to cabling technology or civil work required. E.g. existing cable ducts may be re-used with $C_0(e) = 0$ labor cost, or aerial cables may be cheaper to install than trenching. The cost function hence allows the consideration of infrastructure information, and the cost decreasing (increasing) effect of existing (missing) network elements, which again extends the flexibility of the model, but preserves its formal and abstract nature.

2.2.3. Topology dependence

By summing up network equipment (1) and cable plant (2) costs, the overall network deployment cost is as follows:

$$C_{Total} = C_{CO} + \sum_{Subscribers} C_{SU} + \sum_{Groups} C_{DU} + \sum_{e:F(e)>0} \left(C_0(e) + \sum_{F(e)} C_v(e) \right) \tag{3}$$

Namely the total cost is the sum of the {Central Office costs} + {Subscriber Unit costs} + {Distribution Unit costs} + {Cable plant costs}.

Considering the optimization problem, these costs are divided into topology dependent and topology independent costs. Note that topology independent costs are constants in the optimization problem itself, but these will play an important role during the cost estimation and techno-economic evaluation later.

Regarding equipment, summarized cost of subscriber units ($\sum C_{SU}$) does not depend on the topology (only on the amount of demand points), therefore these may be removed from the optimization objective as constants. Cost contribution of the distribution units ($\sum C_{DU}$) increases with the amount of demand point groups. Cost of the central office (C_{CO}) typically depends on the amount of demand points (topology-independent, constant), and also on the amount of demand point groups. The latter component will be merged with the cost of the distribution units (C_{DU}) into a new variable for this combined cost (C^*_{DU}). Therefore the optimization problem reduces to:

$$C^{Topology\ dependent} = \sum_{Groups} C^*_{DU} + \sum_{e:F(e)>0} (\, C_0(e) + \sum_{F(e)} C_v(e)\,)$$

Table 1 summarizes the main cost factors, indicates their topology dependency, and also their significance or weight relative to other cost factors. The latter needs further explanation, which is given in Section 2.5 – it naturally follows the characteristics of the respective network technologies, e.g. active network elements have higher cost than passive ones.

TABLE 1 SUMMARY OF COST COMPONENTS

Cost component		Topology dependency	Significance
Network equipment	Central Office	Medium	High
	Distribution Unit	Medium	AON, DSL: High PON: Low
	Subscriber Unit	None	Medium
Cable plant	Labor cost	High	Extremely high
	Cable	High	High
	Fiber	High	Low

2.3. TECHNOLOGY DEPENDENT CONSTRAINTS

Once the graph model and its parameters are defined, additional rules constraining the set of "valid" topologies may be specified. The discussed NGA network technologies, according to their physical capabilities, set different constraints on the topology. Two primary limitations have to be considered, namely network range and capacity of the distribution units.

Network range leads to various network deployment rules for every technology, in some cases the overall CO-demand point distance (L_{max}), in other cases the length of the feeder (L^{feed}_{max}) or distribution network segment (L^{dist}_{max}) is constrained. The distribution unit capacity constraints (K) are similar for all technologies, at least on the abstraction level: the maximal splitting ratio of an optical splitter or the switching capacity of an active distribution unit has to be considered.

During the topology design process, not only the previously mentioned costs are to be minimized, but also these constraints are to be fulfilled, which determine the set of *valid* topologies. In the next paragraphs, the specific constraints are discussed for each type of NGA technologies.

Passive Optical Networks have network range for the complete access network segment, i.e. the demand point-CO distances are limited (L_{max}). This L_{max} value depends on the optical splitting ratio. Obviously, the maximal split ratio of the optical splitter (K) is also bounded. Typical values for recent GPON systems are 20-60 km network range with 1:64 splitting ratio [12].

Active Optical Networks (e.g. Active Ethernet systems) are constrained by the capacity of the switching unit (K), by the range of the feeder (L_{max}^{feed}) and distribution network segments (L_{max}^{dist}), respectively [15].

These constraints are more strict for DSL networks, since the length of the copper loop (i.e. the distribution segment, L_{max}^{dist}) decisively affects the available bandwidth. For recent VDSL networks, broadband access limits the DSLAM-demand point distance typically in the range of 300-1000 m [21]. Capacity of the DSLAM (K) also constrains the topology.

The point-to-point dedicated fiber architecture is only limited by the length of the optical fiber (L_{max}). Furthermore, even this single constraint is a loose constraint: it may cover sufficiently long distances in practice.

Table 2 concludes these constraints. We note that we are aiming at a higher level of abstraction in the modeling and problem formulation, in order to keep distance from present NGA technology specifications. These constraints offer realistic restrictions for valid NGA topologies. Flexibility of the graph model and its parameters supports virtually any additional physical or administrative limitations to the optimization problem as specific additional constraints.

TABLE 2 TOPOLOGY DEPENDENT CONSTRAINTS

<table>
<tr><th rowspan="2">Network technology</th><th colspan="2">Network range (segments)</th><th rowspan="2">Distribution unit capacity</th></tr>
<tr><th>Feeder</th><th>Distribution</th></tr>
<tr><td>Passive Optical Networks (PON)</td><td colspan="2">L_{max}, L_{diff}</td><td rowspan="3">K</td></tr>
<tr><td>Active Optical Networks (AON)</td><td rowspan="2">L_{max}^{feed}</td><td rowspan="2">L_{max}^{dist}</td></tr>
<tr><td>Digital Subscriber Line (DSL)</td></tr>
<tr><td>Point-to Point Fiber (P2P)</td><td colspan="2">L_{max}</td><td>-</td></tr>
</table>

2.4. Optimization problem formulation

With the above described representation, the NTD problem can be interpreted as a (minimal cost) subgraph of G, that connects the demand points to the CO in a point-multipoint (or point-to-point) architecture through a subset of network links, using several properly located distribution units, fulfilling all the DU capacity, connection length and network capacity constraints.

Formally, we are given a network graph $G = (V, E)$, consisting of edges E and nodes V representing the available network links, the set of DU locations $\Omega = \{DU_j\} \subseteq V$ and the set of demand points (subscribers) $S = \{s_i\} \subseteq V$.

Objective:

$$minimize\ C_{topology\ dependent} = N \cdot C_{DU}^{*} + \sum_{e:F(e)>0} \left(C_0(e) + \sum_{F(e)} C_v(e) \right)$$

The optimization tends to minimize the topology dependent cost of network deployment. All the edges $e \in E$ have a nonnegative length $l(e)$, a cable deployment cost $C_0(e)$ and fiber cost $C_v(e)$. These costs are typically but not necessarily proportional to the length of the link, and depend on the different cabling technologies and existing infrastructure. $F(e)$ is an auxiliary function indicating the installed capacity upon link e.

Auxiliary variables:

$\forall i \in S, \forall j \in \Omega$	$\omega_j^i \in (0,1)$	Indicator of the demand point-DU location assignment, value is 1 only if demand point i is connected to DU location j.
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j.
	$N \in \mathbb{N}$	The total amount of Distribution Units deployed.
$\forall i \in S, \forall e \in E$	$x_e^i \in (0,1)$	Indicator of edge e on the path between demand point i and its assigned DU location.
$\forall j \in \Omega, \forall e \in E$	$y_e^i \in (0,1)$	Indicator of edge e on the path between DU_j and the CO.

Constraints:

(1) $\forall i \in S$ $$\sum_{j\in\Omega} \omega_j^i = 1$$

(2) $\forall j \in \Omega$ $$\sum_{i\in S} \omega_j^i \le n_j \cdot K$$

(3) $\forall v \in V, \forall i \in S$ $$\sum_{e:v\to} x_e^i - \sum_{e:\to v} x_e^i = \begin{cases} \omega_j^i & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$$

(4) $\forall v \in V, \forall j \in \Omega$ $$\sum_{e:v\to} y_e^j - \sum_{e:\to v} y_e^j = \begin{cases} -1 & v = DU_j \\ 0 & \text{otherwise} \\ +1 & v = CO \end{cases}$$

(5) $\forall i \in S$ $$\sum_{e\in E} l(e) \cdot x_e^i + \sum_{j\in\Omega} \left(\omega_j^i \cdot \sum_{e\in E} l(e) \cdot y_e^j \right) \le L_{max}$$

(5a) $\forall i \in S$ $$\sum_{e\in E} l(e)\cdot x_e^i \le L_{max}^{feed}$$

(5b) $\forall i \in S$ $$\sum_{j\in\Omega}\left(\omega_j^i \cdot \sum_{e\in E} l(e)\cdot y_e^j\right) \le L_{max}^{dist}$$

(6) $\forall e \in E$ $$F(e) = \sum_{i\in S} x_e^i + \sum_{j\in\Omega} y_e^j$$

(7) $$N = \sum_{j\in\Omega} n_j$$

Constraints (1) ensure that every demand point is served by exactly one DU, (2) stands for the DU capacity constraints: the summarized capacity of DUs located at a DU location is an upper bound on the amount of demand points connected to that DU location. The flow conservation (Kirchhoff) constraints (3) and (4) keep the flow problem from splitting, and ensure that every demand point has a dedicated flow from its DU (distribution network), and enforces all DUs to be connected by a dedicated flow to the CO (feeder network). Constraints (5), (5a) and (5b) provide the network reach (distance) limits for the total connection length, feeder and distribution network segments, respectively. Finally, constraints (6) and (7) provide the auxiliary data for the cost function: the $F(e)$ installed capacity for every link, and the amount of demand point groups, i.e. the amount of DUs.

This Quadratic Programming (QP) formulation, its linearization and relaxation opportunities are discussed in Section 6.1. It was necessary due to its clear formulation and model description capabilities. Clearly, the novelty of our approach and model is not the existence of a graph model in itself, but as it was concluded in the introductory on related work (Section 1.3), such detailed and realistic network model, optimization objective and constraints, including all important characteristics of an NGA network deployment, i.e. cable plant and equipment costs, network infrastructure and the map of the service area, was not addressed earlier.

2.5. SPECIAL CASES

The optimization problem was defined by its variables, objective function and constraints in the previous section. In its general form it covers all the discussed NGA technology types: this general approach is beneficial for theoretic modeling and analysis of the problem.

On the other hand, a classification of various present and future access network technologies is possible. Among the permanently evolving network technologies and standards different families are noticeable. Based on the different concepts driving these developments, a primary classification differentiates between Passive Optical Networks (PON), Active Optical Networks (AON), and DSL network in the field of point-multipoint access networks, and a completely different approach of Point to Point (P2P) optical networks, offering a dedicated fiber infrastructure for every demand point.

Without losing the necessary abstraction level or committing ourselves to a specific recent standard, these classes of present and future NGA network technologies will be described as special cases of the optimization problem formulated in the previous section, by identifying the most prioritized parts of the objective (cost) function or the constraint set. This classification, outlined in the following subsections is a significant contribution towards highly efficient, specialized optimization algorithms, therefore serves as a fundamental part of the study.

2.5.1. Special case #1 (Passive Optical Networks, PON)

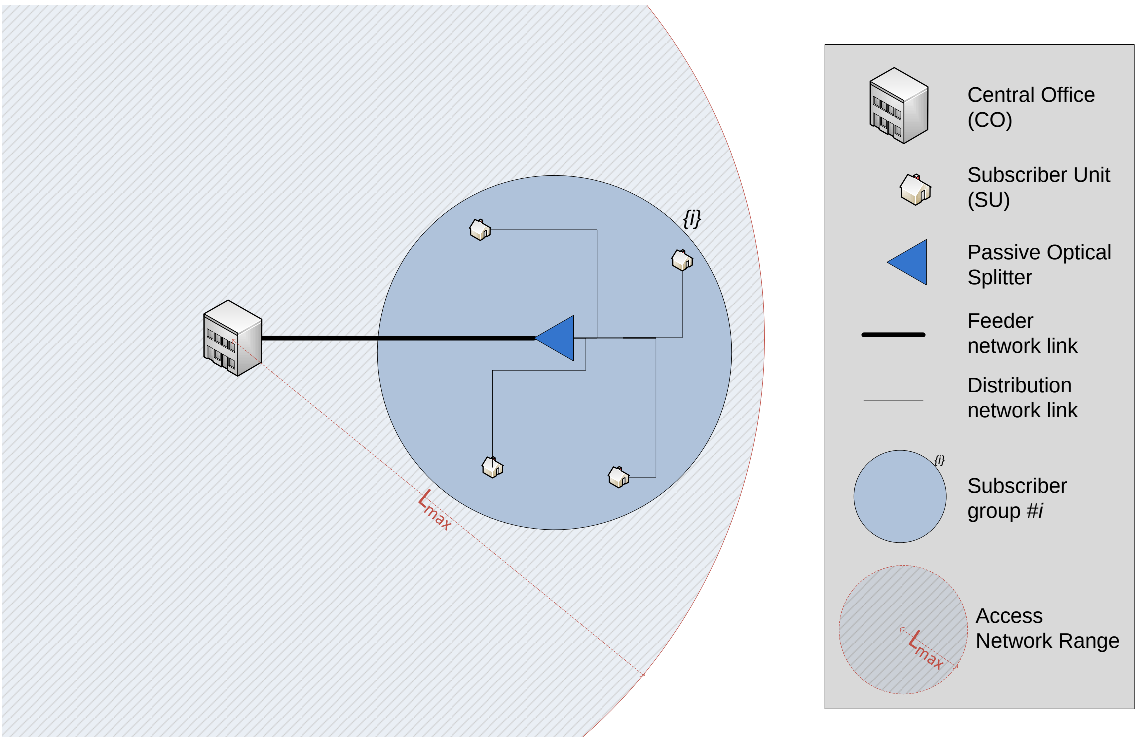


Figure 7 Passive Optical Network (PON)

In a passive optical network, the distribution unit equipment is passive (e.g. a power splitter in TDM PONs or a wavelength switch in WDM PONs), hence the feeder and distribution network segments are in the same optical domain, without signal regeneration at the DU. Therefore the network range limitations stand for the complete optical network segment, i.e. between the CO and the SUs.

However, due to beneficial characteristics of the optical fiber, these network range limitations are fairly permissive for fully optical access networks compared to other access network technologies, e.g. DSL networks. On the other hand, these are not negligible, especially due to the power splitters of passive optical networks: the attenuation depends on the splitting ratio. High capacity distribution units decrease the available network reach.

Capacity constraints of the optical splitters have more significant effect on the topology, in passive optical networks the DU capacities are typically lower than for their active counterparts. Primarily these constraints are affecting the set of valid topologies.

Regarding *topology dependent* network deployment cost, fiber deployment in the distribution network, between every demand point and its corresponding distribution unit (splitter) dominates over the relatively cheap, passive optical splitters, and the less significant feeder network. Moreover, cable plant costs may be further refined. Typically every demand point has to be connected to the access network, thus fiber installation is necessary along the whole street system. Hence cable deployment costs are more or less topology independent. Therefore, during the optimization process, cable plant costs will be incorporated in a single link cost $C_v^*(e)$ which stands for the fiber cost and increased by the respective installation costs.

More formally, in this special case the connection length constraints are relaxed, the $C_0(e)$ and $C_v(e)$ values are substituted by increased $C_v^*(e)$ values, while the C_{DU} distribution unit costs are still considered. This way we get a challenging two-component optimization problem, where the cost function is a combination of two components: DU and cable plant costs. These altogether result in a slightly simplified special case of the NTD problem:

OBJECTIVE:

$$minimize\ C_{topology\ dependent} = N \cdot C^*_{DU} + \sum_{F(e)} C^*_v(e)$$

AUXILIARY VARIABLES:

$\forall i \in S, \forall j \in \Omega$	$\omega^i_j \in (0,1)$	Indicator of the demand point-DU location assignment, value is 1 only if demand point i is connected to DU location j.
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j.
	$N \in \mathbb{N}$	The total amount of Distribution Units deployed.
$\forall i \in S, \forall e \in E$	$x^i_e \in (0,1)$	Indicator of edge e on the path between demand point i and its assigned DU location.
$\forall j \in \Omega, \forall e \in E$	$y^i_e \in (0,1)$	Indicator of edge e on the path between DU_j and the CO.

CONSTRAINTS:

(1) $\forall i \in S$
$$\sum_{j\in\Omega} \omega^i_j = 1$$

(2) $\forall j \in \Omega$
$$\sum_{i\in S} \omega^i_j \leq n_j \cdot K$$

(3) $\forall v \in V, \forall i \in S$
$$\sum_{e:v\to} x^i_e - \sum_{e:\to v} x^i_e = \begin{cases} \omega^i_j & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$$

(4) $\forall v \in V, \forall j \in \Omega$
$$\sum_{e:v\to} y^j_e - \sum_{e:\to v} y^j_e = \begin{cases} -1 & v = SP_j \\ 0 & \text{otherwise} \\ +1 & v = CO \end{cases}$$

(5) $\forall i \in S$
$$\sum_{e\in E} l(e) \cdot x^i_e + \sum_{j\in\Omega}\left(\omega^i_j \cdot \sum_{e\in E} l(e) \cdot y^j_e\right) \leq L_{max}$$
relaxed

(6) $\forall e \in E$
$$F(e) = \sum_{i\in S} x^i_e + \sum_{j\in\Omega} y^j_e$$

(7)
$$N = \sum_{j\in\Omega} n_j$$

2.5.2. Special case #2 (Active Optical Networks, AON)

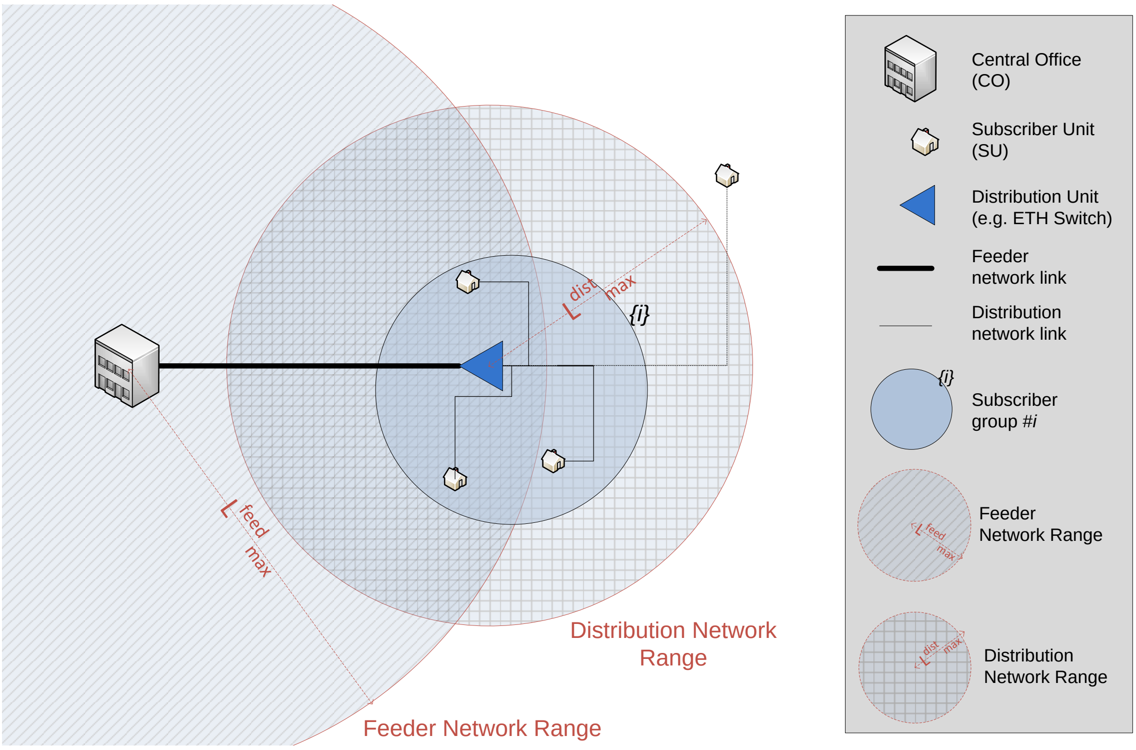


FIGURE 8 ACTIVE OPTICAL NETWORK (AON)

Due to the fully optical cable plant infrastructure, the network range limitations are really permissive, even more then for passive optical networks, since the active distribution units in this case do not increase attenuation, rather make signal regeneration. Moreover, the active equipment divides the optical layer in two, resulting in distinct network range limitations for the feeder and distribution network segments.

As stated earlier, capacity of the active distribution units typically exceeds that of the passive equipment. These altogether move the focus mostly on the cost function, since the constraints permit a considerably wide set of valid topologies.

On the other hand, the active distribution units, compared to the passive ones require significantly higher investment, especially if also the operation and maintenance costs are considered, due to the necessary power and cooling supply, and also due to the higher risk of breakdowns. The cable plant costs, particularly the distribution network has also noticeable contribution to the topology dependent network costs.

According to the problem formulation, in this case the length constraints are relaxed, the DU capacity constraints are kept. The special case is mostly characterized by the cost function: the overwhelming DU costs (C_{DU}), and also the lower distribution network costs are considered, primarily through the fiber (C_v) costs, as described for PON networks.

The formal representation of the special optimization problem:

OBJECTIVE:

$$minimize\ C_{topology\ dependent} = \boldsymbol{N} \cdot \boldsymbol{C}^*_{\boldsymbol{DU}} + \sum_{F(e)} C^*_v(e)$$

AUXILIARY VARIABLES:

$\forall i \in S, \forall j \in \Omega$	$\omega^i_j \in (0,1)$	Indicator of the demand point-DU location assignment, value is 1 only if demand point i is connected to DU location j.
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j.
	$N \in \mathbb{N}$	The total amount of Distribution Units deployed.
$\forall i \in S, \forall e \in E$	$x^i_e \in (0,1)$	Indicator of edge e on the path between demand point i and its assigned DU location.
$\forall j \in \Omega, \forall e \in E$	$y^i_e \in (0,1)$	Indicator of edge e on the path between DU_j and the CO.

CONSTRAINTS:

(1) $\forall i \in S$
$$\sum_{j\in\Omega} \omega^i_j = 1$$

(2) $\forall j \in \Omega$
$$\sum_{i\in S} \omega^i_j \leq n_j \cdot K$$
low significance

(3) $\forall v \in V, \forall i \in S$
$$\sum_{e:v\rightarrow} x^i_e - \sum_{e:\rightarrow v} x^i_e = \begin{cases} \omega^i_j & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$$

(4) $\forall v \in V, \forall j \in \Omega$
$$\sum_{e:v\rightarrow} y^j_e - \sum_{e:\rightarrow v} y^j_e = \begin{cases} -1 & v = SP_j \\ 0 & \text{otherwise} \\ +1 & v = CO \end{cases}$$

(5) $\forall i \in S$
$$\sum_{e\in E} l(e) \cdot x^i_e + \sum_{j\in\Omega} \left(\omega^i_j \cdot \sum_{e\in E} l(e) \cdot y^j_e \right) \leq L_{max}$$
relaxed

(6) $\forall e \in E$
$$F(e) = \sum_{i\in S} x^i_e + \sum_{j\in\Omega} y^j_e$$

(7)
$$N = \sum_{j\in\Omega} n_j$$

2.5.3. Special case #3 (Digital Subscriber Line networks, DSL with optical feeder)

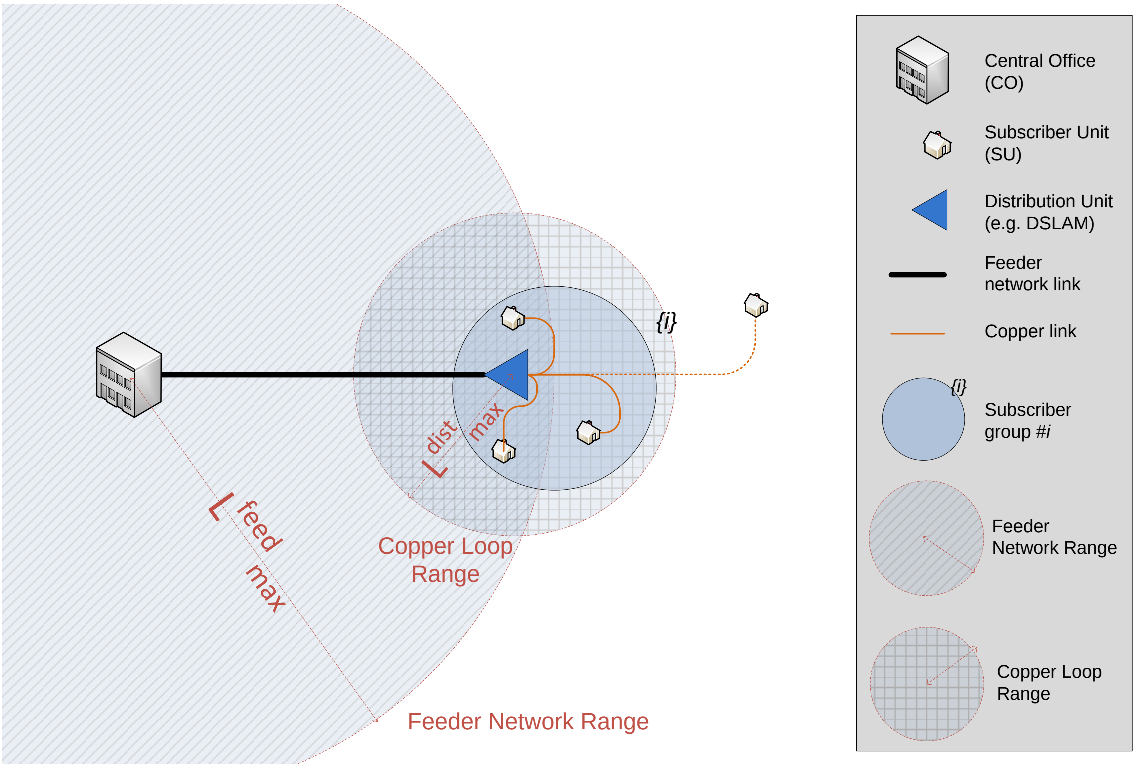


FIGURE 9 DIGITAL SUBSCRIBER LINE NETWORK (DSL)

Physical limitations on the (copper) distribution network segment are dominating the network reach constraints in this case, and obviously the DU capacities are also limited.

Regarding the cost function, it is important to note that these networks are typically installed in areas covered by already existing copper infrastructure, therefore cost of the distribution network is significantly lower due to re-use of copper instead of new optical fiber deployment. The active distribution units, as described in the previous section, require a high capital as well as high operational investment (CAPEX & OPEX), which actually dominates the topology dependent network costs, since the distribution network costs are almost negligible, not like optical access network technologies.

Fulfilling the copper loop constraints, and concurrently considering the capacity of the distribution units is in the heart of the problem in this case; according to the problem formulation, the distribution loop length (L_{max}^{dist}) and DU capacity constraints are kept, and the DU costs (C_{DU}), are considered for minimization.

In terms of optimization problem formulation:

OBJECTIVE:

$$minimize\ C_{topology\ dependent} = N \cdot C_{DU}^{*}$$

AUXILIARY VARIABLES:

$\forall i \in S, \forall j \in \Omega$	$\omega_j^i \in (0,1)$	Indicator of the demand point-DU location assignment, value is 1 only if demand point i is connected to DU location j.
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j.
	$N \in \mathbb{N}$	The total amount of Distribution Units deployed.
$\forall i \in S, \forall e \in E$	$x_e^i \in (0,1)$	Indicator of edge e on the path between demand point i and its assigned DU location.
$\forall j \in \Omega, \forall e \in E$	$y_e^i \in (0,1)$	Indicator of edge e on the path between DU_j and the CO.

CONSTRAINTS:

(1) $\forall i \in S$

$$\sum_{j \in \Omega} \omega_j^i = 1$$

(2) $\forall j \in \Omega$

$$\sum_{i \in S} \omega_j^i \leq n_j \cdot K$$

(3) $\forall v \in V, \forall i \in S$

$$\sum_{e:v\rightarrow} x_e^i - \sum_{e:\rightarrow v} x_e^i = \begin{cases} \omega_j^i & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$$

(4) $\forall v \in V, \forall j \in \Omega$

$$\sum_{e:v\rightarrow} y_e^j - \sum_{e:\rightarrow v} y_e^j = \begin{cases} -1 & v = SP_j \\ 0 & \text{otherwise} \\ +1 & v = CO \end{cases}$$

(5b) $\boldsymbol{\forall i \in S}$

$$\boldsymbol{\sum_{j \in \Omega} \left(\omega_j^i \cdot \sum_{e \in E} l(e) \cdot y_e^j \right) \leq L_{max}^{dist}}$$

decisive

(7)

$$N = \sum_{j \in \Omega} n_j$$

2.5.4. Special case #4 (Point to Point fiber access networks, P2P)

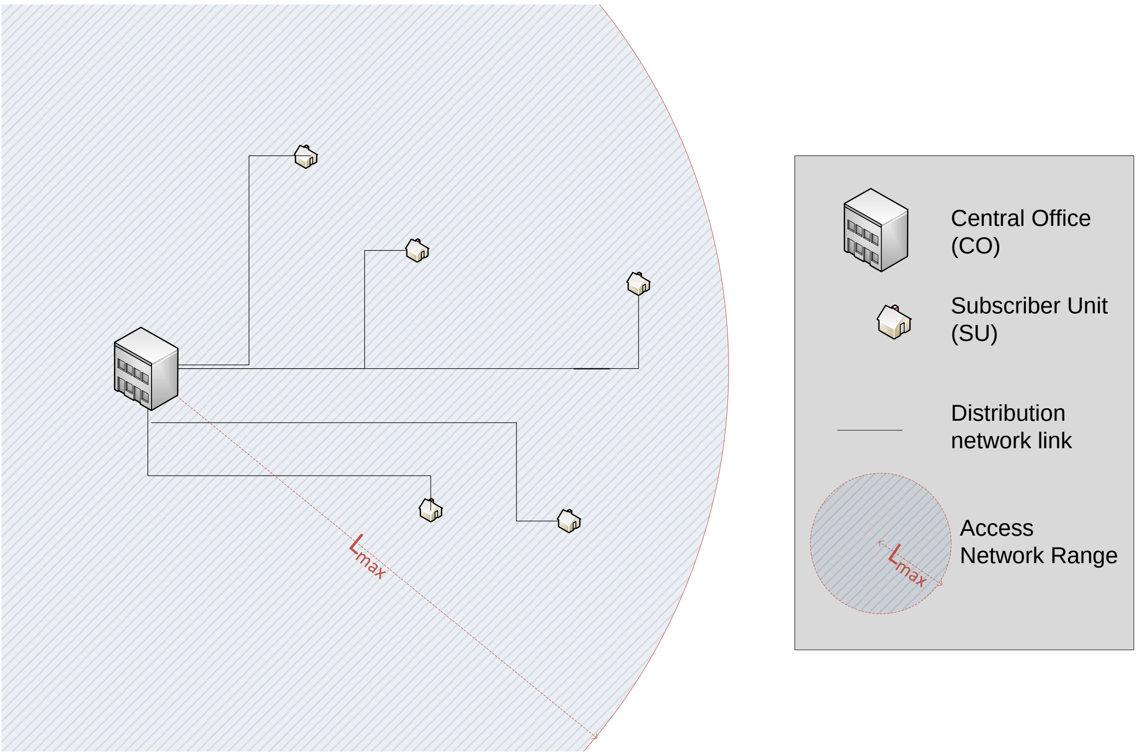


FIGURE 10 POINT-TO-POINT FIBER ACCESS NETWORK (P2P)

Most of the discussed NGA network technologies have point-multipoint architecture. However, dedicated fiber access is a reasonable choice in some cases (e.g. for business users, FTTB systems or mobile backhaul applications). Moreover, the feeder part of any point-multipoint system is treated as a dedicated point-to-point network, connecting the distribution units to the central office. Therefore we need to discuss the optimization of point-to-point network topologies.

In a point-to-point optical access network, we have only one physical constraint for the length of the optical connections, between the CO and the demand points. This is typically a really permissive constraint: the dedicated fiber architecture offers the most robust optical transmission among the presented technologies, in terms of power budget and attenuation aspects.

Absence of DUs restricts the cost function to cable plant costs, without the feeder vs. distribution network distinction. It can be interpreted as the distribution network segment, with the CO serving as a distribution node. Topology dependent cost is reduced to the distribution network: primarily the dominating cable deployment (C_0) costs, and the fiber (C_v) costs with lower priority.

In terms of problem formulation, the problem is significantly reduced, mostly due to the absence of the feeder network:

Objective:

$$minimize\ C_{topology\ dependent} = \sum_{e:F(e)>0} \left(C_0(e) + \sum_{F(e)} C_v(e) \right)$$

Auxiliary variables:

$\forall i \in S, \forall e \in E$ $\quad x_e^i \in (0,1)$ $\quad$ Indicator of edge e on the path from demand point i to the CO.

CONSTRAINTS:

(3) $\forall v \in V, \forall i \in S \qquad \sum_{e:v\rightarrow} x_e^i - \sum_{e:\rightarrow v} x_e^i = \begin{cases} w_j^i & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$

(4) $\forall v \in V, \forall j \in \Omega \qquad \sum_{e:v\rightarrow} y_e^j - \sum_{e:\rightarrow v} y_e^j = \begin{cases} -1 & v = SP_j \\ 0 & \text{otherwise} \\ +1 & v = CO \end{cases}$

(5) $\forall i \in S \qquad \sum_{e} l(e) \cdot x_e^i \leq L_{max}$

(6) $\forall e \in E \qquad F(e) = \sum_{i \in S} x_e$

2.5.5. SUMMARY OF SPECIAL CASES

A graphical overview of these special cases is given in the following figure, highlighting the constraints and cost factors of higher significance. The relaxed constraints and cost (objective) components are represented by clear cells in the table. Significance of the non-relaxed cost components and constraints are indicated by vertical bars and darkness of the table cells at the same time.

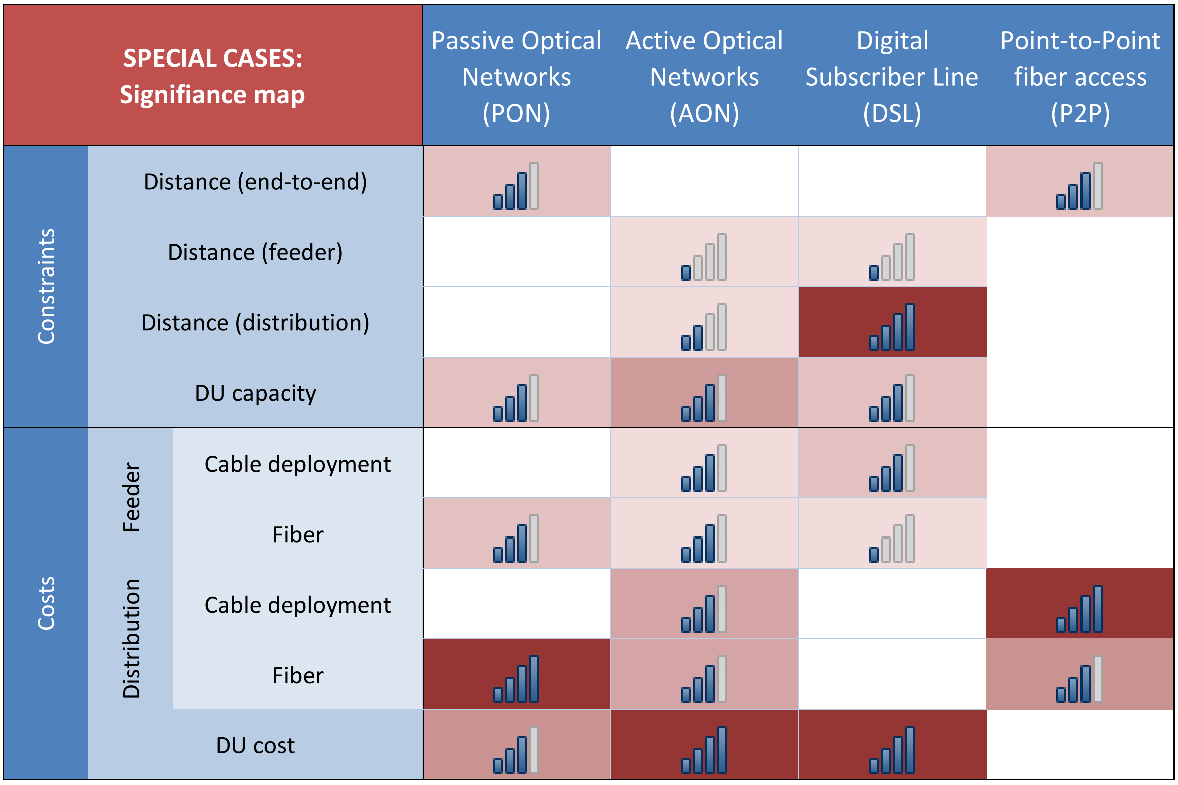


FIGURE 11 SPECIAL CASES OF THE NGA TOPOLOGY DESIGN PROBLEM

3. ALGORITHMIC ANALYSIS

The NGA Topology Design problem was formulated in the previous section, followed by its most important special cases. The next step is the algorithmic analysis of the formulated, mathematical problem. The first straightforward question is on complexity: is it possible to solve the problem in *reasonable time* at all?

The "reasonable time" obviously is not an exact term, it needs further clarification. Network planning or topology design is an offline problem; hence time complexity requirements are not strict. However, the typical problem size is the service area of a single central office, i.e. graphs with thousands or even tens of thousands of nodes. These large scale problem instances do not allow algorithms with exponential complexity. Therefore, the fundamental question is not on the running time, but on general applicability: algorithms with exponential complexity will not provide solutions for the addressed problem size. Hence "reasonable complexity" refers to algorithms that are capable to handle scenarios with 10.000^{+} nodes.

What follows is the algorithmic analysis of the NGA Topology Design problem in its general form and special cases, in order to decide whether it is possible to find the exact optimum in polynomial time or not (Section 3.1). In cases where the exact optimization turns to be intractable, the next important question addresses approximability: how close can we get to the optimum solution within polynomial time (Section 3.2)?

3.1. COMPLEXITY ANALYSIS

Since the problem is now formulated as an optimization problem, the primary question addresses its complexity. The generally used methodology for proving complexity results is to construct reduction schemes. The so-called Karp-reduction supports proof of statements like "the problem is at least as complex as problem P", which P used to be an appropriate known NP-hard problem. Via bi-directional proofs the Karp-reduction also supports NP-completeness statements [54].

More information can be derived from the so-called *linear reduction* (*L-reduction*) scheme that maintains not only complexity, but also approximability features of the problem, being bi-directional in every case. A linear reduction scheme between P_1 and P_2 shows equivalence of two problems: any solution for P_1 directly leads to a solution of P_2 via the L-reduction scheme [55].

In the sequel, such linear reductions will be given for the earlier defined special cases of the NTD problem, in order to prove that their complexity is equal to other already known mathematical problems. The following subsections have a uniform structure: at first, an already known (NP-complete) mathematical problem is defined ("*base problem*"). Then a mutual linear reduction is given, between the respective special case and the base problem, and conclusion is drawn on complexity of the addressed NTD special case, according to the complexity of the *base problem*.

Finally, complexity features for the NTD problem in general are also addressed.

3.1.1. SPECIAL CASE #1 (PASSIVE OPTICAL NETWORKS, PON)

The Capacitated Facility Location (CFL) problem is a well-known optimization problem, with numerous applications, e.g. in the field of logistics, transportation, hub location or even network design problems. In the heart of the problem lies the decision about locations where facilities will be opened, and demand points which will be served through these facilities, e.g. a set of grocery stores for the residents of a city. There is a cost assigned with opening a facility (i.e. the cost of opening a store), and also with the connection between a demand points and a facility (e.g. the distance between a home and the corresponding store) . In the capacitated version, the facilities may serve a limited amount of demand points, even though multiple facilities may be opened at the same location, increasing the facility costs.

***Definition 1** (base problem):* Capacitated Facility Location (CFL) problem [61]

In a capacitated facility location problem we are given two sets: F, the set of *facilities* and C, the set of *clients (i.e. demand points)*. There is a specified *distance* $c_{ij} \geq 0$ between every pair $i, j \in F \cup C$, costs $f_i \geq 0$ for opening facilities $i \in F$, and capacities u_i respectively. Multiple facilities can be opened at the same site, and each copy incurs cost f_i, and serves u_i clients. A multi-set S of facilities has to be identified to serve the clients in C by the facilities in S such that the total facility cost plus the total service cost is minimized. That is, if a client $j \in C$ is assigned to a facility $\sigma(j) \in S$ then we want to minimize $cost(S) = \sum_{i \in S} f_i + \sum_{j \in C} c_{\sigma(j),j}$.

Linear reduction

Lemma 1: The NGA Topology Design (NTD) problem in its special case for PON networks, i.e. when the $C_0(e)$ values are merged into the increased $C_v^*(e)$ values, and the capacitated facility location (CFL) problems are equivalent under linear reductions (L-reductions).

Proof 1: A polynomial time reduction scheme is given for both directions.

CFL→ NTD_{PON}*:* Given a CFL problem instance, including the facilities, clients, distances, etc. An NTD problem will be constructed, which is equivalent to the CFL problem, i.e. it has the same optimum.

The construction ensures that the two problems are identical, i.e. they have coincident optimum. This NTD problem instance can be constructed in polynomial time obviously, and an optimal solution for it also solves the corresponding CFL problem. It both problems, we have a points to be assigned to some nodes in central position. What makes a significant difference is the existence of the graph (that has to be followed by the connections) in the case of the NTD problem, while in a CFL problem the Euclidean space may be used, without restrictions. Figure 12 shows a pair of CFL and NTD problems, in which the clients / demand points, and the facilities / DUs are at the same position. The original graph of the NTD problem is faded but visible at the CFL problem.

The *CFL→* NTD_{PON} construction is composed of the following steps:

- create the NTD graph $G = (V, E)$:
    - $V = F \cup C$, i.e. from the set of clients and facilities
    - $E = \{(i,j) | \forall i, \forall j \in V\}$, i.e. create a full graph,
- the set of DU locations represent the facilities:
    - $\Omega = F$,
- the set of demand points represents the and clients:
    - $S = C$,
- edge cost values are to be assigned as:
    - $C_v^*(e = (i,j)) = c_{i,j}$, i.e. the NTD edge costs will be equal to the CFL costs (distances)
- DU capacities should be the same as the corresponding facility capacities:
    - $K_i = u_i$

This construction ensures equivalence of the two problems, i.e. their optimum coincides.

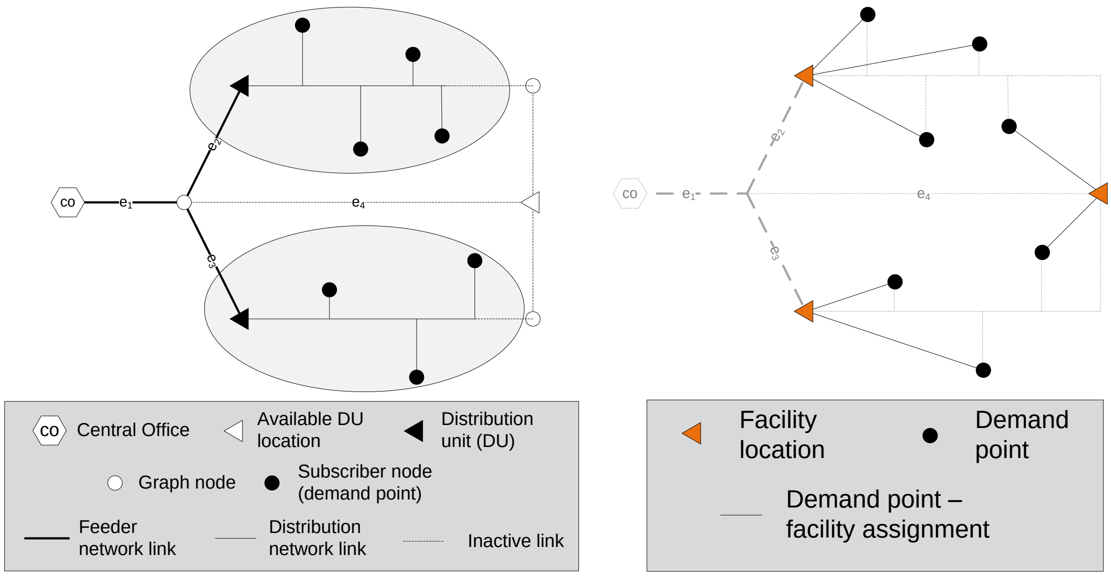


FIGURE 12 FACILITY LOCATION PROBLEM – NETWORK TOPOLOGY DESIGN PROBLEM TRANSFORMATION

NTD_{PON} →*CFL:* In this case, we are given a NTD problem instance, including the graph, demand points, DU locations and edge costs. Now a CFL problem instance with the same optimum value and same solution will be constructed as follows:

- the set of CFL clients will be equivalent to the set of NTD demand points:
  - $C = S$
- the set of facilities will be equivalent to the set of DU locations:
  - $F = \Omega$
- all other nodes of the NTD graph are unnecessary, since the CFL problem does not follow the graph itself: remove nodes of $V \setminus S \cup \Omega$
- the cost of assigning a client to a facility in the CFL problem will be equivalent to the cost of the shortest path (P_G) between the corresponding demand point and DU location in graph G of the NTD problem:
  - $c_{ij} = \sum_{e \in P_G} C_v^*(e)$
- the u_i capacity values of the facilities should be equal to the respective K capacity of the DUs:
  - $u_i = K$
- the f_i cost of the facilities is calculated as the sum of the C_{DU} DU cost and the cost of the feeder link, i.e. the shortest path from the CO to DU location DU_i:
  - $f_i = C_{DU_i} + \sum_{e \in P_{CO \to DU}} C_v^*(e)$

This construction results in a CFL problem instance. Its optimum coincides with the original NTD problem. We note that shortest path algorithms are running in polynomial time, therefore this reduction has polynomial complexity, and it provides identical optimality features for the CFL problem and the original NTD problem.■

Corollary 1a: Since the facility location problem is known to be NP-hard [61], according to the Linear reduction, the NTD_{PON} special case itself is also NP-hard.

3.1.2. Special case #2 (Active Optical Networks, AON)

Obviously there are similarities between the NTD_{AON} and NTD_{PON} problems. The most significant difference is due to the distribution units: in a PON network, the DUs are relatively cheap, having low capacity. In an AON network, the DUs have much higher capacity, and the DU costs are much higher at the same time. It has an important impact on the optimization problem: minimizing the amount of DUs comes into focus. Due to their high capacity, an AON network typically requires just a few DUs to be deployed, not a large set of DUs as for PON networks.

These altogether imply that in NTD_{AON} problem, the amount of necessary DUs (we denote it by p) will be treated as a pre-defined constant. Since the permissive distance constraints allow an extent of demand points being assigned to DUs that completely fills the DU capacity, the minimal amount of necessary DUs is derived from the population (N) and the DU capacity (K): $p = {}^{N}/_{K}$. The overwhelming cost of DUs enforces their maximal utilization, hence (near) minimal amount of DUs leads to the minimal cost topology.

There is a well-known algorithmic problem, namely the capacitated p-median problem (CPMP), which meets the above described interpretation of the NTD_{AON} problem, therefore it will be used in the linear reductions later:

> ***Definition 2*** *(base problem)*: Capacitated p-median problem (CPMP) [64]
>
> In a p-median problem, a set of points N, distance values $\{(d(i,j)|i,j \in N)\}$, and an integer value p is given. The problem is how to select p points of $K \subseteq N$ as *medians*: all other points in $N \setminus K$ have to be assigned to their nearest median. Objective of the optimization problem is to minimize the sum of distances between all points in $N \setminus K$ and their respective medians. In the capacitated version of the problem, each median can have a maximum of C points assigned.

The relationship between the capacitated facility location (CFL) and the capacitated p-median problems (CPMP) is interesting: the amount of medians is given as a constraint for p-median problems, while it is involved in the optimization process for facility location problems. Therefore opening a facility is similar to violating that constraint, and it results in additional cost for that. This technique is known as Lagrangean relaxation: some constraints are integrated in the objective itself with a “penalty” cost for violating them, i.e. the facility cost for facility location problems (see [72], p. 167).

Linear reduction

> ***Lemma 2:*** The NGA Topology Design (NTD) problem in its special case for AON networks, i.e. when length constraints are relaxed, mainly the DU capacity constraints and costs (C_{DU}) are characterizing the problem, and the capacitated p-median problems (CPMP) are equivalent under linear reductions (L-reductions).
>
> ***Proof 2***: A polynomial time reduction scheme is given for both directions.

CPMP→ NTD_{AON}*:* Given a capacitated p-median problem (CPMP) instance, an NTD problem instance will be constructed, which is equivalent to the CPMP problem, i.e. they have a coincident optimum:

- given the set of nodes N for the CPMP problem, the graph $G = (V, E)$ of NTD is constructed:
  - $V = N$,
  - $E = \{(i,j)|\forall i, \forall j \in N\}$, i.e. it will be a full graph

- all nodes of the CPMP problem will be treated as available DU locations in the NTD problem:
    - $\Omega = N$,
- similarly, every node of the graph is a demand point at the same time:
    - $S = N$
- the edge cost values will be equal to the distance between the two nodes of the edge in the CPMP problem:
    - $C_v^*(e = (i,j)) = d(i,j)$,
- the DU capacities will be set to $|N|/p$, according to the explanation above:
    - $K = |N|/p$
- C_{DU} prices will be set to a value which is high enough to dominate fiber costs:
    - $C_{DU} \ll d(i,j)$
- an arbitrary node in N will be chosen as the CO.

This construction leads to an NTD problem in which the DUs are representing the (capacitated) medians. While DUs have a capacity of $|N|/p$, added that the DU costs are dominating the fiber costs, an optimal solution of the NTD problem requires exactly p DUs to serve the N demand points. These DUs will be located at positions minimizing the distribution network costs, i.e. distance sum of the CPMP problem. These altogether ensure coincidence of the optimum for the two problems.

The construction requires polynomial time itself, and a solution for the constructed NTD problem is a solution for the initial CPMP problem as well.

$NTD_{AON} \rightarrow CPMP$: Given an NTD problem instance, a capacitated plant location problem will be constructed with the same optimum.

In order to create the respective CPMP problem, an initial graph transformation step will be made on the NTD problem: the shortest path between every DU location and demand points within L_{max}^{dist} distance will be substituted by a single edge (Figure 13). These length of these edges will be equal to the length of the corresponding shortest paths, with respect to fiber costs in G: $l^*(i,j) = dist_G(i,j)$. This transformed graph will be referred to as $G^* = (V^*, E^*)$.

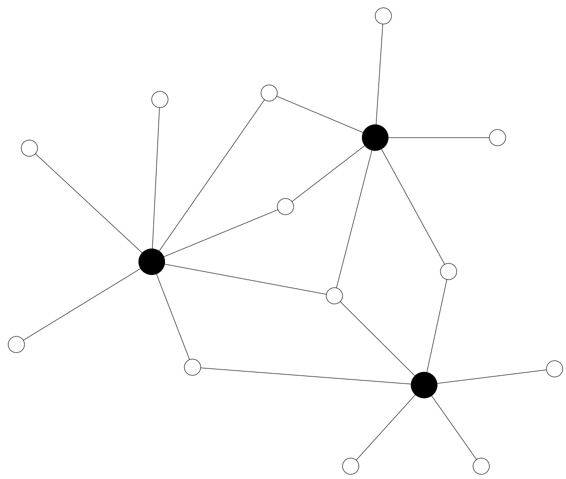


FIGURE 13 GRAPH TRANSFORMATION FOR CPMP

Using this transformation of the NTD problem, the equivalent CPMP problem will be constructed as follows:

- the set of points N for the CPMP problem will be identical to the nodes of G^*:

- $N = V^*$

- distance between nodes of N should be equal to the length of edges between them in G^* if that edge exists, and ∞ otherwise:
  - $d(i,j) = \begin{cases} l^*(i,j) & if\ (i,j)\epsilon E^* \\ \infty & if (i,j)\epsilon E^* \end{cases}$
- The integer constant p, i.e. the amount of medians is calculated based on the DU capacities (K) and the population:
  - $p = |S|/K$
- capacity of the medians will be equal to the capacity of the DUs:
  - $C = K$

This construction leads to a CPMP problem with p medians, i.e. the minimally necessary amount of DUs in the NTD_{AON} problem, minimizing the distribution network fiber costs at the same time.

The transformation clearly works in polynomial time, and the resulting p-median problem has identical optimality features as the original NTD_{AON} problem.■

> ***Corollary 2a***: Since the k-median and the capacitated k-median problems are NP-complete [66], according to the Linear reduction, NTD_{AON} special case of the NTD problem remains NP-complete.

We note that if the assumption about DU costs dominating over link costs does not stand, we get a problem similar to that described for PON networks, which was also NP-hard.

3.1.3. Special case #3 (Digital Subscriber Line networks, DSL)

For the following two special cases, the Steiner-tree problem will be used as the *base problem*. A Steiner-tree seems to be similar to a spanning tree, however it does not have to span all nodes of the graph, just a subset of its nodes, the so-called terminal nodes. Unfortunately this minor difference changes everything: while the minimum cost spanning tree problem can be solved with a quick greedy algorithm, the Steiner-tree problem is NP-hard.

> ***Definition 3** (base problem):* Steiner-tree problem [57]
>
> A Steiner-tree problem instance is defined by a graph $G = (V, E)$, non-negative weights $w(e) \in \mathbb{Z}_0^+$ for each $e \in E$, and a subset of nodes $R \subseteq V$ called terminals. The optimization problem is to find a minimal cost subtree of Gincluding all terminal nodes in R.

The Steiner-tree problem will be used as the *base problem* to prove NP-hardness, with a special graph transformation: the original graph G of the NTD problem is transformed to a G^* graph, as shown on Figure 14. This transformed graph is a 2-level tree, having the central office (CO) as its root, the DU locations (Ω) in the middle level, and the demand points (S) as its leaves. All the DU locations are connected to the CO (with an equal C_{DU} link cost), but only those demand points are connected to a particular ω_i DU location, which are within reach of it, e.g. the distribution loop length does not exceed its limit: $d\left(\omega_i, s_j\right) \le L_{max}^{dist}$.

This graph transformation could be made in polynomial time, and it maintains optimality and validity: the same DU selection and demand point-DU assignment is the optimal result for both the original and the transformed problems, since the optimization objective has the same value (DU costs) and the distribution loop length constraints are fulfilled, through the construction of G^*. What follows is that solution of the NTD problem in G^* and in the original graph is equivalent.

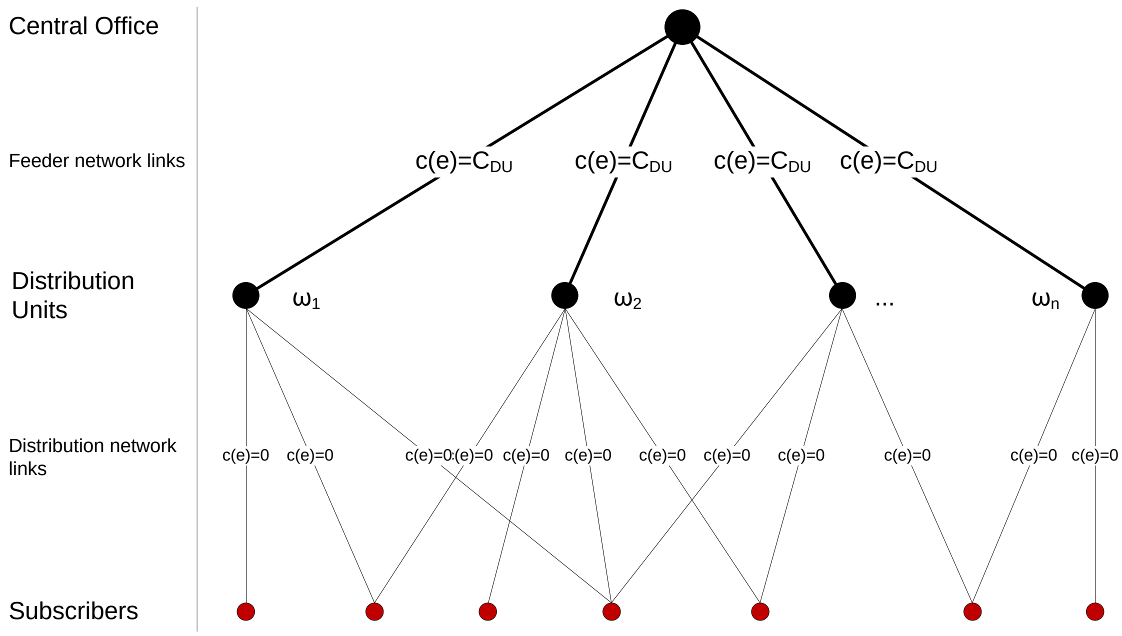


FIGURE 14 STEINER TREE TRANSFORMATION OF DSL NETWORKS

An optimal solution of the NTD problem in G^* requires a minimal set of DUs connecting all the demand points – which is on the other hand a Steiner-tree problem, considering the demand points and the CO as terminal nodes. Adding a DU to the topology means adding its upper level edge to the Steiner-tree with cost C_{DU}, therefore it leads to the same cost increase in both problems. Therefore, the minimal cost Steiner-tree is equivalent to the minimal set of DUs.

Lemma 3: The NGA Topology Design (NTD) problem in its special case for DSL networks, ie. when loop length (L_{max}^{dist}) and DU capacity constraints are kept, and primarily the DU costs (C_{DU}) are considered for minimization, and the Steiner-tree problems are equivalent under linear reductions (L-reductions).

Proof 3: A polynomial time reduction scheme is given for both directions.

NTD_{DSL} → *Steiner-tree*: The above described graph transformation is necessary for the linear reduction. Optimum of the Steiner-tree problem in the transformed graph is identical to that of the original NTD problem; therefore an "oracle" capable to solve the Steiner-tree problem also solves the NTD problem special case for DSL networks.

Steiner-tree → NTD_{DSL}: An NTD_{DSL} problem instance will be constructed, based on the given Steiner-tree problem. The construction will lead to a "degenerate" NTD problem, in which the optimization is restricted to the feeder network: every DU location will have exactly one demand point assigned. It will be achieved by a sufficiently short distribution loop length constraint. The resulting NTD problem will be equivalent to a Steiner-tree problem, with the CO and the DU locations as its terminals, i.e. the Steiner-tree will be feeder network itself.

The NTD_{DSL} problem is constructed in the following steps:

- the graph $G = (V, E)$ of the NTD problem will be identical to the graph of the Steiner-tree problem
- the Central Office will be at an arbitrarily chosen terminal node of the Steiner-tree problem

- the set of available DU locations in the NTD problem will be identical to the terminal nodes of the Steiner-tree problem (except the terminal node which became the CO):
  - $\Omega \cup \{CO\} = R$
- NTD installation costs will be equal to the edge weights of the Steiner-tree problem, while fiber costs will be set to zero:
  - $C_0 = w(e)$
  - $C_v \equiv 0$
- the distribution loop length constraints will be set to a sufficiently small distance, which separates the "zone" of every DU location, i.e. it will be shorter than the distance of the closest pair of DU locations:
  - $L_{dist}^{max} < min_{i \neq j}\left(dist\left(s_i, \omega_j\right)\right)$
- a set of "artificial" demand points will be added in a way that there is only one demand point within L_{dist}^{max} distance from any DU location, and every demand point has only one DU location within L_{dist}^{max} (i.e. it immediately defines a DU – demand point assignment):
  - $\forall i \leq |\Omega|: \ |\{s_i | dist(s_i, \omega_i) < L_{dist}^{max}\}| = 1$

In the optimal topology for this specially constructed the NTD_{DSL} problem, every DU locations (i.e. terminals of the Steiner-tree problem) will have exactly one demand point assigned. The feeder network of the NTD problem will be a minimum cost tree, which connects the CO and the DUs (i.e. terminals of the Steiner-tree problem), minimizing the installation costs C_0, which is identical to the $w(e)$ edge weights of the Steiner-tree problem. Therefore, the feeder network itself gives the optimal solution of the Steiner-tree problem.

Provided that the graph transformations are made in polynomial time, and it maintains optimality and validity of the solution, the two problems are equivalent under linear-reductions. ■

Corollary 3a: Since the Steiner-tree problem is known to be NP-complete [58], according to the Linear reduction, this special case of the NTD problem is also NP-complete.

3.1.4. Special case #4 (Point to Point fiber access networks, P2P)

The NTD_{P2P} problem is more directly related to the Steiner-tree problem than the earlier discussed NTD_{DSL} problem: in a point-to-point access network, several nodes (the demand points) have to be connected to the CO. If we take the CO and the demand points as *terminal* nodes of the graph, we see the relationship with the Steiner-tree problem. Therefore, the Steiner-tree problem will be used as the *base problem* to prove NP-hardness of the NTD_{P2P} problem, but now it requires graph transformation different from the one used earlier for the NTD_{DSL} problem.

Linear reduction

Lemma 4: The NGA Topology Design (NTD) problem in its special case for P2P networks, i.e. when the cable deployment cost dominates all other costs, and the Steiner tree problems are equivalent under linear reductions (L-reductions).

Proof 4: A polynomial time reduction scheme is given here for both directions.

Steiner-tree $\rightarrow NTD_{P2P}$*:* Given a Steiner-tree problem instance, an NTD_{P2P} problem will be constructed, which has the same optimum. The construction follows the above described relationship between the two problems:

- the graph $G = (V, E)$ of the NTDproblem will be identical to the graph of the Steiner-tree problem, i.e. it has the same set of nodes and edges
- the Central Office (CO) of the NTD problem will be located an arbitrarily chosen node which was a terminal node (r_i) in the Steiner-tree problem:
  - $CO \equiv r_i$,
- the set of demand points in the NTD problem will be identical to the set of terminal nodes in the Steiner-tree problem:
  - $S = R$,
- the edge cost values of the NTD problem will be assigned as follows:
  - $C_0(e) = w(e)$, i.e. the deployment costs are equal to the edge costs of the Steiner-tree problem
  - $C_v(e) \equiv 0 \ \forall e \in E$, i.e. the fiber costs are negligible
- finally, the distance limitation are relaxed, i.e. set to infinity:
  - $L_{max} = \infty$.

This NTD problem instance is a (simplified) NTD_{P2P} problem, which is equivalent to the original Steiner-tree problem. Therefore its optimal solution gives a solution of the original Steiner-tree problem at the same time. The construction has polynomial complexity.

$NTD_{P2P} \rightarrow$ *Steiner-tree:* the construction is simple and straightforward. The two graphs will be the same, the Steiner-tree will be "spanning" a subgraph of the CO and the demand points. Formally:

- graph G of the Steiner-tree should be identical to the graph $G = (V, E)$ of the NTD problem
- terminal nodes of the Steiner-tree will be the nodes representing the Central Office (CO) and the demand points (S) in the original NTD problem:
  - $R = S \cup \{CO\}$
- edge weights of the Steiner-tree will be equal to cable deployment costs of the NTD problem:
  - $w(e) = C_0(e)$

The transformation has linear complexity, and maintains optimality features. Moreover, the optimal Steiner-tree itself is identical to the optimal P2P topology, with minimal deployment cost for the original NTD_{P2P} problem.■

Corollary 4a: Since the Steiner-tree problem is known to be NP-complete, even for planar or bipartite $(R, V \backslash R)$ graphs, or with equal edge weights [58], the reduction makes this special case of the NTD problem also NP-complete, even with these more restricted scenarios.

Remark: The feeder network segment of AON and DSL networks may be interpreted as a P2P network: the DUs are connected directly to the CO with dedicated optical fibers.

3.1.5. NTD PROBLEM IN GENERAL

In the previous four subsections, several special cases of the NTD problem were discussed. These were based on the concept of narrowing the set of loose constraints or excluding parts of the objective (cost function). The described special cases lead to "almost valid", "almost optimal" solutions for a subset of NTD problems, with special input settings given in section 2.5.

However, the problem even in its simplified forms remains NP-complete.

Lemma 5: What follows is that the general case of the problem is clearly NP-complete, since any of its relaxations has been proven to be NP-complete.

Proof 5: A general NTD problem, without any restrictions on input values, is composed of the two above discussed optimization problems: For the feeder network part, a Steiner-tree problem has to be solved, for the distribution network part, a capacitated facility location problem is given. Both are NP-complete, what makes the NTD problem also NP-complete. ■

3.2. APPROXIMABILITY STUDIES

Even though the NTD problem and its special cases were proven to be NP-hard, our goal is still to find a solution for them, even for large scale scenarios. When facing such highly complex problems, two fundamental approaches are considered:

- *Relaxation*, when a subset of the constraints is relaxed, eliminated, and the problem is optimized in a wider solution space. The solution may be suboptimal or even invalid for the original problem, but at least it is achieved in a reasonable (typically polynomial) time.
- *Approximation*, in contrary leads to valid solutions. In this case the constraints are kept, but exact optimality is not forced, an approximation is required instead. The running time (complexity) has to be decreased at the expense of losing optimality.

The fundamental requirement of practical applicability does not allow oversimplification. The special cases themselves are acceptable relaxations. Unfortunately, as it was proven, that extent of relaxation still does not allow exact optimization in polynomial time: the special cases are still NP-hard. However, the realistic approach would be lost with a higher level of relaxation, which is contradictory with our fundamental goals.

These altogether imply that a combination of these complexity reduction principles is in demand: the special cases as *relaxations* have to be solved by *approximation* algorithms (heuristics). Such a combination is necessary for the highest quality solution achieved in a reasonable time.

What follows in this subsection is the **approximability analysis** of these special cases.

We will use the term **theoretic bounds** for lower bounds on the best approximation factors obtainable in polynomial time. Typically indirect proofs are known for these, stating "no better approximation factor exists, unless $P = NP$".

The best known approximation algorithms for an equivalent or analogous mathematical problem will be treated as **practical bounds**. These show what is surely possible, since these related algorithms can be interpreted as constructive proofs for some approximation ratio that is obtainable.

It is impossible to construct any better heuristic algorithm than the theoretic bounds, and not reasonable to expect better than the practical bounds.

The presented approximability results are typically corollaries of the linear reductions presented and proven in section 3.1 about complexity analysis, therefore the numbering is consequent to that section.

Beyond the well-known family of NP-hard problems, there are other important approximability classes. APX-hard problems can be approximated within factor of some $1 + \varepsilon$ but not for every $\varepsilon > 0$. There are problems for which a polynomial time approximation scheme (PTAS) exists, which means for every $\varepsilon > 0$ a polynomial time algorithm can be constructed approximating optimum with the factor of $1 + \varepsilon$. For these, at least a constant factor approximation exists (even if the approximation factor may be high), which is not true e.g. for the NP-hard travelling salesman problem.

3.2.1. Special case #1 (Passive Optical Networks, PON)

This special case was proven to be equivalent under linear reductions with the Capacitated Facility Location (CFL) problem (***Lemma 1***).

> ***Corollary 1b***: The linear reduction maintains validity of the approximability results of the facility location problem for the NTD_{PON} problem.

The facility location problem is a well-known mathematical problem, Guha et al. prove [67] that it is NP-hard to approximate the facility location problem with a better constant factor than 1.463, which gives the *theoretic lower bound* also for the NTD_{PON} problem approximation.

The best known approximation of the capacitated facility location problem is from Mahdian et al. providing 2-approximation with a constructive proof, which sets the *practical lower bound* [63].

3.2.2. Special case #2 (Active Optical Networks, AON)

This special case was proven to be equivalent under linear reductions with the capacitated p-median problem (***Lemma 2***).

> ***Corollary 2b***: The linear reduction maintains validity of the approximability results of the k-median problem for the NTD_{AON} problem.

Meyerson et al. [66] prove that it is NP-hard to approximate the k-median problem with better approximation factor than $1 + \frac{2}{e} \approx 1.736$, which gives the *theoretic lower bound* also for the FTD problem special case.

The best known approximation algorithm for the k-median problem however has significantly higher approximation ratio: Arya et al. [65] give a constructive proof for $3 + 2/p$ constant factor approximation, where the value of p is a variable describing complexity of an interior iterative step of the algorithm. It makes the approximation factor scalable, but not lower than 3.

3.2.3. Special case #3 (Digital Subscriber Line networks, DSL)

Partial equivalence under linear reduction with the Steiner-tree problem was proven (***Lemma 3***).

> ***Corollary 3b:*** Through a graph transformation step, equivalence of the NTD_{DSL} and the Steiner-tree problem was proven, therefore not only their complexity, but also approximability features are identical.

The Steiner-tree problem is known to be APX-hard, i.e. a constant factor approximation algorithm exists for it, but not for every $\varepsilon > 0$, or otherwise stated, for a sufficiently small $\varepsilon > 0$, it cannot be approximated within a factor of $1 + \varepsilon$ [57]. Value of the minimal ε is not known, but the best proven value is 0.0062, i.e. in [58] Martin Thimm has proven that it is not possible to approximate the Steiner-tree problem with a constant factor lower than 1.0062 unless $P = NP$. Therefore this value will be handled as *theoretic lower bound*.

On the other hand, after a series of works, the best known approximation ratio (*practical lower bound*) has been reduced to 1.55 by Zelikovsky [59].

3.2.4. Special case #4 (Point to Point fiber access networks, P2P)

This special case was proven to be equivalent under linear reductions with the Steiner-tree problem (***Lemma 4***), therefore it has similar complexity.

Corollary 4b: The linear reduction maintains validity of the approximability results of the Steiner-tree problem for the NTD_{P2P} problem.

Based on published results for the Steiner-tree problem, the theoretic lower bound is 1.0062 [58], while the practical lower bound 1.55 [59] for approximation of the NTD_{P2P} problem.

3.2.1. NTD PROBLEM IN GENERAL

Investigating the NTD_{PON} and NTD_{P2P} special cases reveals the approximability of the NTD problem in general:

The NTD_{PON} special case (section 2.5.1) was describing passive optical networks, by eliminating $C_0(e)$ cost values ($C_0(e) \equiv 0$), while $\mathrm{NTD_{P2P}}$ special case (section 2.5.4), for dedicated optical fiber was minimizing exactly those $C_0(e)$ costs, regardless of other cost factors. Since the NTD_{PON}, NTD_{P2p} (and NTD) problems are solved over the same solution space (constraint set), the NTD problem not only contains them as special cases, but its objective is exactly the sum of these two special cases.

What follows is that the lowest best possible constant approximation factor for the general NTD problem cannot be lower than the maximum of the weighted average of approximation factors for these two partial problems, since a constant α-factor approximation for such a composition of the problems should lead to an α-factor approximation for any of its composers as well.

The *practical lower bound* on the approximation factor (2 and 1.55 for the capacitated facility location and Steiner-tree problems, respectively):

$$\max_{\beta}\{\beta \cdot 1.55 + (1-\beta) \cdot 2\} = 2$$

Regarding the *theoretical lower bound* (1.463 and 1.0062 for the facility location and Steiner-tree problems, respectively):

$$\max_{\beta}\{\beta \cdot 1.0062 + (1-\beta) \cdot 1.463\} = 1.463$$

Corollary 5b: The inherited *practical lower bound* for constant factor approximation of the general NTD problem is 2, and the *theoretical lower bound* is 1.463, i.e. it is not possible to approximate the NTD problem within a factor of 1.463 unless $P = NP$, however the best known heuristic algorithm for the underlying problems provides 2-approximation (2-OPT).

3.3. CONCLUSION

Formulation of the NTD problem, definition of its important special cases, and the exhaustive complexity and approximability analysis completes the theoretic background of the problem addressed in my study.

The following table concludes the results achieved regarding the theoretical background of the NGA Topology Design problem. Complexity and approximability analysis were carried out not only for the general form of the problem, but also for its special cases, motivated and defined by current and future NGA technology and architecture classification.

The proven complexity, i.e. NP-completeness of problems shows necessity for heuristic approximation of the optimum. The approximability results show possibilities and limits of guaranteed approximation: no better approximation can be carried out than the *theoretic lower bound*, and to our knowledge no better approximation algorithm was published over the last decades for the underlying mathematical problems than the *practical lower bound*.

ALGORITHMIC ANALYSIS SUMMARY	NTD in general	NTD_{PON} special case	NTD_{AON} special case	NTD_{DSL} special case	NTD_{P2P} special case
Linear reduction	-	Capacitated facility location problem	Capacited p-median problem	Steiner-tree problem	Steiner-tree problem
Complexity	NP	NP	NP	NP	NP
Approximability (theoretic)	1.463	1.463	1+2/e≈1,736	1.0062	1.0062
Approximability (practical)	2	2	3+2/p > 3	1.55	1.55

TABLE 3 SUMMARY ON THEORETICAL BACKGROUND

4. CRITICALITY: A GRAPH THEORETIC APPROACH

The NGA Topology Design Problem, as it was outlined in Section 0, was translated to the language of graph theory. In this section, we will review the problem from a graph theoretic point of view. Some features of the modeling graph itself may bring us closer to either the optimum or at least upper and lower bounds. In order to find the relevant features, we have to take a look at the constraints and cost factors of the NTD problem, and make an effort to describe what effect these may have on the optimal solution.

Here we focus on the graph, therefore the solution space narrowing effect of constraints, especially the range (length) constraints are important and discussed in this section. Namely the total network range, plus the feeder and distribution network segment limitations will be investigated.

Total network range has an obvious and substantive consequence: any demand point further than $\mathrm{L}_{\max}$ from the CO will remain unconnected. The feeder network range also has a relatively straightforward effect: it constraints the DUs to be close to the CO; in the extreme case, in the CO itself, which degenerates the point-multipoint network to a point-to-point architecture.

The distribution network range limitations have the most complicated effect on valid topologies, i.e. the solution space. These have the most interesting interaction with the graph itself, since short distribution network segments emphasize local features of the graph. Low L_{max}^{dist} values have key importance regarding the clustering subproblem: the topology design problem tends to find coverage of the service area with low diameter groups, and in this case, a properly chosen set of DU locations is fundamental. Several critical nodes of the graph may be identified that determine the location of DUs.

Identification of those critical nodes, the notion of criticality and its applications is the subject of the remainder of this section.

4.1. DEFINITION OF CRITICALITY METRIC AND ORDERING

Some demand points in the graph may have a special position, especially with short distribution network segment constraints. Outlier nodes are the most obvious examples of nodes with significant effect on topology: these may require a dedicated DU in their outlier position. On the other hand, nodes in densely populated, central areas typically do not affect the topology seriously: a DU position may be chosen from a wide set of available DU locations to connect them.

These examples are trivial for the human eye, but need some formal explanation for an algorithm. Such formal representation also helps deeper investigation, beyond trivial observations. In order to identify the necessary DU locations for complete coverage of demand points, I have introduced a metric to measure such significance of a node in the graph, namely *node criticality*.

Definition 1: Node criticality

Measures the amount of DU locations that can serve the demand point, i.e. from which the node is within L_{dist}^{max} (distribution network reach). Formally, if DU_i denotes the i^{th} element within the set of available DU locations, s_j denotes the j^{th} demand point, $d(x, y)$ is the distance of x and y, L_{dist}^{max} is the maximum DU-demand point distance:

$$crit(s_j) = \left|\{DU_i | d(DU_i, s_j) < L_{dist}^{max}\}\right|$$

Simply said, this metric stands for the amount of DU locations, of which at least one must have a DU in the final topology to connect the demand point. The lower this value is, the more critical a demand point is. On Figure 15, the bold numbers top-right from the nodes indicates their criticality value. Clearly, nodes with criticality value of 1 determine that a DU has to be located in their corresponding DU location.

Definition 2: Criticality code

For every node, the DUs within distance L_{dist}^{max} are enumerated, in ascending order of their IDs. Such a list is referred to as the *criticality code* of the node, enlisting the available DU locations for covering the node:

$$criticality\_code(s_j) = \#ASC\{DU_i | d(DU_i, s_j) < L_{dist}^{max}\}$$

On Figure 15, small numbers within parentheses, e.g. {1,2,3} indicate the *criticality code* of the node.

Definition 3: Component

A group of demand points that are reachable from exactly the same set of DU locations, i.e. a group of (neighboring) nodes having the same criticality code form a component.

On Figure 15, such components are indicated with dashed lines, and these components are substituted by single dots on the following schematic figure (Figure 16). Such simplified form of the graph will be used in the following sections, since a group of nodes having the same features regarding criticality will be handled together.

Definition 4: Criticality list

DUs have a criticality list. Criticality values of the demand points within reach of a given DU, in ascending order serve as the *criticality list* of the DU.

$$List(DU_i) = \#ASC\{crit(s_j) | d(DU_i, s_j) < L_{dist}^{max}\}$$

The underlined numbers within parentheses, under the DU locations on the figure give an example that helps better understanding of the criticality lists.

We also define an ordering relation on DU locations with the help of a lexicographical ordering on their criticality lists.

Definition 5: Criticality ordering

This relation is defined on DUs, based on their criticality lists. By notation: $DU_i \leq_c DU_j$, if the criticality list of DU_i precedes that of DU_j in a lexicographical ordering. In the graph it means DU_i has not less nodes with lower criticality value. The criticality relation reflects an ordering driven by importance or necessity of DU locations.

In the given example, the DU locations have the following criticality values:

- DU_1: (1,2,2,2,2,3)
- DU_2: (1,1,1,2,2,2,3)
- DU_3: (1,1,2,2,2,3)

This implies the ordering $DU_3 \leq_c DU_2 \leq_c DU_1$.

Lemma: The criticality relation is an ordering on the set of DUs (Ω).

Proof: Criticality relation is defined as a lexicographic *ordering*. ∎

Corollary: The criticality ordering has the following features:

- $DU_i \leq_c DU_i$ (reflexive)
- $DU_i \leq DU_j \wedge DU_j \leq DU_i \Rightarrow DU_i = DU_j$ (antisymmetric)
- $DU_i \leq DU_j \wedge DU_j \leq DU_k \Rightarrow DU_i \leq DU_k$ (transitive)
- $\forall i, j$: $DU_i \leq DU_j \vee DU_j \leq DU_i$ (total)

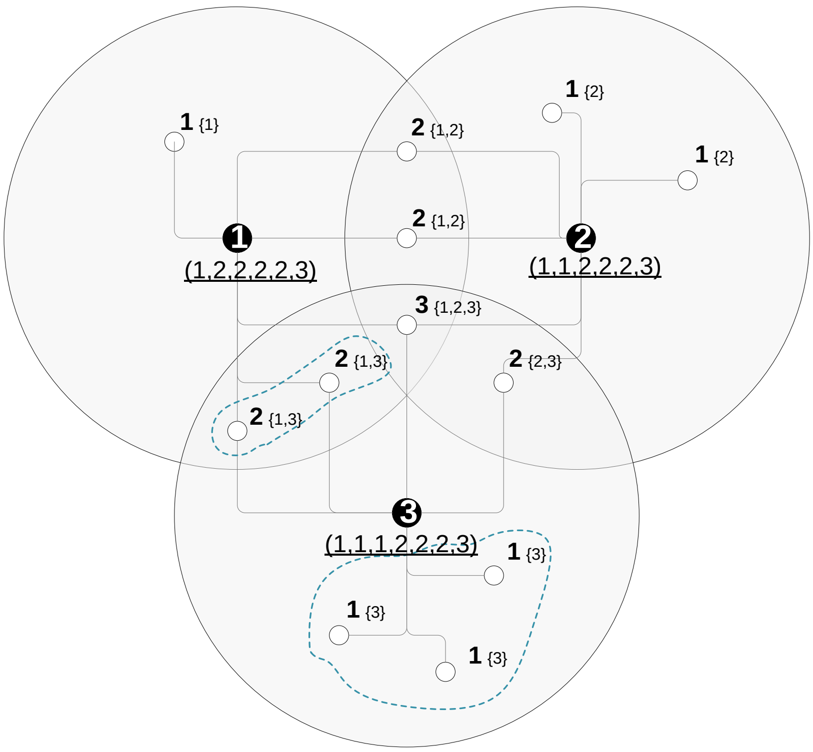


FIGURE 15 EXAMPLE GRAPH WITH CRITICALITY VALUES AND COMPONENTS

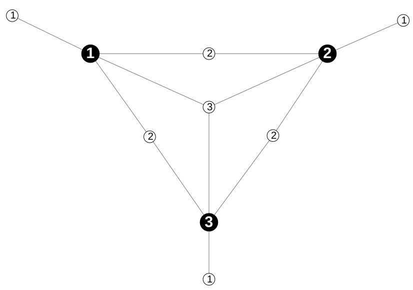


FIGURE 16 SIMPLIFIED GRAPH WITH COMPONENTS

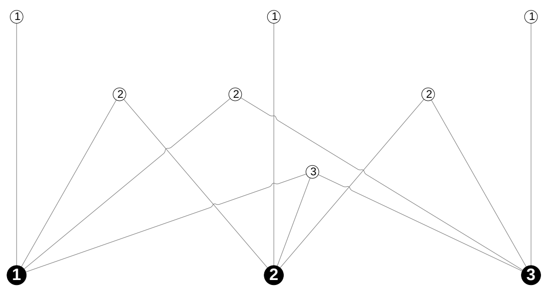


FIGURE 17 BIPARTITE VIEW OF THE SIMPLIFIED GRAPH

4.2. Matching and criticality: an upper bound on DU locations

Based on the notion of *components*, a simplifying graph transformation will be made: every set of nodes having identical *criticality codes* will be substituted by a single node (according to the definition of criticality code, these nodes are connected to the same DUs). The transformation is shown on the simplified graph above (Figure 16), and re-drawn on Figure 17 to highlight the bipartite property of the simplified graph: the components are connected only to DU locations, but no connections exist within the subset of components or DUs. The upper (red) dots represent the components (groups of demand points with same DU connectivity), and the numbers in the dots stand for their criticality value. The black dots represent the DU locations.

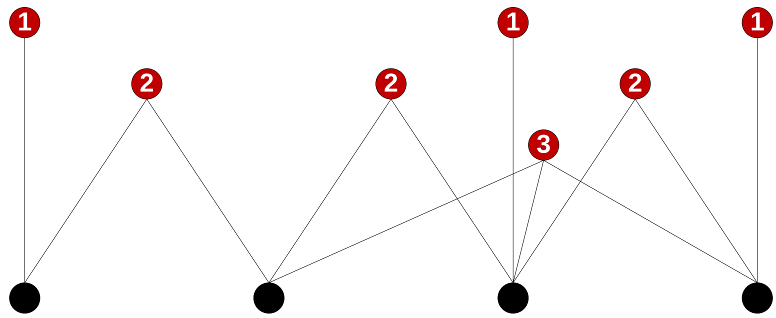


FIGURE 18 BIPARTITE SAMPLE GRAPH

Considering the original topology design problem, a valid solution is a set of DUs "covering" all the components. A DU location will be called "active" if a DU is located there in the access network topology.

Definition 1: In a graph $G = (V, E)$ a *matching* M is a set of pairwise non-adjacent edges, i.e. none of the edges in M share a single common vertex.

Definition 2: a *maximal matching* M is a matching that cannot be extended by adding any more edges to M, since any additional edges would violate its matching property.

Lemma 1: Cardinality of any M maximal matching is an upper bound on the minimal amount of necessary active DU locations to cover all components, i.e. $|M| \geq |DU|_{min}$.

Proof 1: (Indirect)

Any M matching can be translated to a set of active DU locations, by making active all DU locations which are adjacent to any edge of M. This set of active DU locations covers all components in M.

Suppose that there exists a component c_j, which remains uncovered by the above described DU location set, i.e. none of its neighboring DU locations (e.g. DU_i) are adjacent to the edges of M. In the bipartite graph, all edges adjacent to c_j are connected to one of its neighboring DU locations, i.e. M could be extended with an edge (DU_i, c_j), since none of its terminating nodes have adjacent edges in M. Which in itself is contradictory to the initial assumption about M being a maximal matching – and it proves the fact that the translated active DU location set covers all components.

On the other hand, this DU set has exactly $|M|$ elements, since in the bipartite graph every edge is adjacent to one DU location (and one component). These altogether imply that the minimal amount of necessary DU locations is not more than $|M|$.■

We also note that the inequality is tight, e.g. in the following example, a maximal size matching contains 2 edges (A1, B2), while a single active DU location (B) may cover all the components:

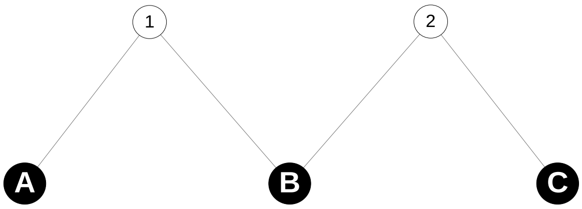


FIGURE 19 TIGHT INEQUALITY EXAMPLE

This first lemma ensures that the amount of necessary active DU locations is bounded from above – by the size of any maximal matching. The next necessary step is to find the strongest upper bound, i.e. the minimal size of a maximal matching.

Lemma 2: Finding the minimal size maximal matching is NP-complete.

Proof 2: This was proven by M. Yannakakis and F. Gavril in [70]■

It unfortunately blocks our attempts to decrease the upper bound and tighten the limits, therefore the best upper bound on the amount of necessary active DU locations is achieved by the size of an arbitrary maximal matching.

This fact also raises a question about the deviation in size of maximal matchings: may we accidentally get an arbitrary maximal matching which is 100 times bigger than the minimal size? It would significantly weaken the upper bound. The next lemma shows that an arbitrary maximal matching is a 2-factor approximation of the minimum size maximal matching (in any graphs):

Lemma 3: The size of an arbitrary maximal matching gives a 2-approximation on the size of the minimum size maximal matching (proven by Z. Gotthilf et al., [69]). ■

Corollary 3: Finding an arbitrary maximal matching leads to a valuable upper bound on the minimum size maximal matching, i.e. also on the amount of necessary DUs.

This last implication shows the reason to deal with matching: it can be applied to the NTD problem. A fast greedy algorithm will be proposed for calculating an upper bound on the necessary amount of DUs, based on the notion of node criticality measure:

Algorithm 1 Upper bound approximation for active DU locations:

- *Step 1*: Create the bipartite graph G of components and DU locations as described above.
- *Step 2*: Select the components with criticality value of 1, and connect them to their single DU; add these DUs to the solution. These undoubtedly are necessary for full coverage of components.
- *Step 3*: Let $i = 2$, and then take the *unconnected* components with criticality value of i. Assign them to an arbitrary adjacent DU location until all of them are covered. If there are any more unconnected components, i++ and *goto Step 3*. Otherwise *goto Step 4.*
- *Step 4*: The selected subset of DU locations is sufficient for connecting all the components, a valid solution is found. STOP.

Lemma 4: Complexity of Algorithm 1 is $O(V^2 \cdot \log(V))$

Proof 4: Individual steps of the algorithm are analyzed regarding their complexity

The first, initial step for creating the bipartite graph is composed of three elements:

- Criticality values are set in $O(V^2 \cdot \log(V))$ steps: the Bellman-Ford algorithm is applied to calculate the distance between any pair of demand points and DUs, and it is compared to L_{max}). The ordered lists of DUs at every node (i.e. the criticality code) are made in $O(V^2 \cdot \log(V))$ steps, since at every node we have at most $O(V)$ DUs within reach.
- Components are created in $O(V^2)$ steps, comparing all nodes pairwise, and contracting the ones with the same criticality code.
- Once the components and DU locations exist, edges of the bipartite graph are added, based on the adjacency information in the component criticality codes, with at most $O(V^2)$ steps (since we have at most $O(V)$ components and DU locations)

These altogether result in $O(V^2 \cdot \log(V))$ complexity for Step 1. Step 2 has a complexity equal to the amount of components: $O(V)$. Step 3 is executed at most $O(V)$ times, since the amount of DU locations bound criticality (therefore i is not increased further), and each step includes at most $O(V)$ assignments: $O(V^2)$ steps for all executions. If we sum up these, we get $O(V^2 \cdot \log(V))$ complexity for Algorithm 1.■

Summarizing this subsection: we have a polynomial time greedy algorithm for computing an upper bound on the amount of necessary DU locations for connecting all demand points, subject to distribution loop length constraints.

4.3. LOWER BOUNDS OF DU SET

Finding a lower bound on the amount of minimal necessary DU locations for connecting all demand points, with respect to distribution loop length constraints could also be a valuable addition.

An upper bound may be used as an obvious measure of quality (if a solution is provided with more DU locations, it definitely may be improved). On the other hand, a lower bound is more difficult to interpret: a solution far from the achieved lower bound still may be of high quality if the lower bound is not tight. However, any solution close to a known lower bound is immediately proven to be a high quality result.

Accordingly, in this section an attempt is made for a theoretic lower bound computation, and in a following chapter we will present a linear programming based lower bound calculation method, which is more of practical interest, and also shows the real power of the proposed heuristics, having a relatively small gap between the achieved results and the known lower bound.

At first, a trivial lower bound is described, based on the notion of components and criticality:

Lemma 5: The amount of components with criticality value of 1 ($n_{c=1}$) makes a lower bound on the amount of necessary DUs (min_{DUs}).

Proof 5: A set of demand points forming a component with criticality value of 1, have to be connected to only one DU location (all of them to the same). That requires that DU location to be active. However, no other components with criticality value of 1 can be connected to the same DU location, otherwise nodes of these two components would have the same criticality code, and the two components should be the same. ■

Similarly, a lower bound is available based on the amount of components with criticality value of 2 ($n_{c=2}$):

Lemma 6: The amount of components with criticality value of 2 ($n_{c=2}$) provides another lower bound on the amount of necessary DUs (min_{DUs}):

$$min_{DUs} \geq \frac{1 + \sqrt{1 + 8 \cdot n_{c=2}}}{2}$$

Proof 6: Supposed we are given ω different DU locations, the maximal amount of different components (with different criticality codes!) is equal to $\binom{\omega}{2}$, since no more mutually different pairs of DU locations exist. Therefore:

$$\binom{\omega}{2} \geq n_{c=2}$$

$$\omega \cdot (\omega - 1) \geq 2 \cdot n_{c=2}$$

$$\omega^2 - \omega - 2 \cdot n_{c=2} \geq 0$$

And then, by solving the quadratic inequality, we get:

$\omega \geq \frac{1+\sqrt{1+8 \cdot n_{c=2}}}{2}$ or $\omega \leq \frac{1-\sqrt{1+8 \cdot n_{c=2}}}{2}$

Considering non-negativity of ω, we get $\omega \geq \frac{1+\sqrt{1+8 \cdot n_{c=2}}}{2}$.■

The computed minimal DU locations for different $n_{c=2}$ values are as follows (ω rounded up to the closest integer):

$n_{c=2} = 1$	$\omega \geq 2$
$n_{c=2} = 2$	$\omega \geq 3$
$n_{c=2} = 3$	$\omega \geq 3$
$n_{c=2} = 4$	$\omega \geq 4$
$n_{c=2} = 5$	$\omega \geq 4$
$n_{c=2} = 6$	$\omega \geq 4$
$n_{c=2} = 7$	$\omega \geq 5$
$n_{c=2} = 8$	$\omega \geq 5$
…	…

According to the fact, that typically the $n_{c=2}$ values do not exceed significantly (if at all) the $n_{c=1}$ values (components with criticality 1), lower bound is typically tighter with $\omega \geq n_{c=1}$, except for small networks and just a few components. Similarly, additional lower bounds are available based on components with 3, 4, … criticality, but for the same reason, these have low practical value. Increasing criticality values and bounds is more about number theory than the graph itself.

5. SPECIALIZED, HIGHLY EFFICIENT HEURISTICS

Beyond clarifying the theoretical background of algorithmic topology design for NGA networks, another fundamental goal of the research was to propose applicable solutions: algorithms that solve not only relatively small sample problem instances under “lab-conditions”, but also realistic, large-scale scenarios, which have practical interest.

Obviously the theoretical studies were necessary before developing algorithms, in order to decide whether the problem can be solved in an algorithmic way at all. The preceding sections support the observation that the problem is NP-complete, exact optimization expectedly cannot be carried out in polynomial time; moreover really tight guaranteed constant-factor approximation is beyond possibilities. The algorithmic analysis lead to reasonable requirements regarding time complexity (scalability) and accuracy of the algorithms.

According to the introductory section, effective (likely polynomial) algorithms are required for sufficient scalability: an NTD problem instance solved at once is typically the service area of a single CO, which is in the magnitude of 1.000s to 10.000s of demand points. On the other hand, topology design is an offline problem, therefore several hours or even days of computation are tolerable. As we have seen, exact optimization has exponential complexity, which excludes practical applications, even with these permissive requirements.

Regarding approximation accuracy of heuristic methods, practical applications raise the bar: deployment of NGA networks requires very high investment; cost-minimization planning and accurate cost estimation is of high importance. Therefore a high extent of suboptimality is not acceptable: even the theoretically existent constant factor approximations with 46% or 73% gap are too loose, if we consider the magnitude of investment at these network deployments. However, by terms of guaranteed approximation, these cannot be outperformed.

Hence, for practical reasons, the approximability requirements will not be set for the theoretic “worst case” scenario, but for the average expected value of suboptimality, tested against various real-world scenarios and case studies. Extent of the allowed optimality gap is difficult to define, and even more difficult to test/check, since the exact optimum is typically not known.

As a conclusion, the requirements we set up for the topology design heuristics:

- scalability: large-scale problems, even for graphs with 10.000+ nodes
- accuracy: suboptimality gap within 10-20% in average

In order to achieve such high performance approximation, highly specialized heuristic algorithms are necessary. According to the well-known *No Free Lunch* theorem [53], specialization allows effective approximation of well-defined problems – this was driving the efforts for identifying special cases, as presented in section 2.5. The heuristics should be adapted to characteristics of the problem, and typical input parameters of the various special cases as far as possible.

In this section, various heuristic algorithms are proposed for special cases of the NTD problem. As a common feature, all of the presented heuristics are based on a decomposition of the topology design problem, which is introduced in the first subsection. We note here, that this section focuses on the three presented point-multipoint technologies, namely PON, AON and DSL. The point-to-point networks were important for the theoretic studies, complexity and approximability analysis, but for topology design, the already published algorithms for solving the Steiner-tree problem may be used (e.g. the Distance Network Heuristic [56], or the algorithm of Zelikovsky [59]).

5.1. SUBPROBLEMS AND DECOMPOSITION

Complexity analysis not only led to the recognition of NP-completeness, but the efforts to identify the included NP-complete subproblems also highlighted the most critical points of the NGA Topology Design (NTD) problem itself.

On an abstract level, optimization for both the general case and special cases of the problem can be seen as a minimum cost, point-multipoint architecture, considering various constraints, connecting demand points to the central office through distribution units. During the design of such topology, three basic questions should be answered:

- In the point-multipoint structure, which demand points have to be connected to the same distribution unit? Or: which demand points should be in the same group (cluster), assigned to a common DU?
- Where should be the distribution units located?
- Which path should the connections follow between the demand points and their assigned DUs, and also between the DUs and the CO?

These *subproblems* will be referred to as *Group formulation*, *DU allocation* and *Connection establishment*, respectively.

In general, *group formulation* belongs to the class of clustering problems, with several specific attributes, e.g. the clustering is carried out on a graph and not in the Euclidean space, diameter of the clusters is bounded by network range constraints, and cardinality of the clusters is fixed due to DU capacity requirements. *DU allocation* is obviously a location problem, more specifically a capacitated location problem defined on a graph. *Connection establishment* belongs to minimum cost path and flow problems, complicated with a non-linear, stepwise cost function of parallel connections.

The NTD problem will be decomposed into these subproblems; however the strong cross-dependence of them has to be taken into consideration. Moreover, that cross-dependence is what makes the problem as complete as it is:

- *Group formulation* strongly affects DU allocation: once the group of demand points is defined, DU allocation reduces to the problem of finding a “centroid” of that group. Therefore a confuse clustering with scattered groups by itself makes “nice” DU allocation impossible, not to mention its effect on the connection establishment, which is intended to minimize the sum of demand point-DU distances.
- *DU allocation*, together with the DU-demand point assignment (i.e. *group formulation*) almost completely determines the optimal connection establishment in the distribution network segment. Hence the optimal distribution network is treated rather as the objective function for the DU allocation, than as an independent optimization step later.

Therefore these problems cannot be solved independently, one after another. In every distinct phase and subproblem, the further steps have to be reflected in the objective. Including later phases into the optimality conditions for any of the subproblems lies in the heart of the heuristics presented in this section.

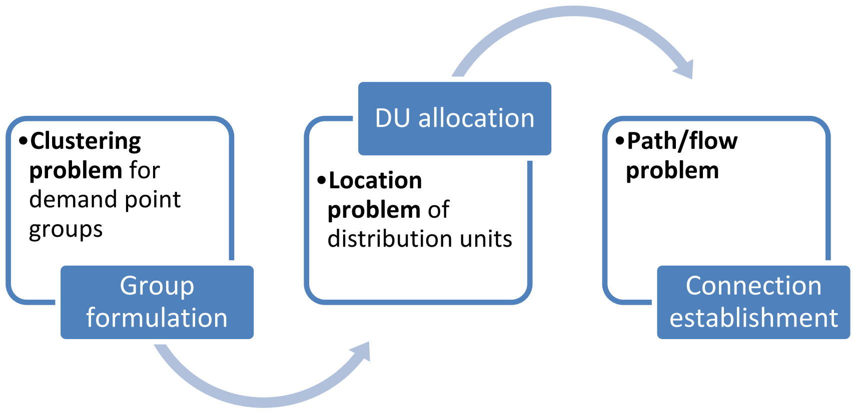


FIGURE 20 DECOMPOSITION OF THE NGA TOPOLOGY DESIGN PROBLEM

5.2. SPECIALIZATION

The optimization problem has its objective (cost function) that differentiates between a "good" and a "bad" solution; and its constraints that differentiate between valid and invalid solutions. The already discussed special cases have in general the same formulation, but the significance or typical values of these cost factors or constraints make the difference between them [37].

The most important difference between the specific heuristics is the different approach for the subproblems driven by these typical values. Therefore at first we briefly review the cost factors and constraints, and highlight the effect of different typical values.

COST FACTORS

DU costs **(**$\mathrm{C_{DU}}$**)**: high distribution unit costs require maximal DU utilization, therefore all the K capacity of them has to be fulfilled. Especially for large K values, it enforces large distribution unit segments. In contrary, DU costs substantially lower than cable plant costs result in simpler distribution unit segments, i.e. small star topologies around the DUs.

The subproblem of formulating groups of demand points around distribution units may be interpreted as a *graph clustering* problem, with a significant specialty: the cluster sizes are bounded from above by the K capacity value. The higher the DU costs are, the closer the cluster sizes should be to this K value. The problem of making almost equal-size segments makes a significant difference between traditional clustering problems, and the group formulation subproblem.

Deployment costs **(**$\mathrm{C_0}$**):** High deployment costs have a well noticeable effect on the optimal solution. Minimizing length of the traces above fiber lengths leads to a clear Steiner-tree problem, as it was discussed at the complexity studies.

Fiber costs **(**$\mathrm{C_v}$**):** Fiber costs, in contrary to deployment costs enforce topologies more like a shortest path tree instead of Steiner trees, where all nodes are connected to the CO along their shortest paths, even if it increases overall trace length.

Access network topologies typically have a tree structure, but these two cost factors make the difference between the two extremities, i.e. trace minimizing Steiner trees, or distance minimizing shortest path trees.

Constraints

DU capacity (K): The effect of DU capacity is not independent from the DU costs, but high DU cost makes the capacity constraints significant. High capacity DUs result in really wide groups and long distribution network segments, that may conflict with distance limitations (see below), and also increases cable plant costs.

On the other hand, low DU capacities lead to really small demand point groups. If the DU capacity is not a magnitude higher than the typical building size, an additional bin packing problem arises, since granularity issues make filling DUs difficult. As an extreme example, splitter with capacity for 64 demand points may be utilized only at around 50% with buildings having 33 demand points.

Network range (L_{max}): This constraint has a straightforward consequence on the graph. The network range is the diameter of the feasible network topology around the CO; any demand points outside this region are excluded from the valid solutions.

Feeder network range (L_{feed}^{max}): The shorter the feeder network segments are constrained, the closer the DUs are to the CO. As an extremity, point-multipoint topologies become point-to-point topologies with zero length feeder network segments, however assuming short feeder, and long distribution network range is not realistic.

Distribution network range (L_{dist}^{max}): As the most interesting and complicated element of the constraint set, the distribution network range, and its consequences were investigated earlier: Section **Hiba! A hivatkozási forrás nem található.** was dedicated to this topic, resulting in the notion of criticality, and a bouquet of valuable observations for short distribution network segment constraints. These are built in the heuristics for DSL networks.

5.3. Passive Optical Networks (PON)

In the case of passive optical networks, we have relatively permissive distance limitations: typically (far) more than K demand points are within L_{dist}^{max} distance from the DU locations. The DU capacity constraints are also not decisive: due to the reasonable price of passive equipment, capacity of a splitting point may be easily increased by adding DU equipment at the same location. Therefore none of the constraints has dominating effect on the solution space.

On the other hand, this special case has a complex objective function: both the DU and the distribution network costs are considered, resulting in a "duplex" function, where the weights assigned to these cost factors (α, β) determine optimal solutions:

$$min\{\alpha \cdot C_{cableplant} + \beta \cdot C_{equipment}\} \sim min\left\{\alpha \cdot \left\{\sum_{j \in S} dist[DU(s_j), s_j]\right\} + \beta \cdot N\right\}$$

The developed heuristic therefore aims at a joint optimization of distribution fiber and distribution unit costs, avoiding two extremities, namely (1) when the DU costs are minimized hence the DUs are completely filled, even if distant demand points are assigned to the DU, significantly increasing distribution network costs, and (2) when the distribution fiber lengths are minimized, even if it requires a distinct DU allocated for every single demand point.

The reason for using a point-multipoint architecture (e.g. PON networks) is the fiber saving achieved in the feeder network segment, where a single fiber carries the traffic from the K demand points connected to the DU. The proposed **Branch Contracting Algorithm (BCA)** in the group formulation phase aims at maximizing these savings. It is achieved by collecting “neighboring” demand points that share a large portion of their path towards the CO (Figure 21).

5.3.1. Description of the Branch Contracting Algorithm (BCA)

All of the proposed heuristic algorithms will be described with a step-by-step description and a flowchart. Moreover, a sequence of figures extends the description of the branch contracting algorithm for PON network, showing the concept of group formulation by path sharing maximization (starting with Figure 21).

Algorithm: Branch Contracting Algorithm (BCA)

- ***Step 1*** (*Initialization*): Construct a tree T in the graph based on the shortest paths between the demand points and the CO. It may be easily seen that this subgraph is a tree [60].

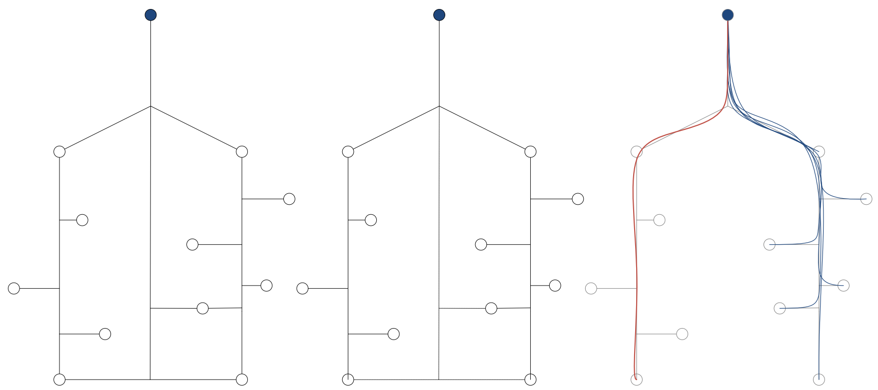


FIGURE 21 SHORTEST PATH TREE

- ***Step 2*** (*Group formulation*): U denotes the amount of not connected demand points in the tree. Start from the farthest demand point on this tree T from the CO, and move from node to node up the tree, towards the CO as its root. If the amount of demand points in the current subtree exceeds a predefined threshold Q, stop moving up. A new demand point group is formed of the demand points in the subtree, i.e. a branch is cut from the tree, contracting its nodes to a group (this step is reflected in the name of the heuristic). Remove these nodes from the set of unconnected demand points (U decreased). The threshold Q is typically a function of DU capacity K, controlling the DU utilization rate. Repeat Step 2 until any unconnected demand point exists ($U > 0$). Figure 22 illustrates the process: the red curved arrow points to the root of the subtree, while the bright blue areas indicate members of the subtree. The upward move process terminates when the subtree is large enough, and the dark blue area shows the final group borders.

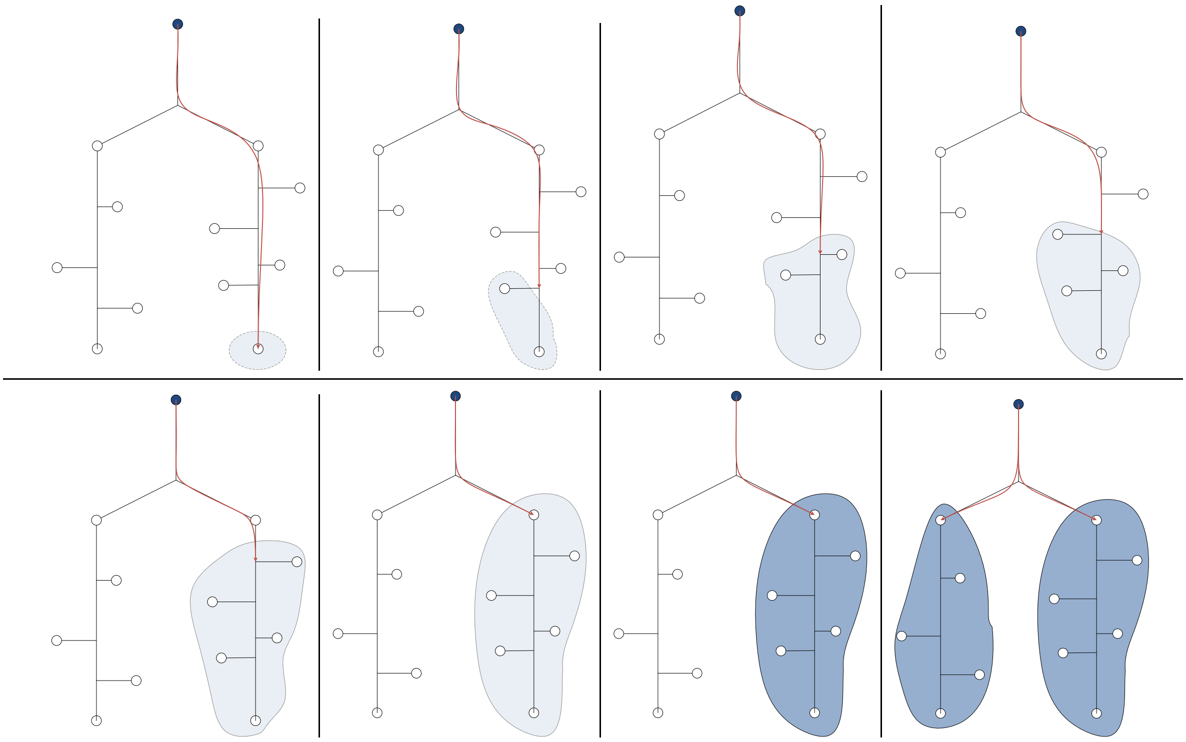


FIGURE 22 GROUP FORMULATION IN BCA

- ***Step 3*** (*DU allocation*): For every group formulated as described in Step 2, find the best DU location, minimizing the sum of distribution network cost. The available DU locations within the boundary of group are evaluated by summarizing lengths of paths to all demand points of the group, by using the Bellman-Ford algorithm. The location with minimal summarized distance is chosen as the DU location for the group.

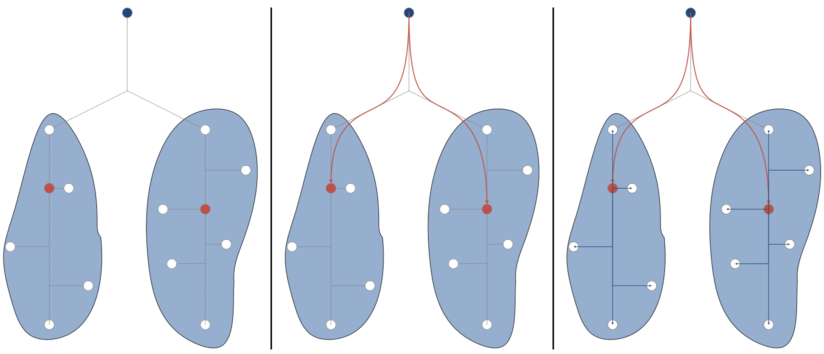


FIGURE 23 DU ALLOCATION AND CONNECTION ESTABLISHMENT

- ***Step 4*** (*Connection establishment*): The demand points should be connected to the assigned DUs via their shortest paths, while connecting the DUs to the CO is a more complicated process. Due to the stepwise behavior of the cable plant costs in the feeder network, minimization of the sum of C_0 costs plays an important role. In order to use the minimal necessary set of links for the feeder network, the problem becomes similar to the Steiner-tree problem. With small modifications, we use a generally accepted approximation of the originally NP-complete problem, the so-called Distance Network Heuristic (DNH, [56]) for designing the feeder network.

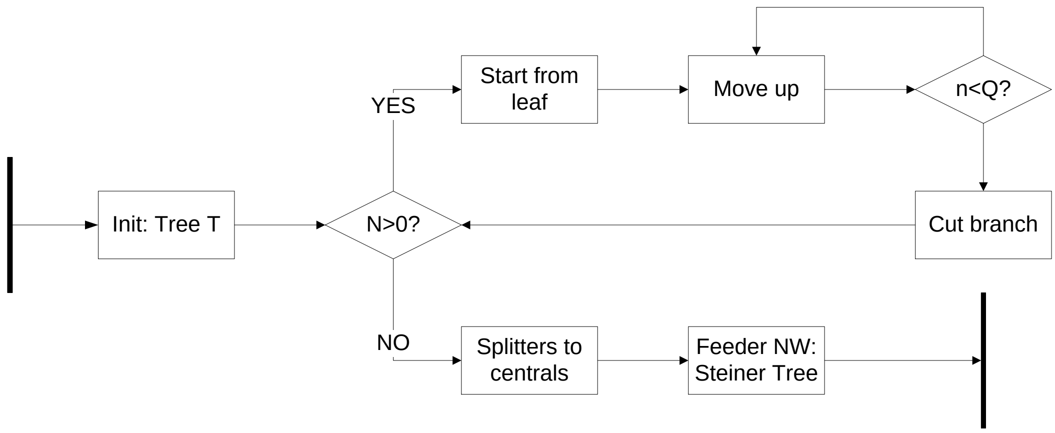


FIGURE 24 FLOWCHART OF THE BRANCH CONTRACTING ALGORITHM (BCA)

5.3.2. COMPLEXITY

Lemma 5.1: The presented BCA algorithm works with time complexity of $\mathrm{O}(\mathrm{V}^2 \cdot \log(\mathrm{V}))$

Proof 5.1:

Preliminary assumptions (these stand for all other algorithms in this section):

- The graph models a map, therefore the maximal degree of a node is bounded from above: a crossing of two streets is a node with degree of 4. Therefore, the amount of network links (graph edges) is bounded by $(maxdegree/2)$ times the amount of nodes, i.e. $O(E) \sim O(V)$
- The amount of DU locations is also proportional to the amount of nodes (i.e. to the population / graph size): $O(\Omega) \sim O(V)$
- For ease of notation, during the complexity analysis here, we use V instead of $|V|$

Based on these assumptions, steps 1-4 of the BCA algorithm will be analyzed, in order to identify their complexity.

- ***Step 1:*** During the initialization phase, the shortest path tree has to be built and the subtree information has to be stored
  - Shortest path tree: the Bellman-Ford algorithm runs in $O(V \cdot E)$ time. After that, along all the demand point-CO shortest paths, the edges have to be added to the tree, in at most $O(V \cdot V)$ steps. While adding the edges to the tree, and proceeding on the paths towards the CO, the demand points are stored at all intermediate nodes in the subtree lists.
  - The subtree lists have to be ordered by the distance of demand points from the root node of the subtree, that needs at most $O(V \cdot \log(V))$ steps.
- ***Step 2:*** The group formulation is composed of two basic operations, in an iterative manner:
  - In order to find the next branch to cut, the furthest leaf has to be found, which needs $O(V)$ steps due to the Bellman-Ford algorithm already executed at the initialization phase. It is followed by upward moves, in at most $O(V)$ steps, towards the CO, and

finally, once the root of the subtree is found, the "deepest" K demand points are selected, in linear time, since the subtree lists are ordered.

  - The branch cut operation requires a refresh operation at all the nodes along the path from the subtree root to the CO. Deletion of the K nodes from the ordered subtree lists is done in linear time at every node, resulting in $O(V \cdot K)$ operations.

- ***Step 3:*** The DU allocation runs in $O(\Omega)$ steps: the sum of distance values have to be compared among all DU locations, while the distance values are already known due to the Bellman-Ford algorithm in the initialization phase.
- ***Step 4:*** Finally, the feeder network is constructed as a Steiner-tree either with the basic Distance Network Heuristic that runs in $O(V^2 \cdot \log(V))$ time, or with the slightly more accurate algorithm of Zelikovsky that runs in $O(V^3)$ time, providing the best known approximation with a constant factor of 1.55.

Steps 1, 3 and 4 run only once, while the Step 2 has to be executed for every group, $O(V)$ times. Therefore, the overall complexity of the BCA algorithm, if the DNH heuristic is used to build the Steiner-tree is:

$$O(V^2) + O\big(V \cdot (V + V)\big) + O(V) + O(V^2 \cdot \log(V)) = O(V^2 \cdot \log(V))$$

If the algorithm of Zelikovsky is chosen to build the Steiner-tree of the feeder network, the BCA algorithm inherits its $O(V^3)$ complexity.■

5.3.3. Approximation performance

Factor of guaranteed approximation for the PON topology design problem is really complex to derive, since the problem itself is a composition of different problems, with varying weights of different cost factors.

The weights themselves are parameters of the proposed heuristic algorithm as well: the predefined threshold Q for the minimal amount of demand points served by a DU has to reflect the significance of DU costs versus the cable plant costs. The capacitated facility location (CFL) problem as an underlying problem has a worst-case approximation factor of 2.

Since this worst-case approximation factor is far from the expected quality of results for practical scenarios, the approximation performance evaluation is focused on average/expected approximation, presented in the numerical results section, via evaluation of case studies, comparing heuristic results against Linear Programming lower bounds.

5.4. ACTIVE OPTICAL NETWORKS (AON)

Active Optical Networks (AON) have their own specific features, in some aspects similar, but from other aspects, contradictory to the already discussed Passive Optical Networks (PON) and Digital Subscriber Line (DSL) networks. Distance limitations are fairly permissive, similarly to PONs, and the DUs are active, complex and expensive equipment, similar to DSL networks. The high DU costs make the AON problem primarily a clustering problem with permissive capacity and length constraints.

A minimal set of groups/clusters is in demand, minimizing the necessary amount of DU equipment. At the same time, a clear structure of DU-demand point assignments ensure low cabling costs. Therefore the resulting topologies are expected to be more "regular", regarding the size and shape of clusters, in order to avoid overlapping of neighboring groups.

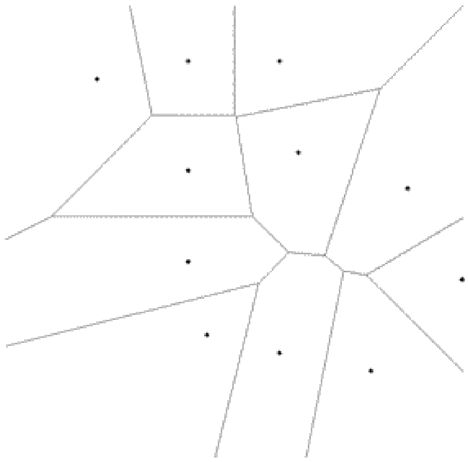


FIGURE 25 VORONOI-DIAGRAM

The concept of Voronoi-diagrams has to be mentioned here. In general, a Voronoi diagram is a decomposition of points in a given space, according to a set of "sites". Every point in the space is assigned to the closest one of these sites, and the points assigned to the same site form a Voronoi cell. In our case, the Voronoi-diagram is interpreted on the graph, with the DUs as its "sites", while the cells are the demand point clusters (groups) around the DUs, even though the DU capacity constraints will slightly distort this ideal structure.

The Iterative Neighbor Contracting Algorithm (INCA) tries to create such a balanced structure of demand point clusters and DU locations, using a bottom-up clustering technique: as an initial step, all demand points are assigned to their closest DU location. In the subsequent steps, these clusters are contracted with their neighbors, in order to match DU capacities, and achieve high utilization of DU equipment.

5.4.1. DESCRIPTION OF THE ALGORITHM

The flowchart of the algorithm is depicted on Figure 26, while the step-by-step description of the algorithm is given below.

Algorithm: Iterative Neighbor Contracting Algorithm (INCA)

- ***Step 1*** (*Initialization*): Given a list of available DU locations, assign every demand point to the closest one (resulting in a Voronoi-diagram around the DU locations). The groups of demand points assigned to these locations form a non-overlapping coverage, however the groups may be smaller/larger than the desired DU utilization.

- ***Step 2a*** (*Pre-filtering*): The desired minimal utilization of a DU equipment is Q, which is typically a function of the DU capacity K. For every DU location with more than Q demand points assigned, allocate a DU, and assign the closest K demand points to it. Remove these from the subsequent calculations, but the residing demand points will be handled later. Step 2a is repeated until no DU locations exist with more than Q demand points.
- ***Step 2b*** (*Aggregation*): Let $\mathcal{A}$ denote the set of clusters containing less demand points than the threshold Q. Find a pair of neighboring groups in $\mathcal{A}$. Merge these groups, and as the cluster center, use the DU location with the smaller summarized distance from the contracted group members (closer to the "centroid" of the group). Repeat this step until any "undersized" group exists, i.e. $|\mathcal{A}| > 0$.
- ***Step 3*** (*Connection establishment*): The distribution network between the DU and the assigned demand points follows their shortest paths. The feeder network is relatively sparse, therefore designed as a Steiner-tree (with the CO and the DUs as its terminals). This Steiner-tree is constructed using the DNH heuristic, similar to the feeder segment of PON networks.

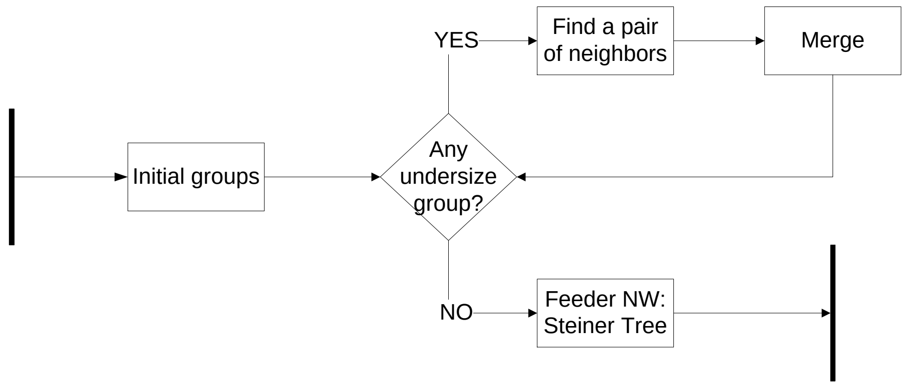


FIGURE 26 FLOWCHART OF THE ITERATIVE NEIGHBOR CONTRACTING ALGORITHM (INCA)

5.4.2. COMPLEXITY

Lemma 5.2: The presented INCA algorithm works with time complexity of $\mathrm{O}(\mathrm{V}^2 \cdot \log(\mathrm{V}))$

Proof 5.2: Based on the preliminary assumptions described at PON networks, Steps 1-3 of the INCA algorithm will be analyzed, in order to calculate their complexity:

- For Step 1, the Voronoi-diagrams have to be created. To assign demand points to their closest DU location, a single execution of the Bellman-Ford algorithm is necessary that runs in $O(V \cdot E)$ time, and then, for every node, the DU location has to be selected with minimal distance value, that requires $O(V \cdot \Omega)$ steps. In total, Step 1 needs $O(V^2)$ operations.
- By a more detailed resolution of Step 2:
    - The DU locations with more than Q demand points have to be found and realized. Finding these needs $O(\Omega)$, while in order to assign exactly the closest K to the new DU, these have to be ordered $(O(V \cdot \log(\mathrm{V})))$, and then the deletion of the connected nodes can be done in linear time. In total, this happens in $O(V \cdot \log(\mathrm{V}))$ time.

- For contracting DUs:
  - The closest neighbor of the chosen DU can be found by a DFS search on $O(\mathrm{V}+\mathrm{E})$ time, and for choosing the centroid of the new group, distance values from the Bellman-Ford algorithm have to summed up and compared, in linear time.
  - The actual DU and group contract operation runs in $O(\mathrm{K}\cdot\Omega)$ time, since all the affected nodes have to be assigned to their closest available DU, that means a simple comparison of all DUs for every node. And there are no more than $2K$ affected nodes, since the two contracted groups had less than K members.
- Finally, the Steiner-tree problem of the feeder network can be solved by any of the following two algorithms: the basic Distance Network Heuristic runs in $O(V^2\cdot\log(V))$ time, while the slightly more complex algorithm of Zelikovsky runs in $O(V^3)$ time.

Steps 1 and 3 are to be run only once, while the operations of Step 2 are used in an iterative manner, in a worst case scenario $O(V)$ times. Therefore, the overall complexity of the INCA heuristic, if the DNH heuristic is used to build the Steiner-tree is:

$$O(V^2)+O(V\cdot[V\cdot\log(V)+V+V\cdot\log(V)])+O(V^2\cdot\log(V))=O(V^2\cdot\log(V))$$

If the algorithm of Zelikovsky is chosen, the INCA heuristic inherits its $O(V^3)$ complexity.■

5.4.3. APPROXIMATION PERFORMANCE

Any guaranteed approximation factor is again very difficult to prove, similar to PON networks. The clustering problem itself contains the capacitated p-median problem (CPMP), supposed that the amount of DUs is fixed ($p={}^{N}/_{K}$). For the CPMP problem, $3+{}^{2}/_{p}$ is the best known approximation factor in the literature, and anything better than $1+{}^{2}/_{e}\approx 1{,}736$ is impossible.

Both of these worst-case approximation factors are far from the expected quality of results for practical scenarios, therefore, regarding approximation performance, we refer to the numerical results and evaluation of case studies, comparing heuristic results against Linear Programming lower bounds (Section 7).

5.5. DIGITAL SUBSCRIBER LINE (DSL) NETWORKS

DSL networks with fiber feeder segment have a very special feature, which makes them different from the completely optical access network types addressed in the study. The re-use of the legacy copper network makes the distribution network cost negligible, at least in comparison with the fiber deployment required by the other technologies. Therefore the cable plant cost reduces to the feeder network segment. At the same time, the physical constraints on the distribution network segment play fundamental role: the copper network is strongly limited by the attenuation characteristics of the copper itself.

The Distribution Unit (DU) costs are significant: the network equipment, on the boundary of the optical feeder and the copper distribution network are expensive. On the other hand, DU capacity constraints do not have significant effect: typically the DU capacity itself is higher than the amount of demand points within the range of the DU due to the limited copper loop length, i.e. not the DU capacities themselves; rather the copper loop length constraints play an important role. Moreover, the incurring DU costs are more related to the amount of active DU locations and not the equipment itself, since the capacity of a distribution unit is typically expandable by additional cards with moderate increase in price. Therefore, the optimization problem formulation allows the minimization of the DU equipment or the active DU locations either: in the latter case, the DU capacity constraints (K) should be relaxed (i.e. set to infinity). These altogether define a special "coverage" problem, in which a minimal amount of DU locations are sought, "covering" all the demand points, i.e. every demand point has a DU located within L_{max}^{dist} distance.

5.5.1. Description of the Stepwise Allocation of Critical DUs (SACD) algorithm

Flowchart of the algorithm may be followed on Figure 27, and the step-by-step description is given below.

Algorithm: Stepwise Allocation of Critical DUs (SACD)

- ***Step 1*** (*Initialization*): Place a *virtual DU* to all available locations. Assign all demand points within L_{dist}^{max} distance to the virtual DU (one demand point may be assigned to multiple virtual DUs). Compute criticality values, and perform the lexicographical criticality ordering of DUs.
- ***Step 2*** (*DU allocation #1*): If any virtual DU exists with more assigned demand points then a predefined threshold T, choose the most critical, with respect to the ordering defined above. The value of T is a function of the DU capacity (K), controlling the desired initial utilization rate. Allocate a single DU at the chosen position, covering the T most critical demand points in the respective demand point list, and remove these T demand points from the graph, and from all other virtual DUs). Repeat this step until virtual DUs exist with at least T demand points, otherwise go to the next step.
- ***Step 3*** (*DU allocation #2*): If no more virtual DUs exist with at least T demand points, choose the one with the highest utilization, and repeat this step until unconnected demand points exist, or until a predefined utilization lower bound is reached. We note that if the amount of active DU locations is minimized instead of DU equipment ($K = \infty$), Step 3 will be skipped.
- ***Step 4*** (*Connection establishment*): The copper network between the DU and the demand points is shortest path based (and a priori defined by the existing and re-used copper network). As we have described it for PON networks, the DNH heuristic will be used to construct the Steiner-tree of the feeder network.

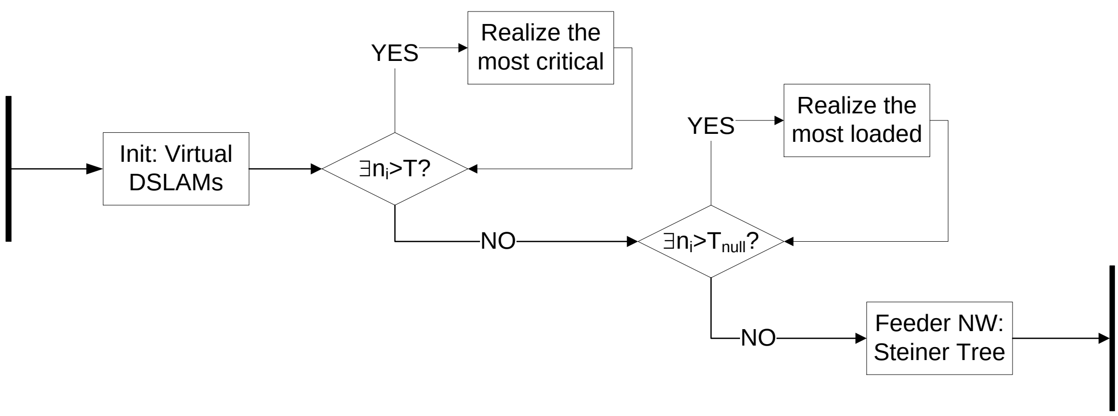


FIGURE 27 FLOWCHART OF THE STEPWISE ALLOCATION OF CRITICAL DUS (SACD) ALGORITHM

5.5.2. Complexity

Lemma 5.3: The presented SACD algorithm works with time complexity of $\mathrm{O}(\mathrm{V}^2 \cdot \log(\mathrm{V}))$

Proof 5.3: Based on the preliminary assumptions described at PON networks, steps 1-4 of the SACD algorithms will be analyzed, regarding their complexity:

- By summing up the above described tasks, Step 1 needs $O\left(V^2 \cdot \log(V)\right)$ steps:
    - Assigning demand points to all DU locations within reach: in order to make this assignment, an execution of the Bellman-Ford algorithm is necessary that runs in $O(V \cdot E)$ time. With respect to our initial assumptions, it is equal to $O(V^2)$.
    - Compute node criticality values: for every node-DU location pair a length-check has to be carried out, based on the distance values obtained from the Bellman-Ford algorithm: $O(\Omega \cdot \mathrm{V}) \sim O(V^2)$.
    - Lexicographical ordering of DUs: at first, the nodes assigned to a DU location have to be ordered in increasing order by their criticality values, that needs $O\left(V \cdot \log(\mathrm{V})\right)$ steps per DU, $O\left(\Omega \cdot V \cdot \log(\mathrm{V})\right) \sim O\left(V^2 \cdot \log(V)\right)$ in total. Finally, the DUs themselves require a lexicographical ordering, based on their (already ordered) node-lists, that needs $O\left(\Omega \cdot \log(\Omega)\right)$ steps, i.e. $O\left(V \cdot \log(\mathrm{V})\right)$ according to our initial assumptions.
- Choosing the next position for DU allocation (Step 2) runs in $O(1)$ time, due to the lexicographical ordering.
- The DU allocation itself (Step 3) needs at the deletion of at most K points from all other virtual DUs in $O(K \cdot \Omega)$ time, and a reordering of the DUs in $O\left(\Omega \cdot \log(\Omega)\right)$ steps. The demand point lists of DUs remain ordered after the deletion. By summarizing these, Step 3 needs $O(K \cdot \Omega + \Omega \cdot \log \Omega) = O(V + V \cdot \log(V)) = O(V \cdot \log(V))$ steps.
- Step 4: For building the Steiner-tree of the feeder network, two heuristics can be used. The simpler Distance Network Heuristic runs in $O(V^2 \cdot \log(V))$ time, while the Zelikovsky algorithm runs in $O(V^3)$ time, providing the best known approximation factor of 1.55.

Steps 1 and 4 are to be run only once, while Steps 2 and 3 are used in an iterative manner, at most $O(V)$ times. Therefore the overall complexity of the SACD algorithm, if the DNH heuristic is used to build the Steiner-tree is:

$$O\left(V^2 \cdot \log(V)\right) + O(V \cdot [1 + V \cdot \log(V)]) + O(V^2 \cdot \log(V)) = O(V^2 \cdot \log(V))$$

If the algorithm of Zelikovsky is chosen, the SACD algorithm inherits its $O(V^3)$ complexity.■

5.5.3. Approximation performance

In Section 4.2 it was proven that the criticality based greedy algorithm provides a 2-approximation of the minimal amount of necessary DU *locations* for a full coverage of demand points. Therefore the SACD algorithm provides a 2-approximation in this sense. However, we can take a little step forward, and evaluate the approximation of the minimal amount of DU *equipment*, with respect to the DU capacity (K) constraints.

Lemma 5.4: Provided that the given maximal matching is of minimal size, the above described algorithm yields to a maximal "additional" 2-approximation inaccuracy of the problem, regarding the amount of DU units used.

Proof 5.4: This suboptimality occurs when the amount of demand points assigned to the DU location leads to an "underutilization" of the DU equipment. The worst case scenario is then the lowest possible utilization of DU equipment on the necessary DU locations, i.e. the case when the k_i distribution units are just not sufficient to connect the demand points assigned to the respective DU location. More formally, the situation when $k_i \cdot K + 1$ demand points are connected to every active DU_i location, requiring $k_i + 1$ DUs there. It is important to see that the since the DU location is active in the optimal solution, at least one DU has to be assigned there ($k_i \geq 1$). Suboptimality occurs when multiple DUs are located there.

Assuming ω active DU locations, the total amount of demand points is as follows:

$$S = \sum_{i=1}^{\omega}(k_i \cdot K + 1) = \omega + K \cdot \sum_{i=1}^{\omega} k_i \geq K \cdot \sum_{i=1}^{\omega} k_i \qquad (1)$$

These demand points require a minimum of $\sum k_i$ DUs, regardless of the actual position of them. However, due to the $k_i \cdot K + 1$ demand points assigned to each location i, the amount of DUs in total will be:

$$\#DUs = \sum_{i=1}^{\omega}(k_i + 1) = \omega + \sum_{i=1}^{\omega} k_i \qquad (2)$$

The proportion of the minimum necessary and actually deployed DUs, according to (1) and (2):

$$\frac{\#DUs}{S/K} = \frac{\omega + \sum_{i=1}^{\omega} k_i}{\sum_{i=1}^{\omega} k_i} = 1 + \frac{\omega}{\sum_{i=1}^{\omega} k_i}$$

Since $k_i \geq 1$ for any location:

$$\sum_{i=1}^{\omega} k_i \leq \sum_{i=1}^{\omega} 1 = \omega$$

This implies:

$$\frac{\#DUs}{S/K} = 1 + \frac{\omega}{\sum_{i=1}^{\omega} k_i} \leq 1 + \frac{\omega}{\omega} = 1 + 1 = 2$$

Therefore in a worst case scenario the amount of DUs actually deployed is two times the minimal necessary amount of DUs.■

Corollary 5.4: The criticality based greedy algorithm leads to a 2-approximation of the necessary active DU locations. The amount of active DU locations is a 2-approximation of the necessary DU equipment. These altogether prove the fact that the greedy SACD algorithm provides in a worst-case scenario a 4-approximation on the minimal amount of DU equipment, and a 2-approximation of the DU locations.

6. EXACT OPTIMIZATION AND METAHEURISTICS

The heuristic solutions proposed in the previous section are highly specialized algorithms, intended to provide high quality approximate results, within reasonable time constraints. In this section, two totally different reference methods are presented: (1) a mathematical programming approach, used to find the optimality gap, i.e. the distance between the heuristic solutions and the exact optimum, and (2) Simulated Annealing, a general (meta)heuristic approach, which will be used to evaluate the scalability of the highly specialized heuristics.

THE OPTIMALITY GAP

The already presented algorithms are heuristics, so we need to know how far they are from the optimum: approximation quality is a primary question. In the first subsection, we will make an effort towards exact optimization, in order to evaluate the gap between the heuristic solutions and the optimum. Through several improvements steps, including linearization and relaxation/decomposition techniques, a linear programming formulation will be derived, which provides a lower bound of the optimum, even for relatively large problem instances.

Such a lower bound then contributes to the initial question: the gap between the heuristic solutions and the optimum. Supposed that we have a lower bound, i.e. a cost value even lower than the optimum ($OPT - \alpha$). The heuristic solution is higher than the optimum ($OPT + \beta$), i.e. the *optimality gap* is β. In such a case, the $\alpha + \beta$ distance bounds the optimality gap from above, and this $\alpha + \beta$ distance will be available even if the optimum itself remains unknown.

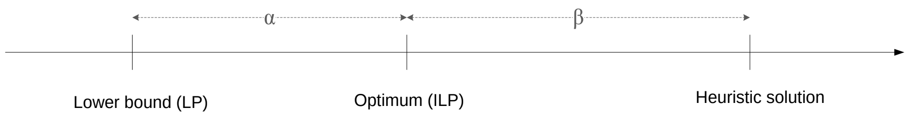


FIGURE 28 BOUNDING THE OPTIMALITY GAP

SPECIALIZATION GAIN

Estimating the optimality gap tells the price of using heuristic approaches instead of exact optimization. Unfortunately, exact optimization is beyond possibilities, as our complexity results have shown. Supposed that an acceptable optimality gap is obtainable, heuristic approaches provide a reasonable solution. However, finding the right heuristic solution is not straightforward. The proposed solutions rely on the specific problem characteristics, i.e. the special cases presented in Section 2.5. As it was discussed in the respective sections, accurate identification of these special cases determines the efficiency of the heuristics (see No Free Lunch Theorem [53]).

A large set of generally adopted metaheuristic approaches exist in the literature, e.g. Random Optimization [75], Genetic Algorithm [76], Simulated Annealing [77], Tabu Search [78], and several nature inspired algorithms: Ant Colony Optimization [79] or the Firefly Algorithm [80] are nice examples for learning from the nature. Many of them were investigated regarding their applicability for the NGA Topology Design (NTD) problem [38], and we found the *Simulated Annealing (SA)* [77] a well suitable approximation scheme. In the second subsection, a Simulated Annealing scheme is developed and adapted to the NTD problem.

These will serve as a "benchmark" general heuristic algorithm for performance evaluation of all presented, highly specialized heuristics, considering both approximation quality and scalability, while the numerical evaluation and the case studies will be presented in the next section.

6.1. Mathematical programming

In this section, the mathematical programming formulation is introduced, beginning with the initial Quadratic Programming (QP) formulation, and then improved through various steps towards a computationally tractable, Linear Programming (LP) formula.

6.1.1. Quadratic Programming (QP) formulation

We recall Section 0, where the formal model of the NGA Topology Design problem was introduced, and the optimization problem formulated. The problem in its most straightforward representation is a quadratic programming (QP) problem.

Formally, we are given a network graph $G = (V, E)$, consisting of edges E and nodes V representing the traces/paths, along which a network link could be built, the set of available DU locations $\Omega = \{DU_j\} \subseteq V$ and the set of demand points $S = \{s_i\} \subseteq V$.

The optimization tends to minimize the topology dependent cost of network deployment. All the edges $e \in E$ have a nonnegative length $l(e)$, a cable deployment cost $C_0(e)$ and fiber cost $C_v(e)$. These costs are typically but not necessarily proportional to the length of the link, and the cost reflects the different cabling technologies and existing infrastructure conditions. The cost of deploying a distribution unit (DU) is C_{DU}^*.

Constants

$\forall e \in E$	$C_0(e)$	Cost of cable deployment on link *e*
$\forall e \in E$	$C_v(e)$	Cost of fiber on link *e*
$\forall e \in E$	$l(e)$	Length of link *e*
	C_{DU}^*	Cost of Distribution Unit (DU)s
	K	Capacity of DU units

Variables:

$\forall i \in S, \forall j \in \Omega$	$\omega_j^i \in (0,1)$	Indicator of the demand point-DU location assignment, value is 1 only if the demand point i is connected to DU location j.
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j.
$\forall i \in S, \forall e \in E$	$x_e^i \in (0,1)$	Indicator of edge e on the path between demand point i and its assigned DU location (Distribution network)
$\forall j \in \Omega, \forall e \in E$	$y_e^j \in (0,1)$	Indicator of edge e on the path between DU_j and the CO (Feeder network)
$\forall e \in E$	$F(e)$	Number of connections over edge e in the feeder and in the distribution networks altogether
$\forall e \in E$	$I_e \in \{0,1\}$	Indicator variable for existence of edge *e* either in the feeder or in the distribution network segments
	N	Total amount of DUs deployed in the network

OBJECTIVE:

$$minimize\ C_{topology\ dependent} = N \cdot C_{DU}^{*} + \sum_{e} (I_e \cdot C_0(e) + F(e) \cdot C_v(e))$$

CONSTRAINTS:

(1) $\forall i \in S$ $$\sum_{j\in\Omega} \omega_j^i = 1$$

(2) $\forall j \in \Omega$ $$\sum_{i\in S} \omega_j^i \leq n_j \cdot K$$

(3) $\forall v \in V, \forall i \in S$ $$\sum_{e:v\to} x_e^i - \sum_{e:\to v} x_e^i = \begin{cases} \omega_j^i & v \in \Omega \\ +1 & v = s_i \\ 0 & \text{otherwise} \end{cases}$$

(4) $\forall v \in V, \forall j \in \Omega$ $$\sum_{e:v\to} y_e^j - \sum_{e:\to v} y_e^j = \begin{cases} -n_j & v = DU_j \\ 0 & \text{otherwise} \\ +n_j & v = CO \end{cases}$$

(5) $\forall i \in S$ $$\sum_{ee\in E} l(e) \cdot x_e^i + \sum_{j\in\Omega} \left(\omega_j^i \cdot \sum_{e\in E} l(e) \cdot y_e^j \right) \leq L_{max}$$

(5a) $\forall i \in S$ $$\sum_{e\in E} l(e) \cdot x_e^i \leq L_{max}^{dist}$$

(5b) $\forall i \in S$ $$\sum_{j\in\Omega} \left(\omega_j^i \cdot \sum_{e\in E} l(e) \cdot y_e^j \right) \leq L_{max}^{feed}$$

(6) $\forall e \in E$ $$F(e) = \sum_{i\in S} x_e^i + \sum_{j\in\Omega} y_e^j$$

(7) $\forall e \in E$ $$F(e) \leq I_e \cdot (S + \Omega)$$

(8) $$N = \sum_{j\in\Omega} n_j$$

Constraints (1) ensure that every demand point is served by exactly one DU, (2) represents the capacity constraints for DUs: the amount of demand points connected to a DU location is bounded from above by the summarized total capacity of the DUs located at the given location. The flow conservation (Kirchhoff) constraints (3) and (4) keep the flow from splitting. These ensure that every demand point has a dedicated unitary flow towards its DU (distribution network), and every DU locations are connected to the CO by a flow of n_j units (feeder network). Constraints (5), (5a) and (5b) provide the network reach (distance) limits for the overall, feeder and distribution network segments, respectively. We note that constraints (5) and (5b) make the formulation quadratic. Finally, constraints (6), (7) and (8) provide the auxiliary data for the cost function: the $F(e)$ installed capacity values for every link, the indicator variable for link deployment and the amount of DUs.

We introduce some basic assumptions for calculating the $|Variables \times Constraints|$ dimensions of the formulation:

- for the sake of simplicity, in this section we use E instead of $|E|$, V instead of $|V|$, etc. during the problem dimension calculations
- the amount of edges (E) and nodes (V) is in the same order of magnitude: $E = O(V)$. This initial assumption is supported by the fact that our graph represents the map (street system) of a given service area, at which the nodes are degree-constrained: most of them have exactly two adjacent edges, and street crossings typically have a degree of four
- the amount of DU locations and demand points is proportional to the number of nodes: $\Omega = O(V)$

This Quadratic Programming (QP) formulation has the following dimensions:

VARIABLE SET DIMENSIONS

$$Var(QP) = S \cdot \Omega + \Omega + \mathrm{S} \cdot \mathrm{E} + \Omega \cdot \mathrm{E} + \mathrm{E} + \mathrm{E} + 1 = \mathrm{S} \cdot \mathrm{E} + \Omega \cdot (1 + \mathrm{S} + \mathrm{E}) + 2 \cdot \mathrm{E} + 1 \sim$$
$$\sim \mathrm{O(V)} \cdot \mathrm{O(V)} + \mathrm{O(V)} \cdot [1 + \mathrm{O(V)} + \mathrm{O(V)}] + O(V) = 2 \cdot \mathrm{O(V^2)} + \mathrm{O(V)} = \boxed{\mathrm{O(V^2)}}$$

CONSTRAINT SET DIMENSIONS

$$Con(QP) = S + \ \Omega + \mathrm{V} \cdot \mathrm{S} + \mathrm{V} \cdot \Omega + \mathrm{S} + \mathrm{S} + \mathrm{S} + \mathrm{E} + \mathrm{E} + 1 =$$
$$= S \cdot (V + 4) + \Omega \cdot (1 + \mathrm{V}) + 2 \cdot \mathrm{E} + 1 \sim \ \mathrm{O(V)} \cdot \mathrm{O(V)} + \mathrm{O(V)} \cdot \mathrm{O(V)} + 2 \cdot \mathrm{O(V)} + 1 =$$
$$= 2 \cdot \mathrm{O(V^2)} + \mathrm{O(V)} = \boxed{\mathrm{O(V^2)}}$$

In summary, dimensions of QP:

$$Dim(QP) = Var(QP) \times Con(QP) = \mathrm{O(V^2)} \times \mathrm{O(V^2)}$$

6.1.2. MIXED INTEGER PROGRAMMING (MIP) FORMULATION

Linearization of the quadratic length constraints (5,5b) of QP is possible: both ω and y are binary variables, therefore their product is the AND operation of Boolean algebra. A new variable z is introduced, and by two constraints, $z = \omega \cdot y$ is enforced, as follows:

$$z = \omega \cdot y \Leftrightarrow \begin{cases} \omega + y \geq 2z \\ -1 + \omega + y \leq z \end{cases}$$

In our case, the new variable will be:

$\forall i \in S, \forall j \in \Omega, \forall \mathrm{e} \in \mathrm{E}$	$z_e^{i,j} \in (0{,}1)$	Indicator of edge e being on the path between $\mathrm{DU_j}$ and the CO if demand point i is assigned to $\mathrm{DU_j}$ (Feeder network!)

Constraints (5) and (5b) are substituted by (A1)-(A4) as follows:

(A1)	$\forall i \in S$	$\sum_{\mathrm{e} \in \mathrm{E}} x_e^i + \sum_{j \in \Omega} \sum_{\mathrm{e} \in \mathrm{E}} z_e^{i,j} \leq L_{max}$
(A2)	$\forall i \in S$	$\sum_{j \in \Omega} \sum_{\mathrm{e} \in \mathrm{E}} z_e^{i,j} \leq L_{max}^{feed}$
(A3)	$\forall i \in S, \forall j \in \Omega, \forall \mathrm{e} \in \mathrm{E}$	$\omega_j^i + y_e^j \geq 2 \cdot z_e^{i,j}$
(A4)	$\forall i \in S, \forall j \in \Omega, \forall \mathrm{e} \in \mathrm{E}$	$-1 + \omega_j^i + y_e^j \leq z_e^{i,j}$

(A3)-(A4) ensures that $z_e^{i,j} = \omega_j^i \cdot y_e^j$, and then (5) is transformed to (A1):

$$\sum_{e \in E} x_e^i + \sum_{j \in \Omega} \left(\omega_j^i \cdot \sum_{e \in E} y_e^j \right) = \sum_{e \in E} x_e^i + \sum_{j \in \Omega} \sum_{e \in E} \omega_j^i \cdot y_e^j = \sum_{e \in E} x_e^i + \sum_{j \in \Omega} \sum_{e \in E} z_e^{i,j} \leq L_{max}$$

Similarly, (A3)-(A4) ensures that $z_e^{i,j} = \omega_j^i \cdot y_e^j$, and then (5b) is transformed to (A2)

$$\sum_{j \in \Omega} \left(\omega_j^i \cdot \sum_{e \in E} y_e^j \right) = \sum_{j \in \Omega} \sum_{e \in E} \omega_j^i \cdot y_e^j = \sum_{j \in \Omega} \sum_{e \in E} z_e^{i,j} \leq L_{max}^{feed}$$

Unfortunately, the problem dimensions are increased with this linearization. Both the variable and the constraint sets became V times larger, i.e. the problem dimensions are multiplied by V^2:

VARIABLE SET DIMENSIONS

$$Var(ILP) = Var(QP) + S \cdot \Omega \cdot E = O(V^2) + O(V^3) = \boxed{O(V^3)}$$

CONSTRAINT SET DIMENSIONS

$$Con(ILP) = Con(QP) + (S + S + 2 \cdot S \cdot \Omega \cdot E) - (S + S) =$$

$$= Con(QP) + 2 \cdot S \cdot \Omega \cdot E = O(V^2) + O(V^3) = \boxed{O(V^3)}$$

In summary, dimensions of LP:

$$Dim(ILP) = Var(ILP) \times Con(ILP) = \mathrm{O}(\mathrm{V}^3) \times \mathrm{O}(\mathrm{V}^3)$$

6.1.3. AGGREGATED FLOWS: A REDUCED LINEAR PROGRAMMING FORMULATION

Linearization of the originally quadratic problem was a significant step forward; even if the further increased dimensions, namely the $\mathrm{O}(\mathrm{V}^3)$ variables and $\mathrm{O}(\mathrm{V}^3)$ constraints results in a really challenging problem size: an $O(1000) \times O(1000)$ size matrix representation of the problem even for a graph of 10 nodes - an enormous matrix with approximately one million elements, for a pretty small graph.

Therefore, the problem has to be reduced in size, without losing its linearity.

In this section, a new LP formulation is introduced, which is based on the aggregation of the distinct network segments into a single feeder and distribution network flow problem, respectively:

- the feeder network flow originates from the CO as its *source*, and terminates in the DU locations as its *sinks*, serving every DU location with a flow of n_j units, i.e. one unit per distribution unit
- the distribution network flow originates from the DU locations as its *sources*, and terminates at the demand points as its *sinks*, serving every demand point with a flow of 1 unit

These flow problems are not independent: the DUs are treated as sinks of the feeder flow and sources of the distribution flow, while the traffic terminating in the feeder flow gives a bound on the traffic originating in the distribution flow, according to DU capacity constraints.

Such an aggregated flow problem hides the individual connections, hence the access network topology cannot be derived from it, but it gives a **lower bound** on the optimal value of the cost function: if the original topology design problem has a solution with cost C, it is also a solution for the flow problem with the same cost C, even though the flow problem *may* have a solution with even lower cost.

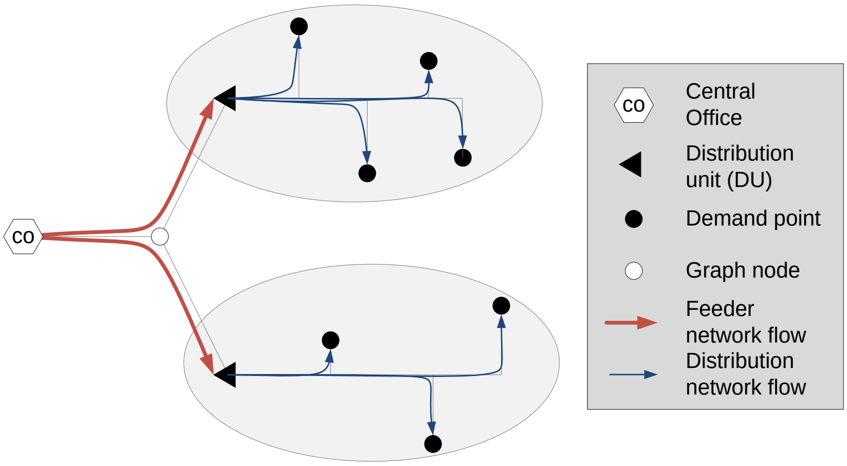


FIGURE 29 AGGREGATED MIP: THE FLOW PROBLEM

The length constraints are relaxed in this MIP_{aggr} formulation: the individual connections are not traceable, therefore their lengths are not considered. Problem complexity is further reduced by taking the linear relaxation of the problem: the flow integrality constraints are relaxed. In the aggregated flow formulation, integrality does not prevent flow splitting anyway: the variables stand only for the sum of distinct flows, not for the individual flows themselves.

According to the above described concept, the linear programming formulation for aggregated flows is as follows:

Variables

$\forall e \in E$	$x_e \in \mathbb{N}$	Feeder network flow over edge e
$\forall e \in E$	$y_e \in \mathbb{N}$	Distribution network flow over edge e
$\forall j \in \Omega$	$n_j \in \mathbb{N}$	The amount of DUs at location j
$\forall e \in E$	$I_e \in \{0,1\}$	Indicator variable for edge e either in the feeder or the distribution network flows
	N	Total amount of DUs

Objective

$$minimize\ C_{flow} = N \cdot C_{DU}^{*} + \sum_{e} [I_e \cdot C_0(e) + (x_e + y_e) \cdot C_v(e)]$$

Constraints

(1) $\forall v \in V$
$$\sum_{e \in v \rightarrow} x_e - \sum_{e \in \rightarrow v} x_e = \begin{cases} N & v = CO \\ -n_v & v \neq CO \end{cases}$$

(2) $\forall v \in V$
$$\sum_{e \in v \rightarrow} y_e - \sum_{e \in \rightarrow v} y_e \begin{cases} = -1 & v \in S \\ \leq n_v \cdot K & v \notin S \end{cases}$$

(3) $\forall e \in E$
$$x_e + y_e \leq I_e \cdot (S + \Omega)$$

(4)
$$N = \sum_{v \in V} n_v$$

Constraints (1) and (2) are the flow conservation constraints for the feeder and distribution network flows, respectively, while (3) and (4) sets the auxiliary variables for the cost function: the indicator values I_e, and the amount of DUs. We note that the value of n_j, i.e. the amount of DUs deployed at location j is implicitly constrained by (2): the DUs must "absorb" the flow originated from the demand points in S, then these n_j values are acting as a lower bound on the feeder network flow, which is forced towards its minimum by the cost functions.

This aggregated linear programming formulation has the following dimensions:

VARIABLE SET DIMENSIONS

$$Var(MIP_{aggr}) = E + E + \Omega + \mathrm{E} = 3 \cdot \mathrm{E} + \Omega \sim \boxed{\mathrm{O(V)}}$$

CONSTRAINT SET DIMENSIONS

$$Con(MIP_{aggr}) = V + V + E + 1 = 2 \cdot V + E + 1 \sim \boxed{O(V)}$$

In summary, dimensions of MIP_{aggr}:

$$Dim(MIP_{aggr}) = O(V) \times O(V)$$

This way we have constructed a mixed integer, linear formulation with significantly lower dimensions. Obviously, this formulation does not solve the original problem directly, it rather gives a lower bound. However, even a lower bound was extremely difficult to find, which makes this MIP_{aggr} formulation a valuable addition – as we will see at the validation and evaluation section later.

6.1.4. Overview of Mathematical Programming Formulations

The formulations presented in this section are compared and summarized in Table 4. The initial Quadratic Programming (QP) formulation was the direct formal interpretation of the topology design problem, as it was described in Section 0. The following Integer Linear Programming formulation (ILP) was the linearization of QP, with the same conditions, solution space and optimum. The complexity challenges were then addressed by an aggregated MIP formulation (MIP_{aggr}), which has significantly lower dimensions.

Even if the latter, aggregated MIP formulation gives only a lower bound on the optimum instead of an exact solution, it was implemented and applied for evaluation, due to the heavy complexity challenge the topology design problem poses. The significantly lower dimension, linearization and the linear relaxation of the majority of its variables altogether made the MIP_{aggr} mathematical programming approach scalable enough, at least for mid-size problem instances (see Section 7 for numerical results).

TABLE 4 OVERVIEW OF MATHEMATICAL PROGRAMMING FORMULATIONS

Formulation	Problem type	Dimensions	Features
QP	Quadratic, Integer	$\mathrm{O}(\mathrm{V}^2) \times \mathrm{O}(\mathrm{V}^2)$	Exact optimization
ILP	Linear, Integer	$\mathrm{O}(\mathrm{V}^3) \times \mathrm{O}(\mathrm{V}^3)$	Exact optimization with linearization
MIP_{aggr}	Linear, Mixed Integer	$O(V) \times O(V$	Reduced complexity, linearization, linear relaxation

6.2. METAHEURISTICS

Several metaheuristic approaches and optimization strategies are known in the literature, e.g. genetic algorithms, tabu search, branch and bound methods, hill climbing or even the greedy algorithm. Many of these methods have been investigated, considering their applicability for the NGA Topology Design problem [38]. Simulated Annealing turned out to be the most promising alternative, among others due to its scalability, the ability to avoid local optima, and the possibility to influence the convergence speed by the temperature function [38]. It does not require numerous problem/solution instances (like e.g. Genetic Algorithm), neither enormous amount of visited previous states (like e.g. Tabu Search) to store.

In contrary to mathematical programming, Simulated Annealing does not provide exact optimum. On one hand, it will be used as a "benchmark" for the proposed heuristic algorithms. However, on the other hand, it is not used only as a benchmark, but the really flexible parameter-setting features of SA make it a promising alternative for general applications, with future technologies or under unknown circumstances, where the highly specialized heuristics suffer performance degradation. Therefore the Simulated Annealing approach is a significant part of the proposed algorithm set.

6.2.1. SIMULATED ANNEALING

The concept of Simulated Annealing comes from metallurgy: the metal is heated at first, and then the molten metal material goes through a controlled cooling process. The initial high temperature causes the atoms to move dynamically around their initial position, letting them to reach a minimum energy state later. With decreasing temperature this motion is also reduced, and finally stopped: the atoms get stuck to their position in the crystal structure.

The cooling process ensures convergence of the simulated annealing, while the high temperature range helps to avoid local minima, by allowing state transitions through lower and higher energy states. Allowing moves toward higher energy states (which is a "backward" step in a minimization process) is the most important feature of SA.

The flowchart of a general Simulated Annealing process is given on Figure 30.

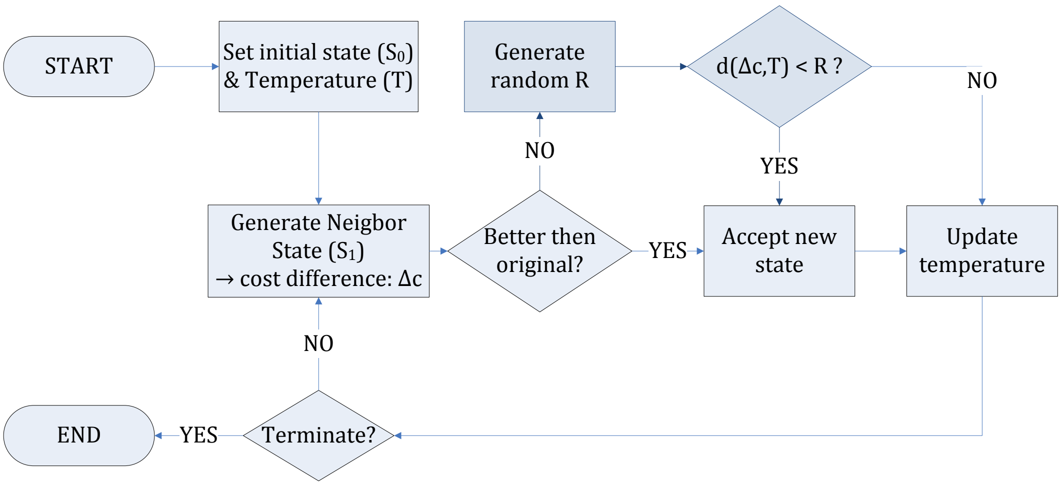


FIGURE 30 FLOWCHART OF THE SIMULATED ANNEALING (SA) PROCESS

6.2.2. Simulated Annealing for the NGA topology design problem

Applying a SA scheme is not a straightforward process. SA itself just gives a strategy, but it needs customization and adaptation to the addressed optimization problem. This adaptation itself makes the difference between highly effective and totally useless applications of SA.

Namely, the following features have to be defined:

- cooling (temperature control) strategy
- neighbor state generation
- state evaluation
- decision function
- initial state generation
- termination

State evaluation

During the SA process, we will mostly concentrate on the set of Distribution Units (DUs). Assuming an a priori given DU allocation (including the amount and location of DUs), the network topology is relatively easily derived, in the following steps:

- ***Step 1***: We recall the notion of Voronoi diagrams. As an initial step, we assign all demand points to their closest DU.
- ***Step 2***: In order to fulfill capacity constraints, this assignment is refined. For DUs with more than K demand points ("overloaded" DUs), the closest K demand points will be assigned to the DU, while the rest remain unconnected. Obviously, if n DUs are located at the same location, the closest $n \cdot K$ demand points will be assigned to them.
- ***Step 3***: After filling these "overloaded" DUs, the remaining demand points will be connected to the remaining DUs having spare capacity.

Based on the assignment procedure a network topology is created with respect to the given DU allocation, for which the cable/fiber costs will be calculated, and added to the cost of the DU units. The resulting overall cost will be used for state evaluation and comparison.

Neighbor state selection

Convergence and scalability is mostly influenced by neighbor state selection, therefore it is probably the most important part of the SA adaptation. It should support the process to walk effectively across the solution space: the amount of necessary steps between the two most distant states defines the "diameter" of the solution space.

We propose a neighbor state selection method based on the amount and location of DUs. A neighbor state is a network topology, which is created via modification of the current topology by any of the following operations:

- ***ADD***: Add a new DU to the current solution by assigning it to one of the available DU locations. The new DU location may be randomly selected, or in order to speed up convergence, it may be influenced by distribution of demand points: the new DU should be located with higher probability in a region where overloaded DUs exist.
- ***DELETE***: Remove a DU from the current solution. A DU may be randomly chosen for deletion, or it may be again influenced by the distribution of demand points: a DU should be removed

with higher probability from a region where underutilized DUs exist (with significantly less than K demand points assigned).

- ***MOVE***: Move a DU from its location to a neighboring DU place, in geometric sense: the closest available DU location in the graph is selected. We note that this operation can be interpreted as a combination of an ADD and a DELETE operation of DUs.

The neighbor state is the result of a random choice among these operations. Combined with the demand point assignment process described at the state evaluation, adding a DU splits oversized groups, deleting a DU contracts undersized groups, while movement slightly rearranges the affected groups. At initial, high temperatures, multiple DUs may be involved in these operations, i.e. multiple DUs may be added, removed or moved in a single step, in order to speed up the convergence. This extension helps to reduce the effect of a possibly incorrect initial state, being far from the optimum.

The amount of necessary DUs in an optimal solution is typically close to $S/_K$, i.e. the population per DU capacity ratio. The diameter of the solution space is the maximum of necessary steps to move from one solution to another one. With the above described neighbor states, removing all DUs of solution Y_1, and adding all DUs of solution Y_2 requires approximately $2 \cdot S/_K$ steps, therefore the ***diameter*** of the solution space is proportional to that amount:

$$d \sim 2 \cdot \frac{S}{K}$$

In a typical PON network scenario, with a few thousand demand points ($S\sim 3.000$), and 1:64 splitting ratio ($K = 64$), diameter of the solution space is in the magnitude of 100 steps.

Temperature control

The temperature controls the probability of a “backward” step during the minimization process, which increases overall cost. It lies in the heart of Simulated Annealing, prevents it from being stuck in a local minimum. The schedule and rate of temperature reductions give a tradeoff between quality of the solution and scalability, i.e. the convergence speed. Typically processes with faster convergence may stop earlier, with higher probability in a local minimum, while slower processes have a higher probability to reach a global optimum (or at least a “better” local minimum). Figure 31 shows an example for this phenomenon (“Kecel” case study, VDSL network design).

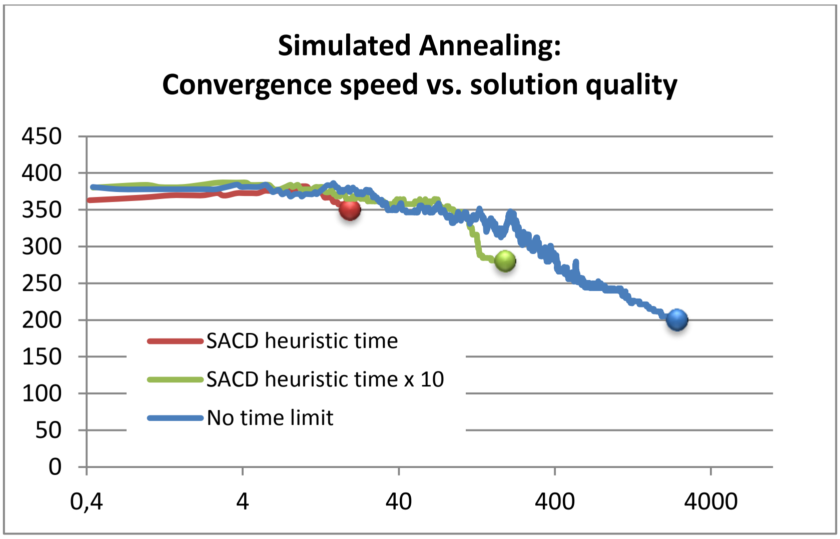


FIGURE 31 CONVERGENCE SPEED VS. OPTIMALITY

The temperature should obviously decrease monotonically. Here I have chosen an exponentially decreasing function from $T = 1\ 000$, multiplying it by an constant $\alpha < 1.0$ in every loop, which leads to a fast decrease in the beginning, and slower process at the end, around the presumed optimum (Figure 32).

Decision function

If the evaluated neighbor state (N_1) is better (has lower cost) than the actual state (N_0), the SA accepts it, and takes the step towards N_1. However, even if N_1 has higher cost (i.e. it is a *worse state*), a probabilistic decision is made, which allows move to N_1. This probabilistic decision is controlled by the temperature. For every decision, a random number (R) is chosen, and compared to a predefined decision function d of the temperature (T) and the difference between the cost of N_0 and N_1, i.e. $\Delta c = c(N_1) - c(N_1)$:

$$d(T, \Delta c) < R \ \Rightarrow accept\ even\ a\ worse\ state$$

Therefore, the temperature decreasing strategy, and also the decision function has to be determined.

Decision function d is monotonic in T: with decreasing temperature, the probability of a "backward" step is also decreased. When applying SA for the topology design problem, the following decision function was used:

$$e^{\frac{-\Delta c}{T \cdot \mu_{price}}} > R$$

Here μ_{price} is a scaling factor for compensating the difference between the price and temperature values, which can differ by several orders of magnitude.

Initial state generation

For any iterative heuristic approach, choosing the right initial state has significant impact on convergence speed and quality of results. In the case of the NTD problem, and the above discussed realization of SA, the DUs play central role. Namely, the amount and the location of distribution units must be determined at the initial state.

As a "naïve" approach, the initial DU allocation follows the distribution of demand points. An optimal DU allocation that allows maximal utilization of DUs requires $|S|/K$ distribution units; therefore the algorithm will start with 20% more, i.e. $1.2 \cdot |S|/K$ DUs, in order to allow a "random walk" in the solution space at the beginning.

For the initial allocation of these DUs, a weighted random selection is applied among the available DU locations (Ω). The weights of the respective available DU locations reflect the local demand point density, i.e. the amount of demand points within an r radius of the given DU location.

Termination

The iterative process needs an "exit condition", which stops the process. For this realization of SA, I have added two terminating conditions. The Simulated Annealing process stops:

- when T falls below 1.0
- if the cost remains unchanged for a sufficiently long period of time, i.e. no neighbor states are accepted for a number of iterations

PARAMETER SETUP

The decision function and the temperature control strategy together controls the "dynamics" of the SA process. Fine tuning its parameters has key importance. In the realization of SA which is described in this section, the parameters were set in a way that keeps the decision "alive" throughout the cooling process. On Figure 32 everything is put together: the temperature decrease, the probability of an ADD/DELETE and a MOVE state transition. These state transition probabilities have a "slow start" curve, which supports the initial steps across the solution space, in order to avoid local optima. The convergence speed is increased during the SA process. The ADD/DELETE transitions have a more pronounced effect on the topology, while a MOVE operation causes just moderate changes, therefore the latter has higher probability of acceptance: it is more like a fine-tuning step. Since an ADD/DELETE operation typically results in higher cost increase/decrease than just moving a DU, different μ_{price} scaling factors are assigned to them. The curves are controlled via the above described parameters, namely T, α and μ_{price}:

- with a lower initial T values, the "slow start" phase gets shorter, while higher initial temperature makes it longer
- the μ_{price} scaling factor affects the height of the curves: higher μ_{price} values result in higher acceptance probabilities
- the α temperature decrease coefficient affects the number of iterations: the closer it is to 1.0, the more iterations we get

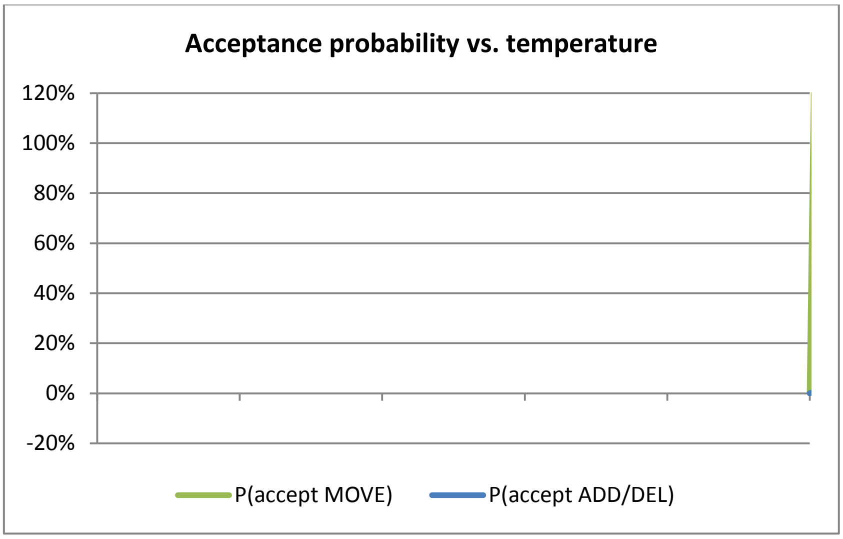


FIGURE 32 SIMULATED ANNEALING PROCESS DYNAMICS

In this subsection I have proposed a metaheuristic solution, which meets the requirements and specifications of the addressed NTD problem; the distinct "building blocks" were designed to provide a reasonable search for the optimum over the solution space. This way we got a universal heuristic approach, which looks for an optimal solution within the domain of valid topologies "independently": in contrary to the earlier presented, highly efficient heuristics, it is not intended to follow the network designers' way of thinking. For this reason, SA will be a valuable "benchmark" for evaluation, and a universal approach to solve any future NTD problems which do not fit any of the presented special cases.

7. Numerical results: Validation and Evaluation

In this section, the proposed methods and algorithms are evaluated thoroughly. In the first section, the algorithms are validated with problem instances which have a regular structure. Therefore, the optimum can be calculated analytically. Results of the heuristics are then compared to that optimum.

7.1. Validation: calculations on grid topology

For validation, a regular graph structure is needed which allows analytic calculation of the optimal solution. Therefore grid topologies were used, since these are able to model a regular street system. Figure 33 is an example grid topology. Such grid topologies will be defined by four parameters:

- N: amount of squares along one side of the grid, i.e. the grid has $N \times N$ squares, $(N-1) \times (N-1)$ street crossings
- L: length of the edges of each square, i.e. the complete grid is $L \times N$ wide
- k: the amount of demand points on a single side of an edge in the grid, with equal distance between their respective drop points and the street crossings
- d: length of the drop cable connecting the end points to the streets, i.e. edges of the grid

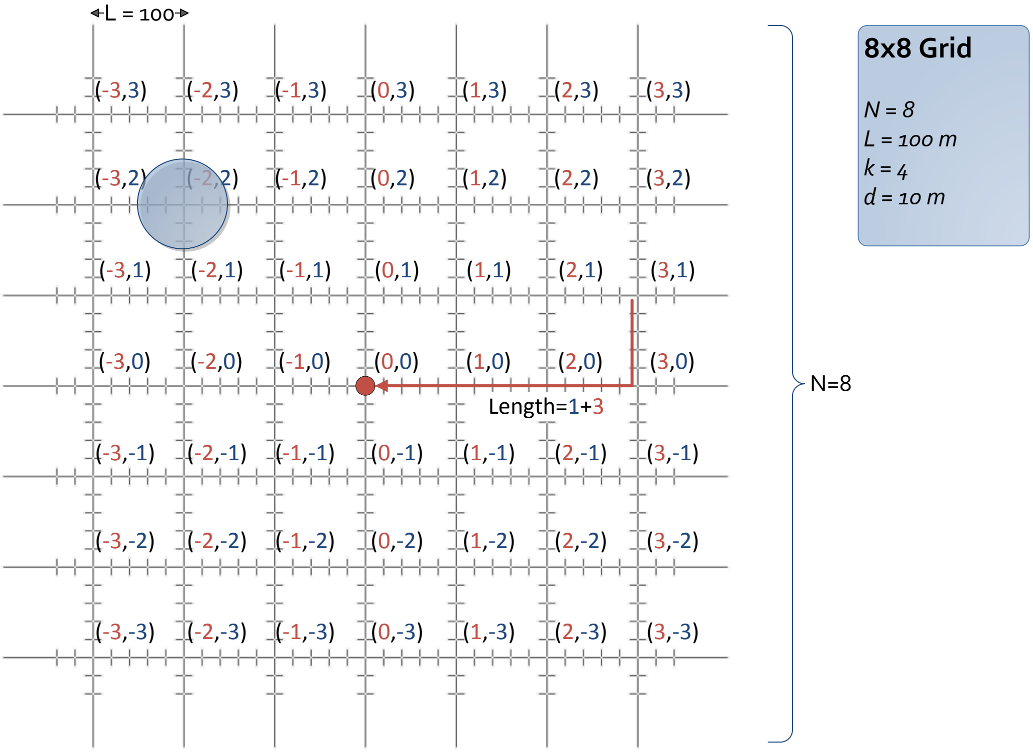


Figure 33 Grid topology (N=8, L=100, K=4, D=10)

The amount of demand points (S) is expressed with these parameters. The "neighboring zone" of a street crossing is shown on Figure 33 by the colored circle: these are the demand points which have the center of the circle as their closest street crossing. There are $2k$ demand points along a street segment (k on both sides), half of which belong to the same "neighboring zone". Summarizing the four directions, $4k$ demand points fall into every zone, and there are $(N-1)^2$ street crossings:

$$S = (N-1)^2 \cdot 4k \qquad \text{(A)}$$

7.1.1. OPTIMAL SOLUTION: ANALYTIC CALCULATIONS

Our intention is to define a scenario at which we can easily calculate the optimal solution. Assuming that the street crossings are the available DU locations (Ω), a proper setting of the DU capacity allows all demand points to be connected to their closest DU location (street crossing), minimizing the distribution fiber (DF) usage. The $4k$ demand points within the neighboring zone have to be served by one or multiple DUs, therefore the DU capacity has to be a divisor of $4k$:

$$K = \frac{4k}{\alpha} \qquad \text{(B)}$$

With this setup, in an optimal topology the DUs will be located at the street crossings, exactly α at each location. Components of the network deployment cost for the optimal topology will be calculated in the following paragraphs.

DISTRIBUTION UNITS

The minimal necessary amount of DU units is given by dividing the amount of demand points by the DU capacity, according to (A) and (B):

$$\#DUs = \frac{S}{K} = \frac{(N-1)^2 \cdot 4k}{\frac{4k}{\alpha}} = \alpha \cdot (N-1)^2 \qquad (1)$$

DISTRIBUTION FIBER (DF)

Figure 34 shows the "neighboring zone" of a DU location: the fiber usage connecting all these demand points to the DU location at the street crossing will be calculated now. Assuming k demand points on a single edge of the grid, the distance between neighboring drop points (indicated by s on Figure 34) is expressed by the parameters of the grid:

$$s = \frac{L}{k+1} \qquad \text{(C)}$$

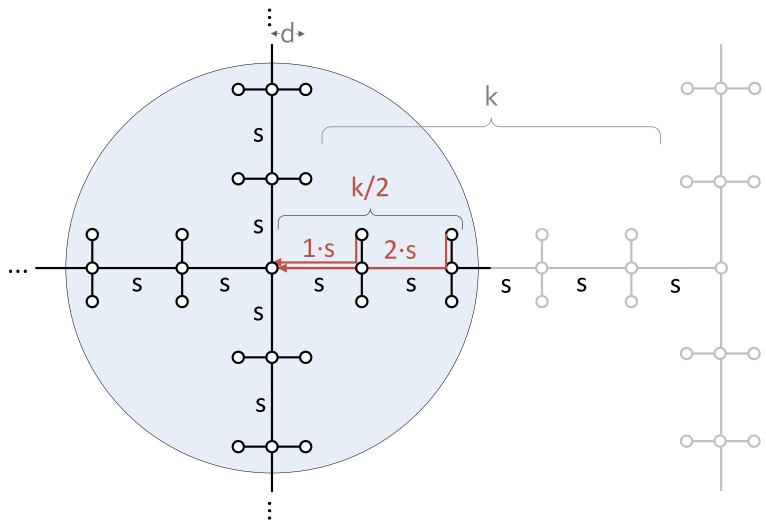


FIGURE 34 DISTRIBUTION FIBER IN THE GRID

Then it is easy to see that we have to connect $k/2$ demand points on each side of every edge adjacent to the street crossing. These drop points have $(s, 2 \cdot s, 3 \cdot s, \dots, \frac{k}{2} \cdot s)$ distance from the DU location. The distribution fiber for a single DU region is the sum of the four directions, and two sides per direction, therefore:

$$DF = 8 \cdot (s + 2s + 3s + \cdots + \frac{k}{2} \cdot s) \ = 8 \cdot \sum_{i=1}^{k/2} i \cdot s = 8 \cdot s \cdot \sum_{i=1}^{k/2} i = 8 \cdot s \cdot \frac{\frac{k}{2} \cdot \left(\frac{k}{2} + 1\right)}{2}$$
$$= 8 \cdot \frac{L}{k+1} \cdot \frac{\frac{k}{2} \cdot \frac{k+2}{2}}{2} = L \cdot \frac{k \cdot (k+2)}{k+1}$$

The drop fiber is ignored here, since the drop fiber usage is independent of the topology optimization; it would only add a constant d for all demand points.

In total, we are given $(N-1)^2$ street crossings, therefore the total distribution fiber length in the network with this setup is:

$$\sum DF = (N-1)^2 \cdot L \cdot \frac{k \cdot (k+2)}{k+1} \qquad (2)$$

FEEDER FIBER (FF)

In this regular setup, n DUs are located at every street crossing. The minimum necessary amount of feeder fiber will be then α times the sum of distances between DU locations and the CO, which should be in a central position (see Figure 33 again). The distance between every DU location and the CO is equal to L times the sum of the absolute value of its "coordinates", taking the CO as the origin $(0,0)$.

The x and y coordinates are then easy to read from the figure. In the middle line, the y coordinates are equally 0. In the first lines below and above, we have y coordinates equally 1, and so on – until the bottom and top of the grid, i.e. the $(\frac{N}{2} - 1)th$ lines, and in every line we have $N - 1$ points. If we sum up just the x coordinates at first, we get:

$$\Sigma^x = 2 \cdot (N-1) \cdot \left(1 + 2 + \cdots + \frac{N}{2} - 1\right) = 2 \cdot (N-1) \cdot \sum_{i=1}^{\frac{N}{2}-1} i$$
$$= 2 \cdot (N-1) \cdot \frac{\left(\frac{N}{2} - 1\right) \cdot \left[\left(\frac{N}{2} - 1\right) + 1\right]}{2} = 2 \cdot (N-1) \cdot \frac{\frac{N-2}{2} \cdot \frac{N}{2}}{2}$$
$$= \frac{N \cdot (N-1) \cdot (N-2)}{4}$$

Obviously $\Sigma^x = \Sigma^y$. If we have α DUs at each DU location, the feeder fiber usage is α times the sum of x and y coordinates, i. e. $\sum FF = \alpha \cdot (\Sigma^x + \Sigma^y) = 2\alpha \cdot \Sigma^x$:

$$\sum FF = \alpha \cdot \frac{N \cdot (N-1) \cdot (N-2)}{2} \qquad (3)$$

DEPLOYMENT COSTS

The minimal necessary cable deployment defines a subgraph, i.e. a subset of network links that preserves connectivity. It has to be a minimal set: removing any more links from the network makes it unconnected. A minimal, connected graph is a tree, by definition.

Clearly, the drop cables connecting the demand points to the street are necessary, otherwise the demand points would be disconnected from the network, i.e. there is no reason to minimize the drop cables. Therefore, here we will concentrate on links of the street system, in order to find a minimal connected set of them.

On every edge of the street system grid between two street crossings, at most one small segment s may be removed from the graph. Otherwise, there will be an endpoint between the two removed segments, being disconnected from both neighboring street crossings, therefore, unreachable from any DU locations (e.g. the four central demand points on Figure 35):

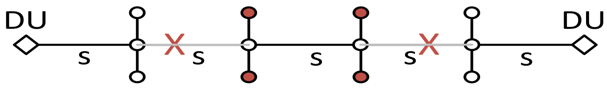


FIGURE 35 CONNECTIVITY OF DEMAND POINTS

The feeder network should connect all DUs to the CO, i.e. connectivity of the street system grid is necessary. Now if we take a look at the original graph (Figure 36), we can divide it in two different regions. The *external region* is on the boundary of the grid. Here, outside of the last demand point, $\frac{k}{2}+1$ small segments are unnecessary on every external edge (bold, red segments on the figure). In the *internal region*, the street system should remain connected. In an NxN grid we have $(N-1) \times (N-1)$ street crossings. A tree connecting $(N-1)^2$ nodes contains $(N-1)^2-1$ edges, therefore $(N-1)^2-1$ edges of the feeder network must remain intact. The remaining edges of the street system are not required for the feeder network, however due to the distribution network connectivity, not more than one segment s may be removed from each of them, as we have seen earlier.

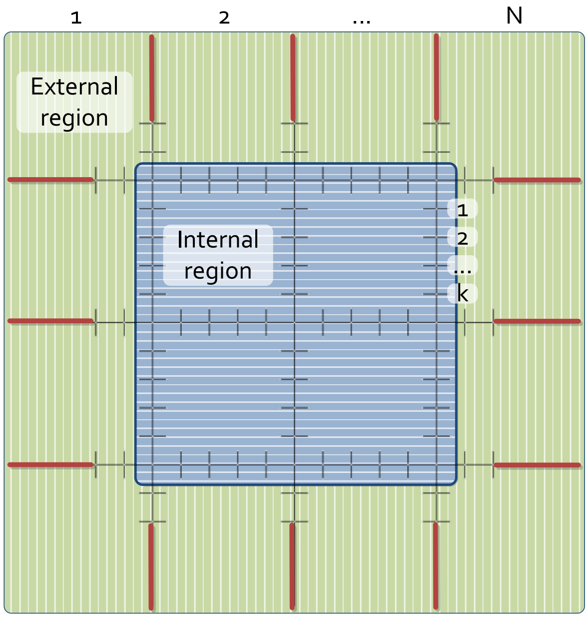


FIGURE 36 MINIMAL DEPLOYMENT CALCULATIONS

We get minimal network deployment (DP) by subtracting these two “savings” from the complete street system length, i.e. segments removed in the internal and external regions:

$$\sum DP = DP^{total} - DP^{saving}_{internal} - DP^{saving}_{external}$$

The complete street system length is easy to sum up: the graph contains $(N-1)$ “streets” both horizontally and vertically with length of $N \cdot L$:

$$DP^{total} = 2 \cdot L \cdot N \cdot (N-1)$$

The external region savings are the bold red segments on Figure 36. According to the above findings, there are $\frac{k}{2}+1$ unnecessary small segments at the “external end” of those $N-1$ street system edges in all four directions:

$$DP^{saving}_{external} = 4 \cdot (N-1) \cdot s \cdot \left(\frac{k}{2}+1\right) = 4 \cdot (N-1) \cdot \frac{L}{k+1} \cdot \frac{k+2}{2} = 2 \cdot L \cdot (N-1) \cdot \frac{k+2}{k+1}$$

In the internal region, there are in total $2 \cdot (N-1) \cdot (N-2)$ edges of the street system. In order to maintain connectivity of the feeder tree, $(N-1)^2 - 1$ of them have to remain intact (as it was explained above). On the rest of the street edges, one small segment s per each edge is unnecessary. Therefore, the internal region savings are:

$$DP^{saving}_{internal} = s \cdot \{[2 \cdot (N-1) \cdot (N-2)] - [(N-1)^2 - 1]\}$$
$$= s \cdot (2N^2 - 6N + 4 - N^2 + 2N - 1 + 1) = \frac{L}{k+1} \cdot (N^2 - 4N + 4)$$
$$= \frac{L \cdot (N-2)^2}{k+1}$$

In summary, the minimal cable deployment:

$$\sum DP = DP^{total} - DP^{saving}_{internal} - DP^{saving}_{external}$$
$$= 2 \cdot L \cdot N \cdot (N-1) - \frac{L \cdot (N-2)^2}{k+1} - 2 \cdot L \cdot (N-1) \cdot \frac{k+2}{k+1}$$
$$= L \cdot \left\{2 \cdot N \cdot (N-1) - \frac{(N-2)^2}{k+1} - 2 \cdot (N-1) \cdot \frac{k+2}{k+1}\right\}$$
$$= L \cdot \frac{2 \cdot N \cdot (N-1) \cdot (k+1) - (N-2)^2 - 2 \cdot (N-1) \cdot (k+2)}{k+1}$$
$$= \frac{L}{k+1} \cdot (2N^2k + 2N^2 - 2Nk - 2N - N^2 + 4N - 4 - 2Nk - 4N + 2k + 4)$$
$$= \frac{L}{k+1} \cdot (2N^2k + N^2 - 4Nk - 2N + 2k) = \frac{L}{k+1} \cdot (2N^2k + N^2 - 4Nk - 2N + 2k)$$
$$= \frac{L}{k+1} \cdot [(N^2 - 2N + 1) \cdot (2k+1) - 1]$$

Finally we get for the minimal deployment:

$$\sum DP = \frac{L}{k+1} \cdot [(N-1)^2 (2k+1) - 1] \qquad (4)$$

Equations (1)-(4) give the optimal value for the amount of DUs, feeder and distribution fiber usage, and cable deployment, respectively.

7.1.2. Example grids

A set of different regular grid structures were chosen for the validation of the heuristic algorithms. The main parameters of these grid networks are concluded in Table 5. The table also contains the calculated dimensions of the optimal network topology, according to the formulas of the previous section.

TABLE 5 GRID NETWORKS FOR VALIDATION

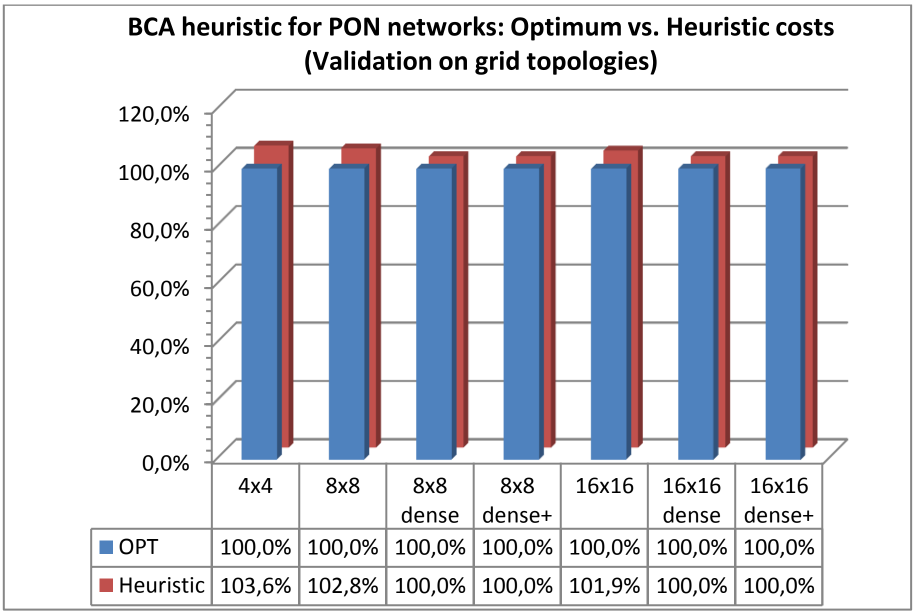
Parameters	Grid types						
Name	4x4	8x8	8x8 dense	8x8 dense+	16x16	16x16 dense	16x16 dense+
N	4	8	8	8	16	16	16
L	100	100	200	1000	100	200	1000
k	4	4	16	64	4	16	64
d	10	10	10	10	10	10	10
K	16	16	16	16	16	16	16
α	1	1	4	16	1	4	16
Demand points	144	784	3136	12544	3600	14400	57600
	Optimal solution						
Deployment	1 600	8 800	19 012	97 231	40 480	87 341	446 523
Feeder Fiber	1 200	16 800	134 400	2 688 000	168 000	1 344 000	26 880 000
Distribution Fiber	4 320	23 520	166 024	3 184 246	108 000	762 353	14 621 538
Distribution Units	9	49	196	784	225	900	3 600

7.1.3. Numerical results

Three completely different heuristic approaches were presented for three different types of Next Generation Access network technologies, namely Passive Optical Networks (PON), Active Optical Networks (AON) and Digital Subscriber Line (DSL) networks. First of all, we recall Section 5.2 about the specialization of the heuristics. For these three network types, the various cost factors are represented in the total cost with different weights, and the given physical constraints are also significantly different for each network type.

During the validation, these three heuristics, namely the Branch Contracting Algorithm (BCA), the Iterative Neighbor Contracting Algorithm (INCA) and the algorithm for Stepwise Allocation of Critical DUs (SACD) were applied for optimizing PON, AON and DSL network topologies on the grids, respectively. The calculated total costs are normalized: the analytically calculated optimum is used as 100.0%, the heuristic results are compared to it, showing the cost surplus, i.e. the difference from the optimum.

Starting with PON networks, Figure 37 shows a comparison of total cost: for the “densely populated” grids the heuristic has almost optimal results, but even the “worst case” 4x4 grid leads to only 3,6% higher cost than the optimum, which is surprisingly good from a really fast heuristic algorithm.



	4x4	8x8	8x8 dense	8x8 dense+	16x16	16x16 dense	16x16 dense+
OPT	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%
Heuristic	103,6%	102,8%	100,0%	100,0%	101,9%	100,0%	100,0%

FIGURE 37 VALIDATION OF THE BCA HEURISTIC ON GRIDS: TOTAL COST

The total cost figures may hide the details, therefore on the following figures we look behind the curtain. Figure 38 shows the weight (contribution) of distinct cost factors to the total cost: the fiber costs are clearly dominated by the build and equipment costs. Figure 39 shows a comparison of distinct cost components of the heuristic solution to that of the optimal topology.

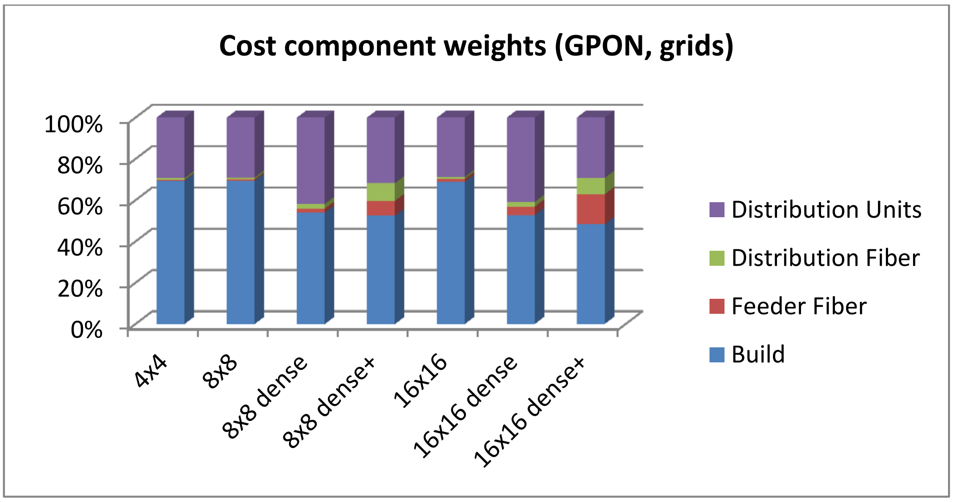
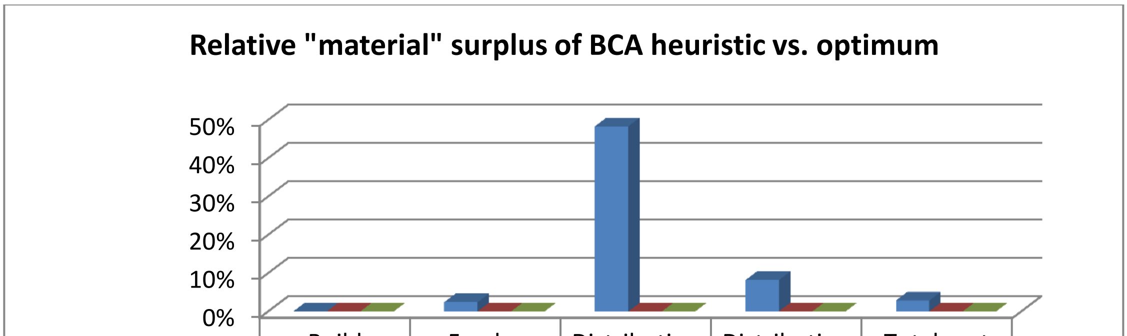


FIGURE 38 COST COMPONENT WEIGHTS (GPON ON GRIDS)

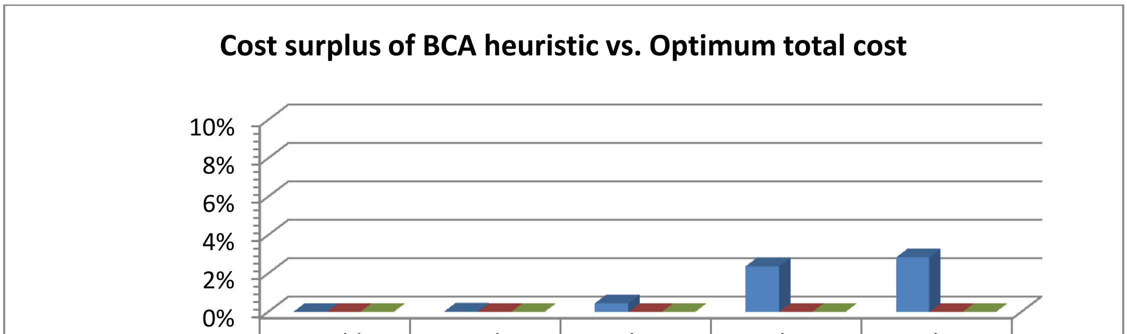
On Figure 40, these differences are weighted by their contribution to the total cost. We can observe that in the lowest density 8x8 grid, there is a somewhat significant difference in distribution fiber costs, while at the two other 8x8 grids with higher density of demand points, the distribution network is optimal (and we have observed the same phenomenon with 16x16 grids).

The reason is a tricky artifact or "side-effect" of the highly regular grid structure (Figure 41): if the heuristic creates a group which is not identical to the optimal "neighboring zone" (see Section 7.1.1), it automatically creates a malformed neighboring group. An example is the bottom-right group on the figure: four of its demand points should have been connected to its left neighbor street crossing. Because of these four points, the neighboring group will be similarly malformed, and this irregularity propagates from group to group, all across the gird. Fortunately, with real-life scenarios, or even with more demand points and DUS within the same “neighboring zone”, this side-effect becomes negligible (see “8x8 dense”, and “8x8 dense+” grid results on Figure 39).



	Build	Feeder Fiber	Distribution Fiber	Distribution Units	Total cost
8x8	0,0%	2,4%	48,3%	8,2%	2,8%
8x8 dense	0,0%	0,0%	0,0%	0,0%	0,0%
8x8 dense+	0,0%	0,0%	0,0%	0,0%	0,0%

FIGURE 39 PON HEURISTIC VS. OPT – DETAILS I.



	Build	Feeder Fiber	Distribution Fiber	Distribution Units	Total cost
8x8	0,0%	0,0%	0,4%	2,4%	2,8%
8x8 dense	0,0%	0,0%	0,0%	0,0%	0,0%
8x8 dense+	0,0%	0,0%	0,0%	0,0%	0,0%

FIGURE 40 PON HEURISTIC VS. OPT – DETAILS II.

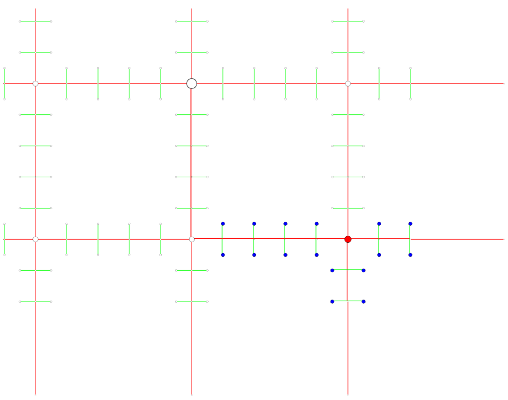


FIGURE 41 GRID STRUCTURE SIDE-EFFECT

If we investigate the INCA heuristic for AON (Figure 42 & Figure 43) and SACD heuristic for DSL networks, we get even better results: the heuristics produce network topologies almost identical to the optimal solution, both in terms of total cost and every component of the total cost.

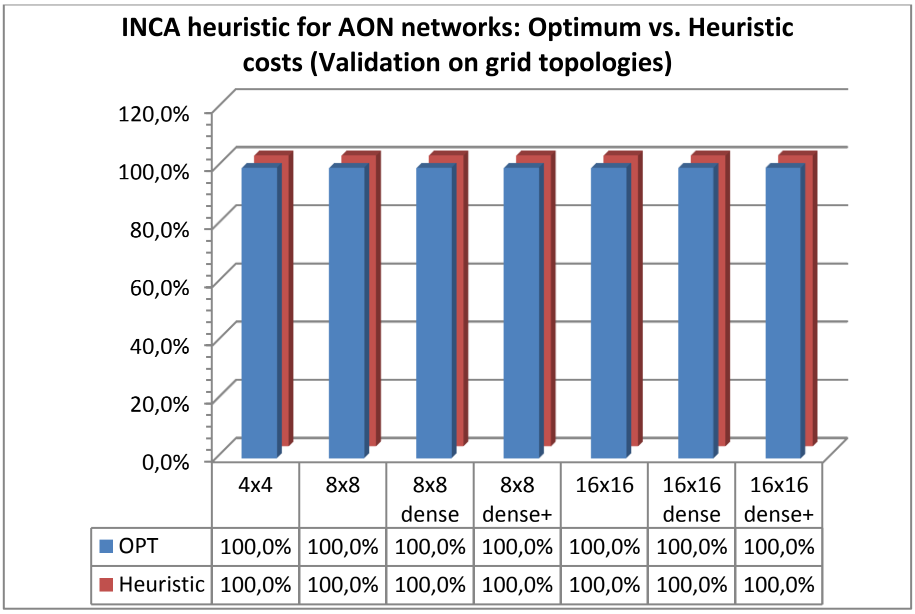


FIGURE 42 VALIDATION OF THE INCA HEURISTIC ON GRIDS

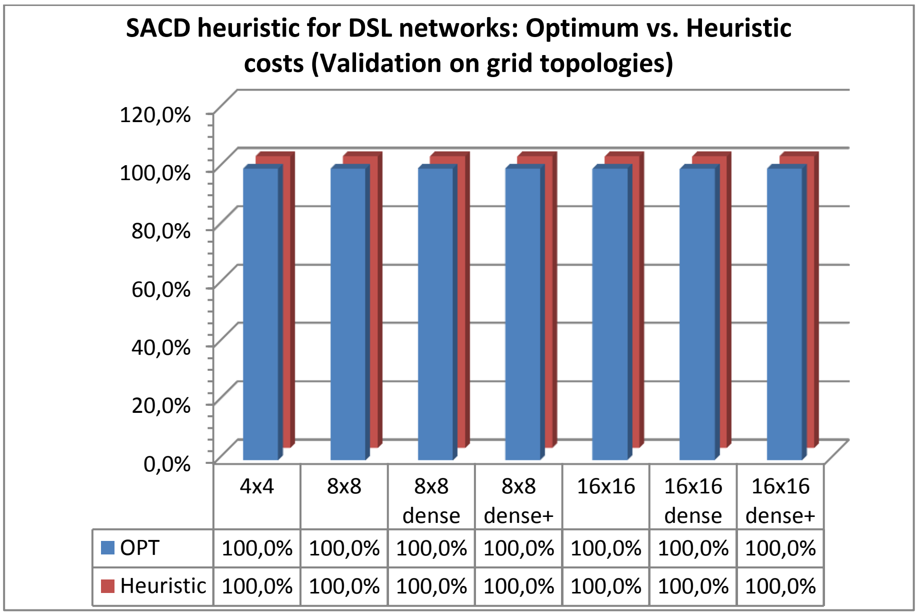


FIGURE 43 VALIDATION OF THE SACD HEURISTIC ON GRIDS

As a conclusion, on these regular grid structures the proposed heuristics performed well, having a very moderate or even no difference from the optimum at all. In the next section, the heuristics will be evaluated on large scale real world scenarios.

7.2. EVALUATION

The proposed heuristic algorithms will be evaluated against realistic scenarios in terms of approximation performance and scalability. Now we are not facing artificial problems with regular structures; hence we immediately lose the ability to calculate the optimum. At this point, we turn to the reference methods presented in Section 6. Here we recall the figure from Section 6:

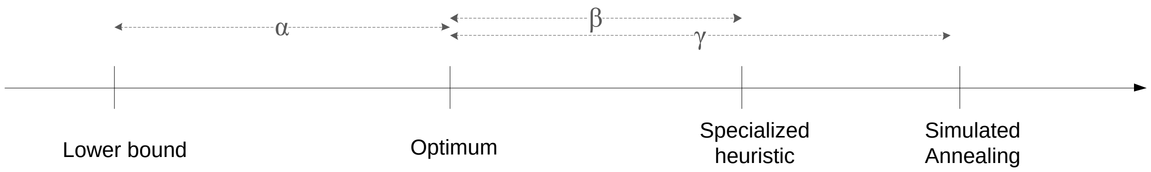


FIGURE 44 REFERENCE POINTS FOR EVALUATION

The optimal solution is the minimal cost network that serves all the demand points with the given constraints. The Integer Programming solution provides a lower bound on it, with a cost lower by α than the optimum (see Section 6.1.3 for details). The Simulated Annealing provides an approximation of the optimum, with γ distance from it, while the proposed, specialized heuristics will approximate the optimum by β. What makes evaluation difficult is the fact that we do not know the optimum itself. Therefore, we can only prove that the heuristic solution does not exceed the optimum higher than $\alpha + \beta$ – this will be the way to evaluate accuracy of the heuristics.

The case studies will be presented in Section 7.2.1. Section 7.2.2 presents the evaluation of "solution quality", i.e. the ability of the heuristics to approximate the optimum. Section 7.2.3 is devoted to scalability aspects: resource requirements, namely running time and memory consumption of the presented methods.

7.2.1. CASE STUDIES

A set of reference areas were chosen for evaluation purposes; these will be presented in the first subsection. The proposed methodology for topology design of Next Generation Access (NGA) networks has four main input data: (1) the map, i.e. graph of the street system, (2) location of demand points, i.e. buildings and demand points, and (3) existing infrastructure information defines the reference area itself, while (4) the cost database and the physical constraints derived from technology specifications complete the scenario (Figure 45).

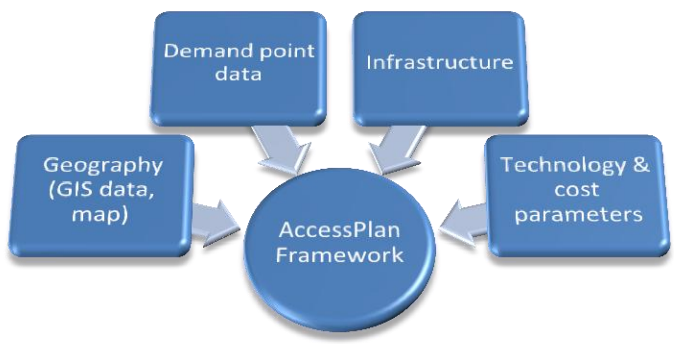


FIGURE 45 INPUT DATA FOR A CASE STUDY

For the presented case studies, the digital maps were derived from the publicly available OpenStreetMap database [81], and the demand point data from the Hungarian Statistical Institute. Table 6 concludes the main geographic characteristics of the chosen scenarios. The topology design methodology is part of our AccessPlan Framework (see later), which applies a detailed and well-structured cost database. However, the cost data is confidential information, therefore not published in details, but we can reveal the "aggregate" cost parameters used for the topology optimization process (Table 7). The technology specifications used throughout the case study evaluations are based on international standards for Gigabit Passive Optical Network (GPON, [9]), Active Ethernet (AETH, [15]) and Very High Bitrate DSL (VDSL, [18]) access networks (Table 8). The infrastructure information and the cost databases for the case studies are from joint projects with industrial partners, therefore these are not publicly available.

TABLE 6 SCENARIOS / CASE STUDIES

SCENARIO NAME	KŐSZEG	KECEL	SOLYMÁR	SASHEGY	ÚJPALOTA
Type	Small city central	Countryside town	Agglo-meration	Suburban	Urban
Area (km2)	0,5	7,21	4,42	5,89	1,18
Buildings	367	3 071	2 134	1 080	1 080
Demand points	367	3 165	2 714	4 366	6 994
Avg. demand points / building	1,00	1,03	1,27	4,04	6,48
Buildings / km2	730	430	480	180	920
Demand points / km2	730	440	610	740	5 930
Graph nodes	1 075	6 712	4 783	11 205	2 265
Graph edges	1 104	6 850	4 846	8 040	2 319
Street system length	18 040	97 858	55 703	152 556	26 660

TABLE 7 COST PARAMETERS

Access Network Technology	Distribution Unit Cost	Cable plant costs (C_0: cable deployment, C_v: fiber costs) [HUF/m]								
		New trenching		Re-use of existing substructure		New aerial cable		Re-use of existing aerial cabling		Re-use copper cable
		C_0	C_v	C_0	C_v	C_0	C_v	C_0	C_v	C_v
GPON	150 000	8000	10	600	10	4000	10	800	10	-
AETH	4 000 000	8000	10	600	10	4000	10	800	10	-
VDSL	3 000 000	8000	10	600	10	4000	10	800	10	5

TABLE 8 TECHNOLOGY SPECIFICATIONS AND PHYISICAL CONSTRAINTS

Network technology	Network range (segments)		Distribution unit capacity
	Feeder	Distribution	
GPON	$L_{max} = 20\ 000\ m$		$K = 64$
AETH	$L_{max}^{feed} = 20\ 000\ m$	$L_{max}^{dist} = 3\ 000\ m$	$K = 300$
VDSL	$L_{max}^{feed} = 20\ 000\ m$	$L_{max}^{dist} = 300\ m$	$K = 96$

The first scenario is the central area of a countryside city (“Kőszeg”), which is the smallest case study, used mainly for test and visual verification, assigning exactly one demand point per building (i.e. FTTB architecture). The second case study is a complete countryside town (“Kecel”), mainly with single family houses (i.e. one demand point per building). The third area (“Solymár”) is located in the agglomeration of Budapest, the capital of Hungary. It has slightly different settlement structure, but the majority of the buildings is still owned by a single family. The last two scenarios are from Budapest: one “suburban” region, with a mix of different building types, containing also “empty” areas of a cemetery and a small nature reservation area, therefore it has a low population density in average (“Sashegy”). The last area is from the densely populated urban region of Budapest, typically with large apartment houses (“Újpalota”): this one has the highest number of demand points among the five different reference areas.

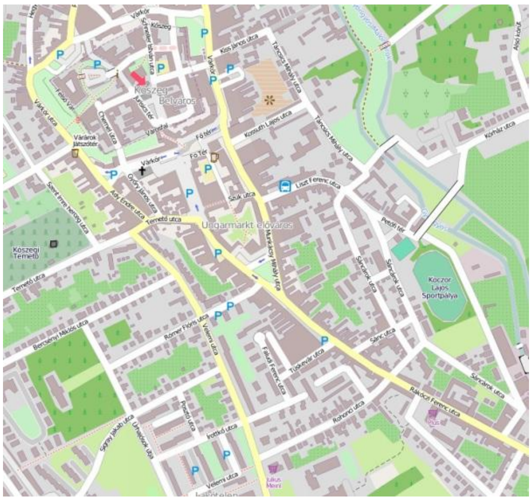


FIGURE 46 KŐSZEG MAP

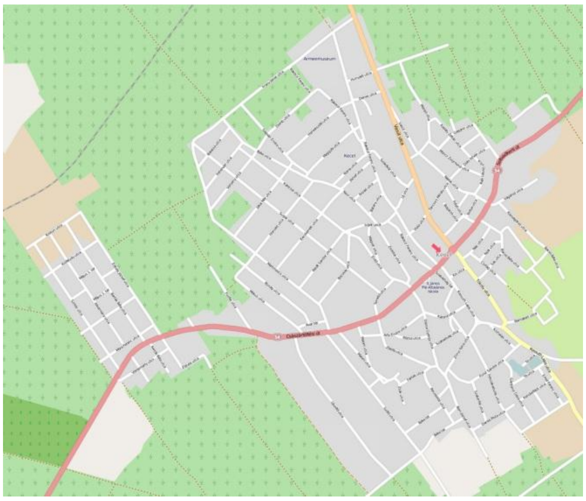


FIGURE 47 KECEL MAP

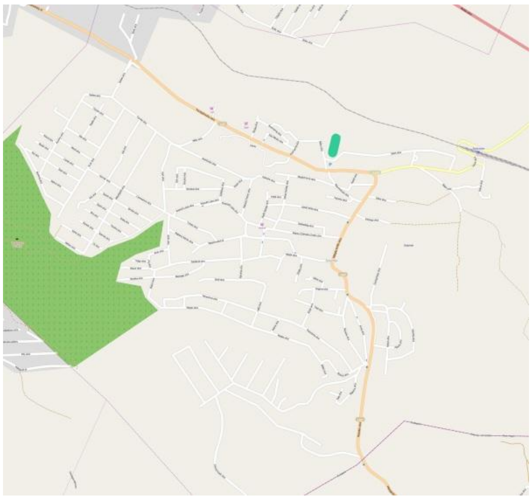


FIGURE 48 SOLYMÁR MAP

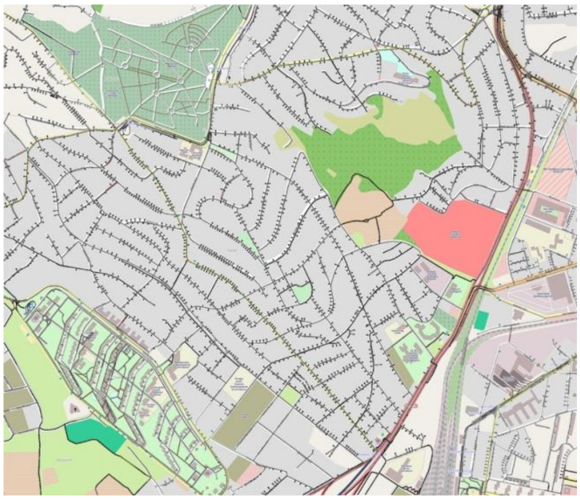


FIGURE 49 SASHEGY MAP

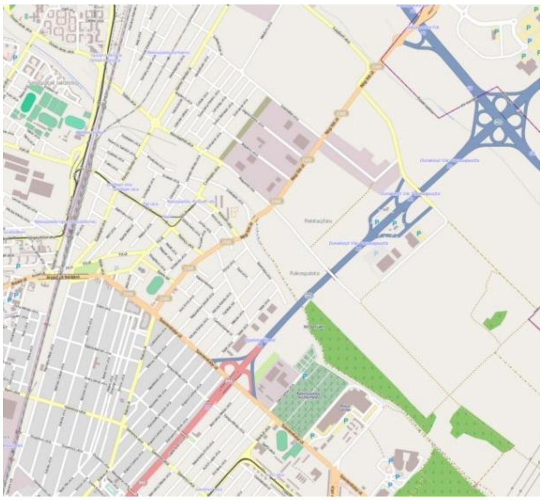


FIGURE 50 ÚJPALOTA MAP

7.2.2. Approximation performance

This section is devoted to evaluation of the solution quality, i.e. ability of the presented methods to minimize cost function of the NGA Topology Design (NTD) problem. Once we leave the “lab conditions”, evaluation and interpretation of the results becomes complex and sometimes not easy to understand. In this subsection, the proposed highly efficient heuristics presented in Section 5 are compared to the Mixed Integer Programming (MIP, Section 6.1) lower bound, in order to evaluate the quality of approximation, i.e. the gap between the heuristic solution and the optimum.

7.2.2.1. Mixed Integer programming (MIP) results

Results of the MIP formulation need some little explanation. Any MIP solver, during its operation, maintains an upper and a lower bound. In a cost minimization problem, the upper bound is the best known feasible, integral solution. The assumed optimal solution is somewhere between the upper and lower bounds, therefore the MIP solver tries to converge them. For large-scale, highly complex problems, the resource constraints may not allow the solver to achieve the exact optimal solution, therefore it returns the best known optimal solution, and the gap between these bounds, which adds some kind of uncertainty to the achieved results.

The MIP solver was used with settings that enforce faster upper bound convergence, i.e. finding the best known feasible integral solution. Two examples are shown here: Figure 51 presents an MIP solver process, in which the lower and upper bounds converge in approximately 15 minutes, therefore the MIP problem was solved successfully. The difference between upper and lower bound convergence speed may be observed also: the upper bound quickly approaches the optimal solution (in approx. 100 seconds), while the lower bound increases slower. Figure 52 shows a more complex optimization process: after 72 hours of computation, there is still more than 20% gap between the upper and lower bounds, even if the best known integral solution was found in the very beginning of the process.

This phenomenon was observed for all of the presented MIP results: whenever the calculations were terminated due to time or memory constraints, the upper bounds had been unchanged for a long time, i.e. the best known feasible solution was found quickly in the beginning of the process, and then the solver spent most of the time with the lower bound improvement.

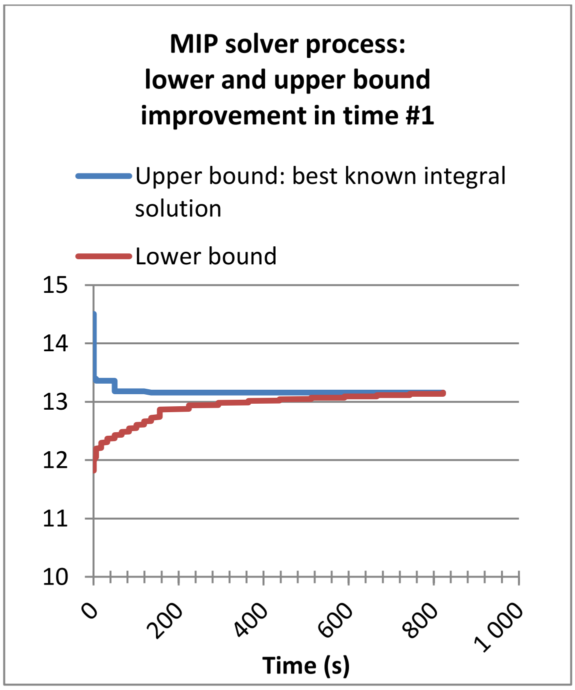


FIGURE 51 MIP LOWER AND UPPER BOUND DYNAMICS #1

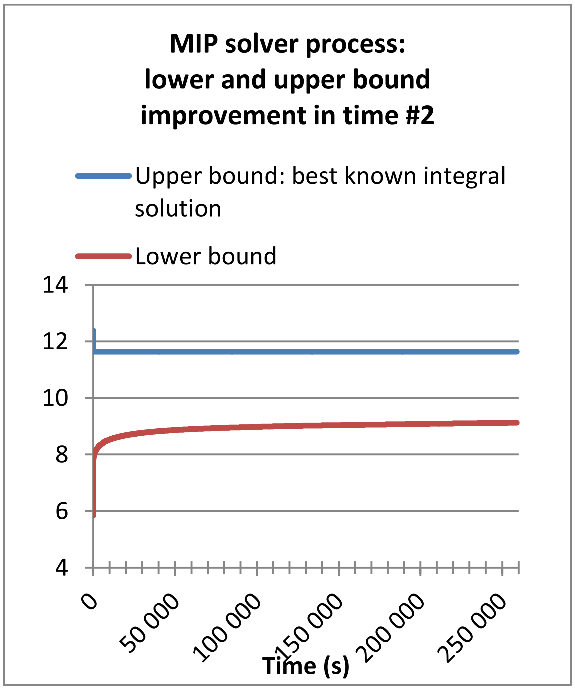


FIGURE 52 MIP LOWER AND UPPER BOUND DYNAMICS #2

The MIP results were carried out by using the most powerful MIP solver on the market: ILOG CPLEX, now developed by IBM (version 12.2). Specifications of the computer running the calculations are summarized in Table 9 – we just have to emphasize that CPLEX 12 supports parallelization, therefore all CPU cores were used during the calculations.

In the following subsections, all heuristics will be compared to MIP results. Which MIP results cannot be treated as optimal results for the original topology design problem, but they give a lower bound of it. Due to the simplified, relaxed reformulation of the complex quadratic problem, the MIP formulation may lead to even lower cost results, since the length constraints are relaxed. Therefore the distance between heuristic and MIP results is an upper bound on the distance between heuristic results and the optimum.

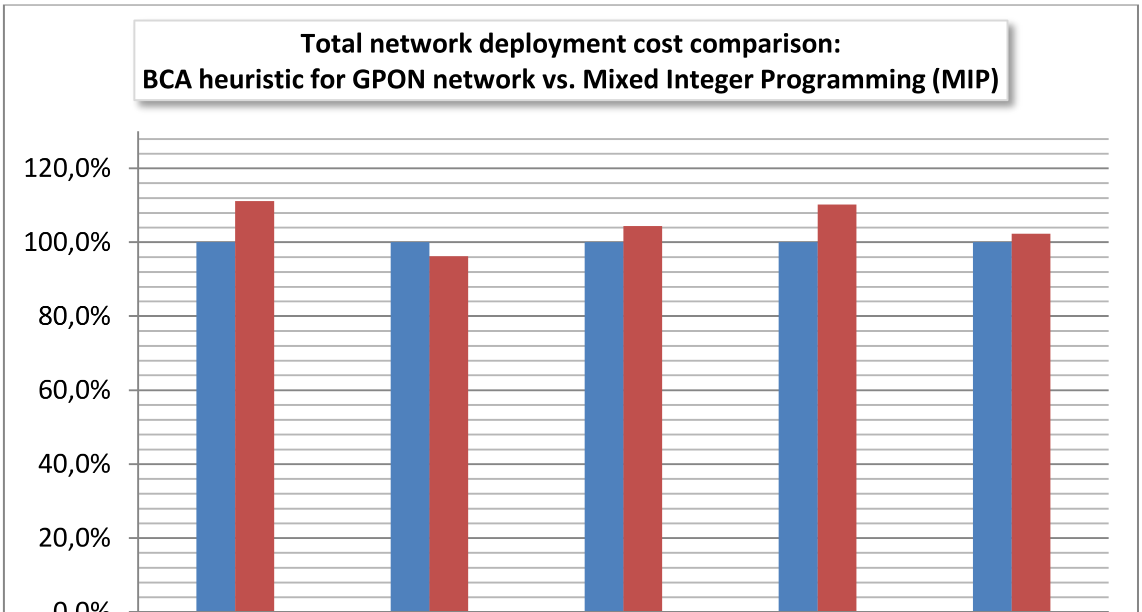
TABLE 9 CPLEX & COMPUTER SPECIFICATIONS

CPU	Intel Core i5 2500K
CPU speed	3.3 GHz
CPU cores	4
RAM	8 GB
CPLEX version	12.2
CPLEX search tree size	500 GB

7.2.2.2. BCA HEURISTIC FOR PASSIVE OPTICAL NETWORKS

The total cost of the network deployment is the first and foremost aspect of evaluation and comparison. On Figure 53, the BCA heuristic results and the MIP lower bounds are depicted as a pair of columns for all presented case studies. The results are normalized such that MIP lower bound is 100% in any case. The results show that the Branch Contracting Algorithm (BCA heuristic, Section 5.3) provides an approximation of the MIP solution within 11.2% for all scenarios, and since the MIP solution is not the optimum but a lower bound, BCA may approximate the optimum even more.

The second pair of columns requires further explanation: in the “Kecel” scenario, the BCA heuristic outperforms MIP, which is normally not possible. The reason is the extreme complexity of even a simplified MIP formulation, and “Kecel” is the scenario with the highest number of buildings, i.e. demand point locations. The MIP solver has found a valid solution after 3 minutes, while it had 60% relative gap. After 24 hours of parallel calculations on four CPU cores this gap was still 57%. In the meantime, BCA has finished in 34 seconds on one CPU core, and provided better results, falling “within the gap” between upper and lower bounds of MIP. This notably scalability gain of BCA heuristic explains the results for the “Kecel” scenario.



	Kőszeg	Kecel	Solymár	Sashegy	Újpalota
MIP	100,0%	100,0%	100,0%	100,0%	100,0%
BCA	111,2%	96,2%	104,5%	110,2%	102,3%

FIGURE 53 TOTAL COST OF GPON NETWORKS: BCA HEURISTIC VS. MIP LOWER BOUND

The aggregated total cost may hide differences in various components of the cost function, especially for components with lower weight. Which is in fact correct: the BCA algorithm was designed in a way that it preferably minimizes cost factors with higher weight. The contribution of the various factors to the total cost in a typical PON scenario is drawn on Figure 54. The distribution unit and distribution fiber costs are more or less balanced, both play an important role – this differentiates between PON networks, and AON/DSL networks. Build costs have the highest contribution: this is typical for an access network deployment. On the other hand, build costs are less dependent on network design than fiber or DU costs, since build costs are more predictable: optical cables will be deployed along all the streets where demand points exist. The amount of DUs and optical fiber is more dependent on the structure of network, i.e. clustering of demand point groups. Therefore on Figure 55, the more variant cost factors are depcicted, i.e. the equipment and fiber cost for the five different scenarios.

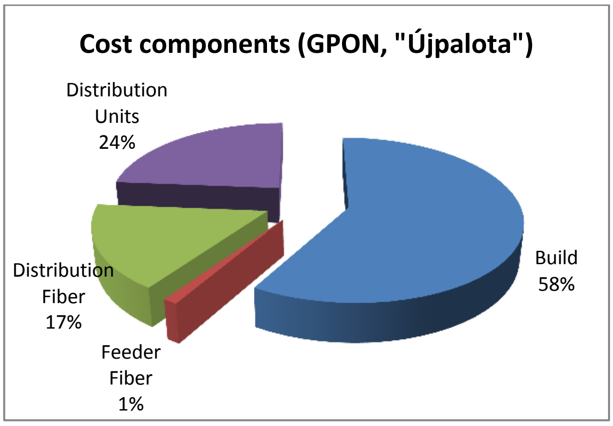


FIGURE 54 GPON COST COMPONENTS I.

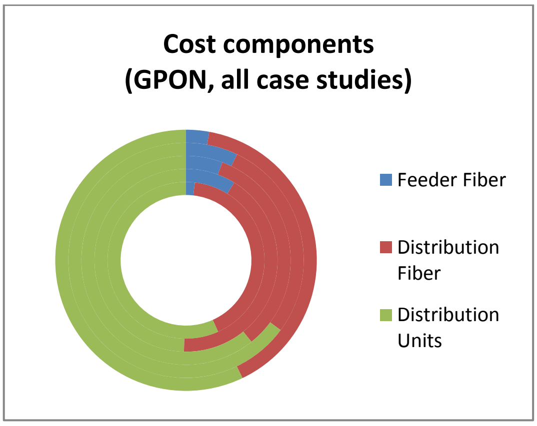


FIGURE 55 GPON COST COMPONENTS II.

Figure 56 and the following figures depict an in-depth comparison of the results, showing only a moderate difference between various components for MIP and BCA. It is not surprising: according to the cost breakdown figures, both fiber and equipment costs have a non-negligible weight, and it does not allow a significant sub-optimality in any cost component, i.e. none of them could be preferred over the other. As we will see, it makes the difference between PON, and the following AON/DSL networks. Concluding these results, the BCA heuristic has proven its ability to approximate the optimum within a reasonable 10% range, both in terms of total cost and significant cost factors.

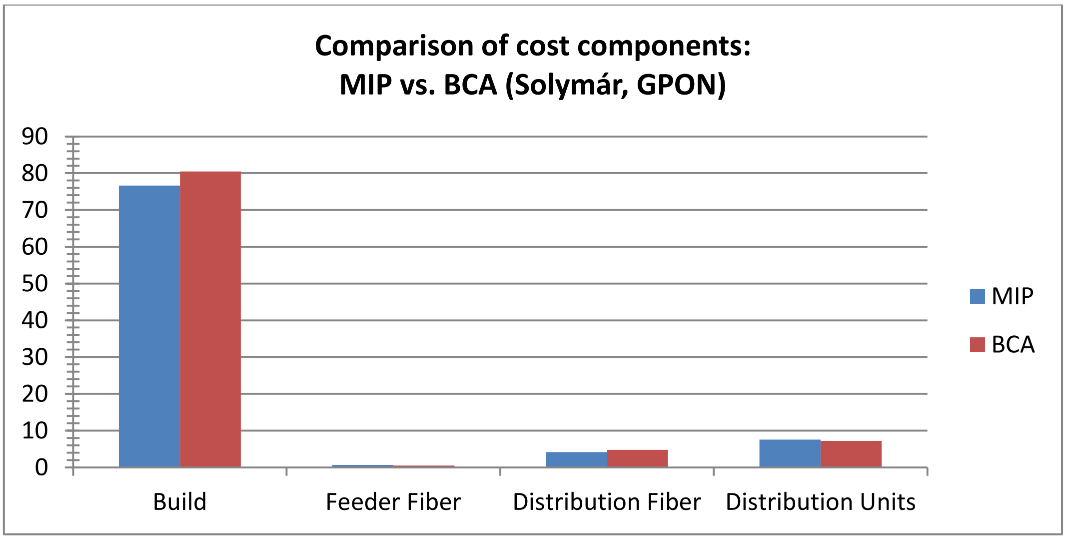


FIGURE 56 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. BCA HEURISTIC I. FOR GPON NETWORKS (SOLYMÁR)

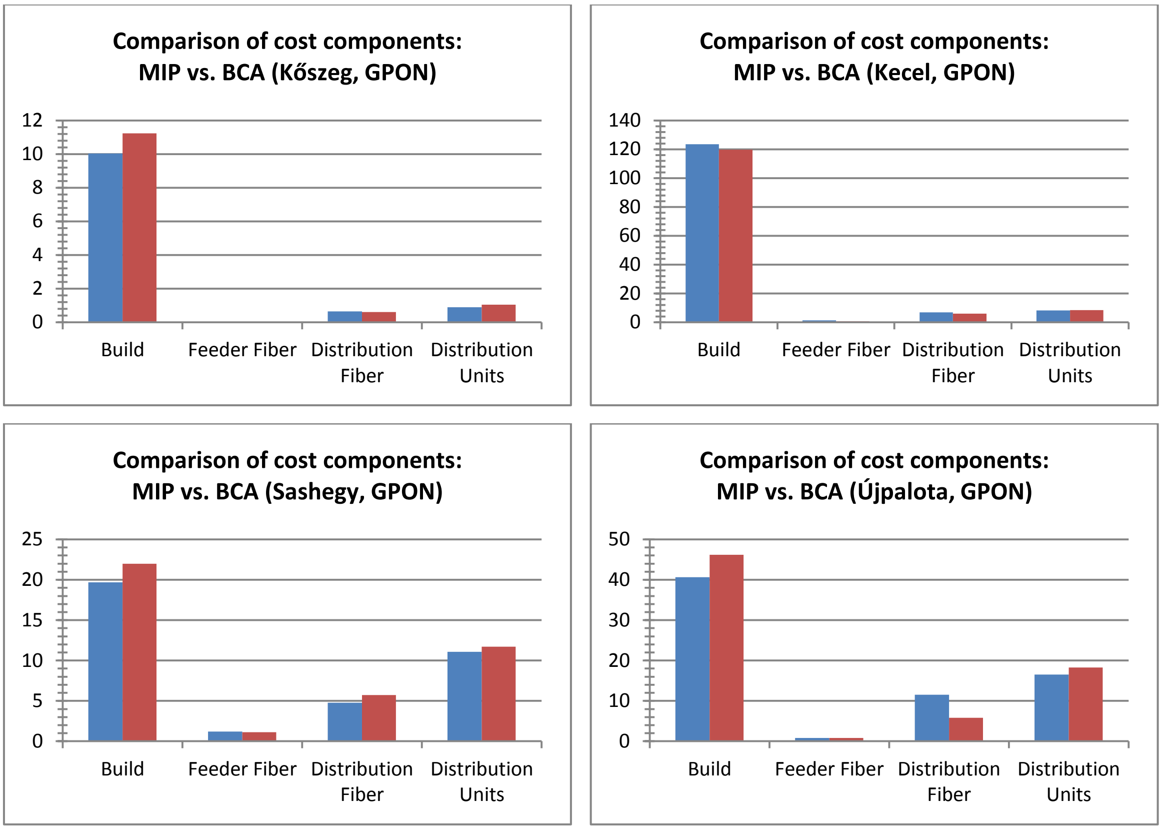
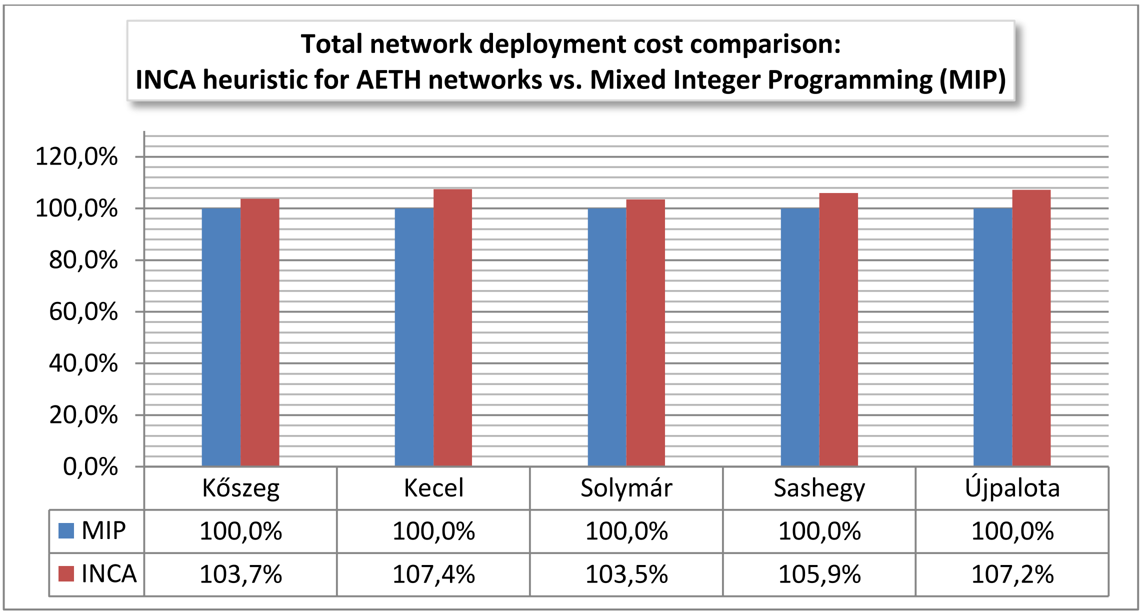


FIGURE 57 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. BCA HEURISTIC FOR GPON NETWORKS II.

7.2.2.3. INCA HEURISTIC FOR ACTIVE OPTICAL NETWORKS

The Iterative Neighbor Contracting Algorithm (INCA, Section 5.4) is compared to the MIP results, as we did for BCA heuristic earlier. The total cost comparison is depicted on Figure 58. The difference between the INCA heuristic results and the MIP lower bound values are between 3.5% - 7.4%, while it may be even closer to the exact optimum. It opens promising possibilities for practical applications.



	Kőszeg	Kecel	Solymár	Sashegy	Újpalota
MIP	100,0%	100,0%	100,0%	100,0%	100,0%
INCA	103,7%	107,4%	103,5%	105,9%	107,2%

FIGURE 58 TOTAL COST OF AETH NETWORKS: INCA HEURISTIC VS. MIP

Besides total cost comparison, the different cost components were also investigated: the total cost summary may hide interesting details. First of all we see the weight of various cost factors on Figure 59. As expected, lower population density increases the weight of fiber plant costs, while in urban and suburban scenarios ("Sashegy" and "Újpalota", the last two columns) equipment costs are more significant. Within fiber plant costs, build costs are dominating over fiber costs, especially over the feeder fiber costs, which are almost negligible.

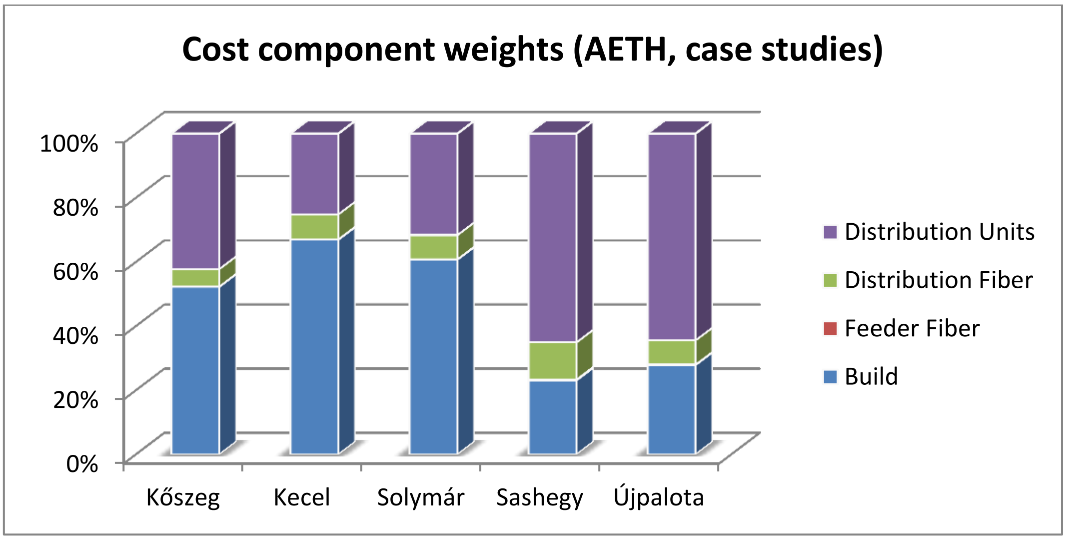


FIGURE 59 COST COMPONENT WEIGHTS FOR AETH NETWORKS

Figure 60 and Figure 61 shows cost component comparison of INCA and MIP. Fiber costs are dominated by equipment and build costs, in these two cost factors a significant difference has visible effect on the total cost. INCA closely approximates MIP for both of these cost components (and also the fiber costs), which leads to high quality approximation of the total cost as well. We have to note that INCA performs well both with lower and higher population density scenarios, i.e. for Solymár/Kecel where build costs dominate over DU costs, and Sashegy/Újpalota, where DU costs are more emphasized. The adaptation to different scenarios is an attractive ability of the INCA heuristic: such a multi-faceted optimization algorithm should not only focus on a single cost component (e.g. build costs or DU costs), but also on an arbitrary weighted combination of these.

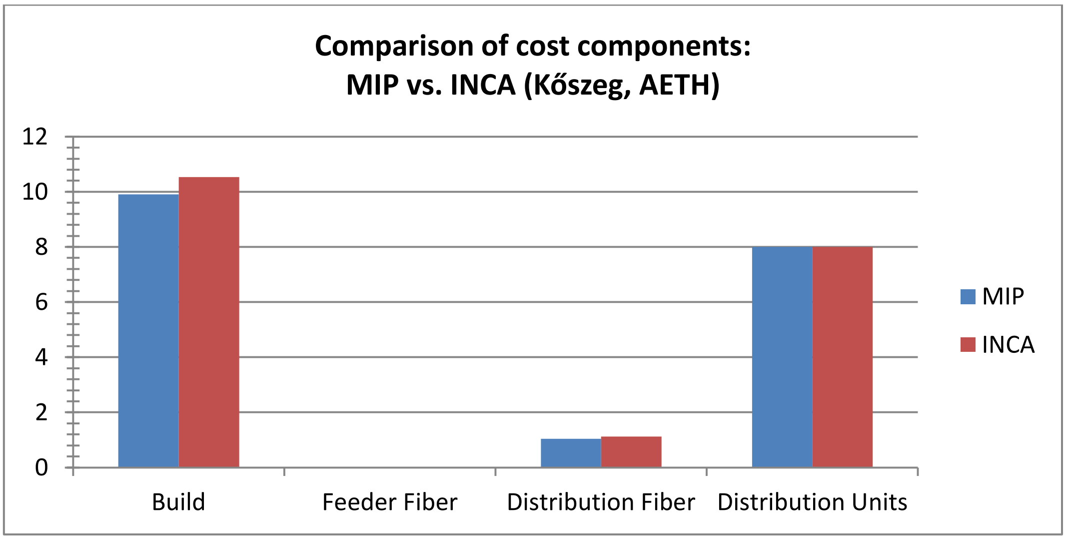


FIGURE 60 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. INCA HEURISTIC FOR AETH NETWORKS I. (KŐSZEG)

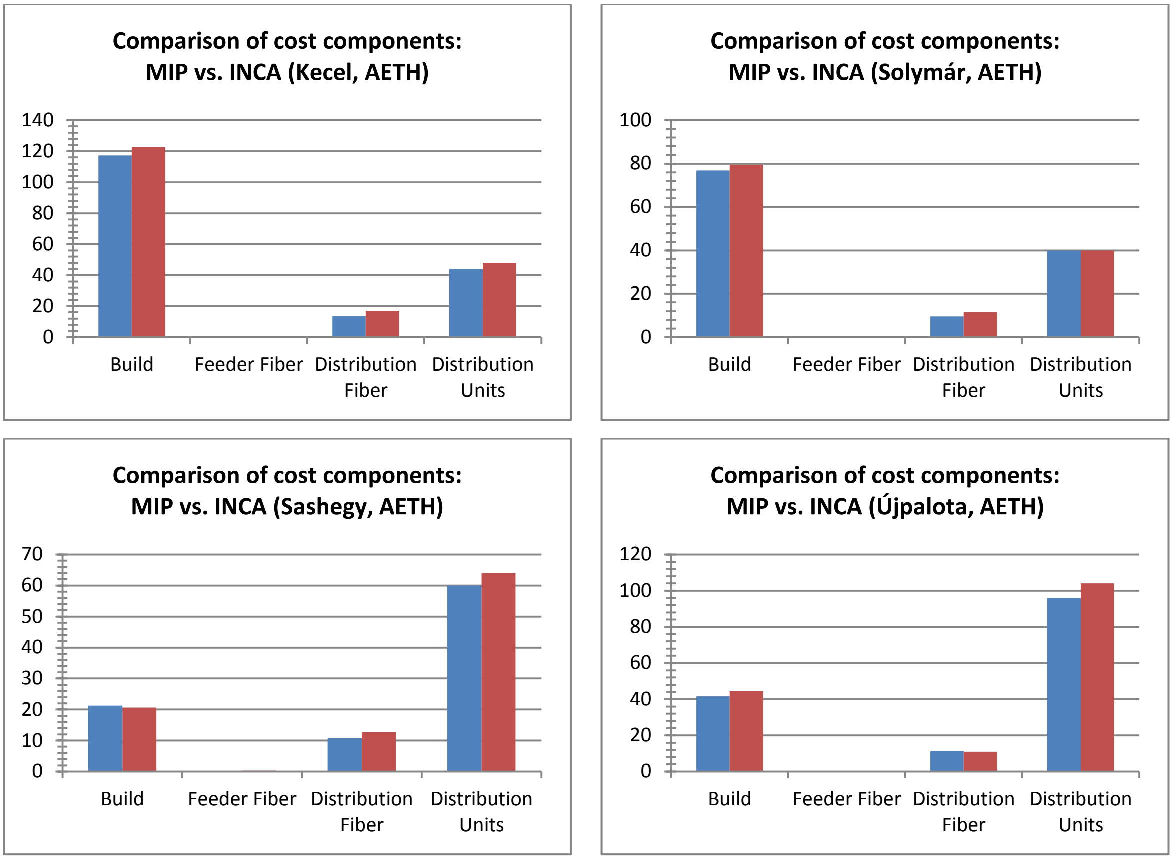
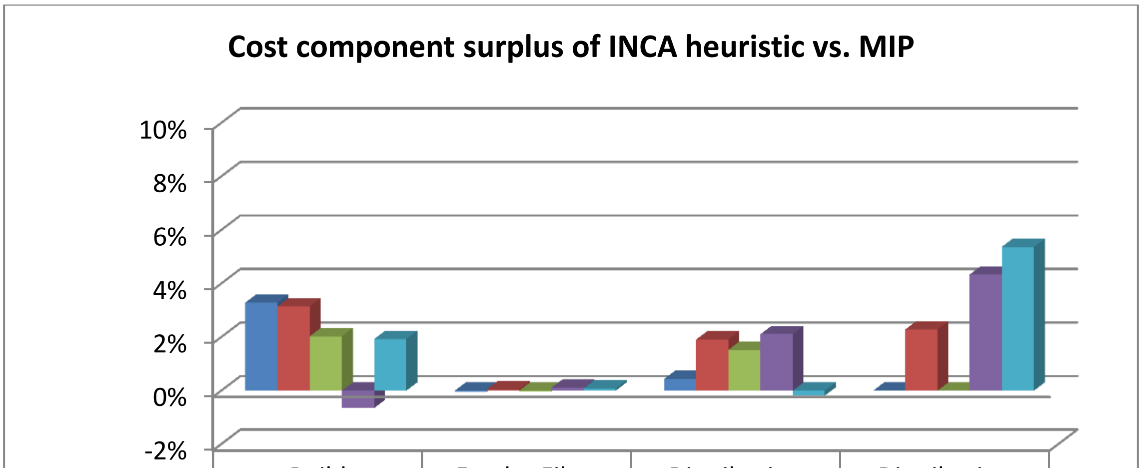


FIGURE 61 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. INCA HEURISTIC FOR AETH NETWORKS II.



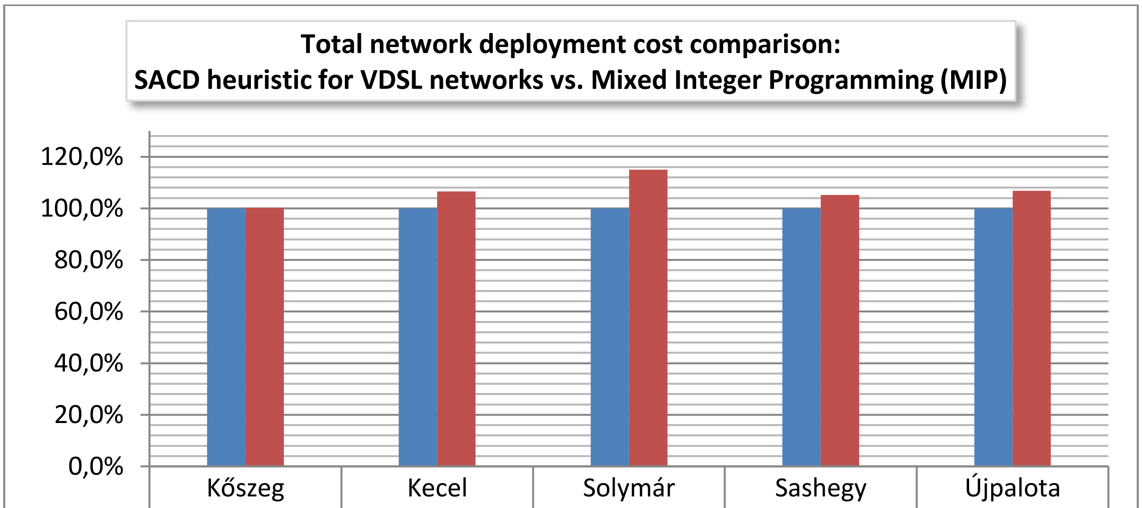
	Build	Feeder Fiber	Distribution Fiber	Distribution Units
Kőszeg	3,3%	0,0%	0,4%	0,0%
Kecel	3,1%	0,1%	1,9%	2,3%
Solymár	2,0%	0,0%	1,5%	0,0%
Sashegy	-0,6%	0,1%	2,1%	4,3%
Újpalota	1,9%	0,1%	-0,2%	5,4%

FIGURE 62 INCA VS. MIP: EXCESS COST BY DISTINCT COMPONENTS

Figure 62 is a more complex figure: the difference between e.g. build cost results of MIP and INCA are weighted by their contribution to the total cost, and this relative addition to the cost is depicted on the figure. For example if the build cost has 25% weight in the total cost, and INCA build costs are 10% higher than MIP build costs, it means 2,5% cost surplus with respect to total cost. Now we recall Figure 58: the last, urban scenario of "Újpalota" had a MIP vs. INCA difference of 7.2%. Now, if we take a look at Figure 62, we find the reason: the equipment costs are responsible for 5.4% out of that 7.2%. However, this 5.4% is the largest difference value in the table, which is still acceptable: the scalable INCA heuristic causes not more than 5.4% excess cost (relative to the total network deploy cost) in any cost components, i.e. build, fiber or equipment costs.

7.2.2.4. SACD HEURISTIC FOR DSL NETWORKS

Evaluation of the Stepwise Allocation of Critical DUs (SACD, Section 5.5) heuristic begins with the comparison of total cost figures versus the MIP lower bound (Figure 63). Even if for one scenario, namely for Solymár we can observe an "above average" 15% difference between the MIP lower bound and the heuristic results, it is still close in the acceptable range – and for the other four scenarios we got even more promising results (0.2% - 6.8%).



	Kőszeg	Kecel	Solymár	Sashegy	Újpalota
MIP	100,0%	100,0%	100,0%	100,0%	100,0%
SACD	100,2%	106,5%	115,0%	105,2%	106,8%

FIGURE 63 TOTAL COST OF VDSL NETWORKS: SACD HEURISTIC VS. MIP

While the total cost values are similar, if we look at the details, namely the distinct cost components of the MIP and SACD solutions (Figure 64 and Figure 65), we can observe a more significant difference (see e.g. the build and DU costs for the "Kecel" scenario). The SACD heuristic, and the MIP approach leads slightly different results: during the minimization process: what is save by SACD on build costs was saved by MIP on the DU costs.

Even if we did not see such difference between heuristic and MIP results for PON and AON networks earlier, the SACD heuristic still provides a high quality approximation in the total cost. The DSL topology design problem had the most complex and tight constraint set due to the copper network reach limitations. However, the simplified MIP formulation for lower bound calculations has relaxed the length constraints, i.e. it may lead to "invalid topologies", where demand points are assigned to DUs falling outside of the copper network range. It may be the reason for SACD and MIP leading to different topologies and different split of the total costs among the distinct cost factors. The relaxation of length constraints also explains the fact that we did not see that difference for PON/AON networks, where length constraints were not that tight.

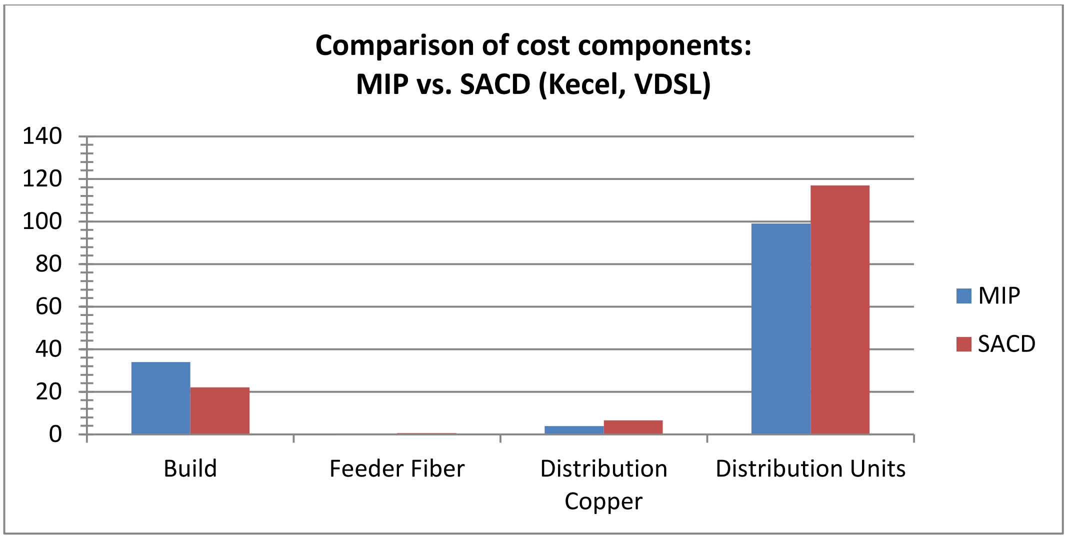


FIGURE 64 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. SACD HEURISTIC FOR VDSL NETWORKS I. (KECEL)

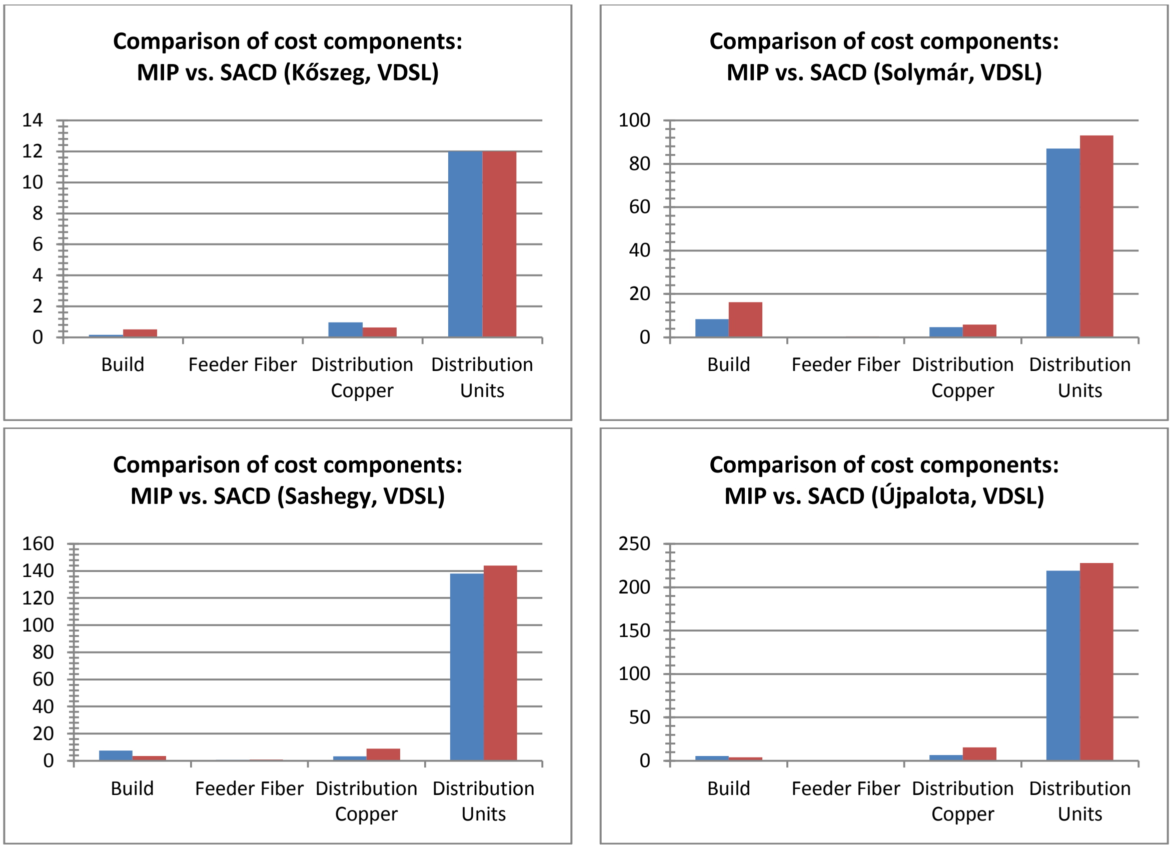


FIGURE 65 COMPARISON OF VARIOUS COST COMPONENTS: MIP VS. SACD HEURISTIC FOR VDSL NETWORKS II.

Moreover, the "clear topology structure" was not a formal constraint, even if it is a fundamental requirement for practical applications. The topologies designed by the SACD heuristic (Figure 66 for Kecel) fulfill this requirement (the distinct colors represent the clustering of demand points).

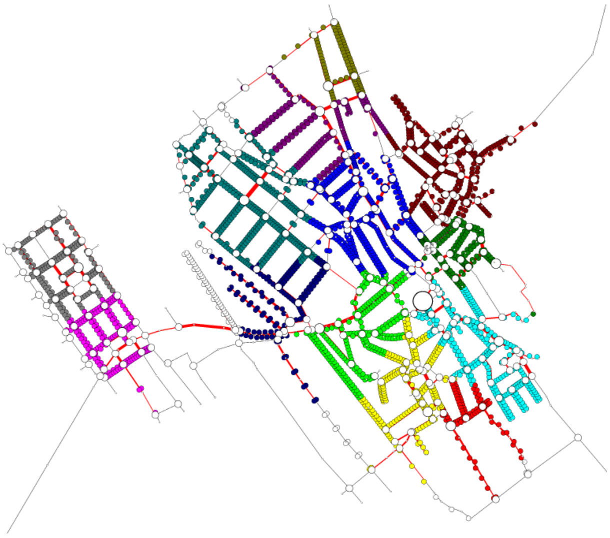


FIGURE 66 VDSL TOPOLOGY DESIGNED BY SACD HEURISTIC (KECEL)

If we take a look at the cost component weights (Figure 67), the reason for focusing on equipment cost minimization becomes clear: DU costs dominate all other components of the cost function. This is mainly the consequence of the re-use of the existing (copper) infrastructure: cabling costs become almost negligible.

The weighted difference in the distinct cost components between the SACD heuristic and the MIP solution is shown on Figure 68, as it was shown also for AETH networks above. In general, we see moderate differences not only for the total cost, but also for the various cost components. The colored circles indicate the two most interesting case studies again: "Kecel" and "Solymár", where the build costs (in the feeder network) had somewhat higher weight (see Figure 67). In "Kecel", the MIP lower bound and the heuristic solution leads to a different balance between equipment and cable plant build costs.

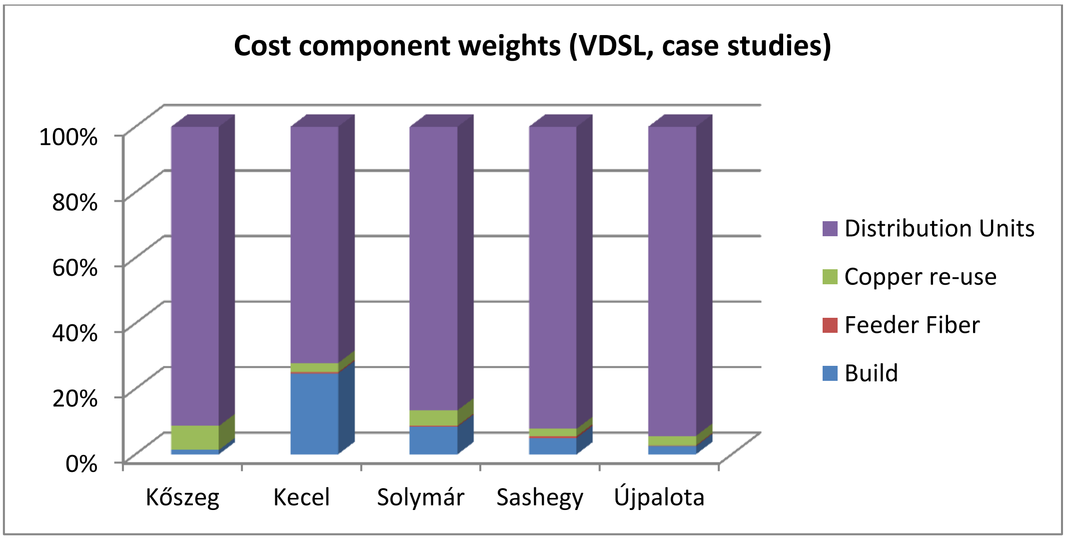
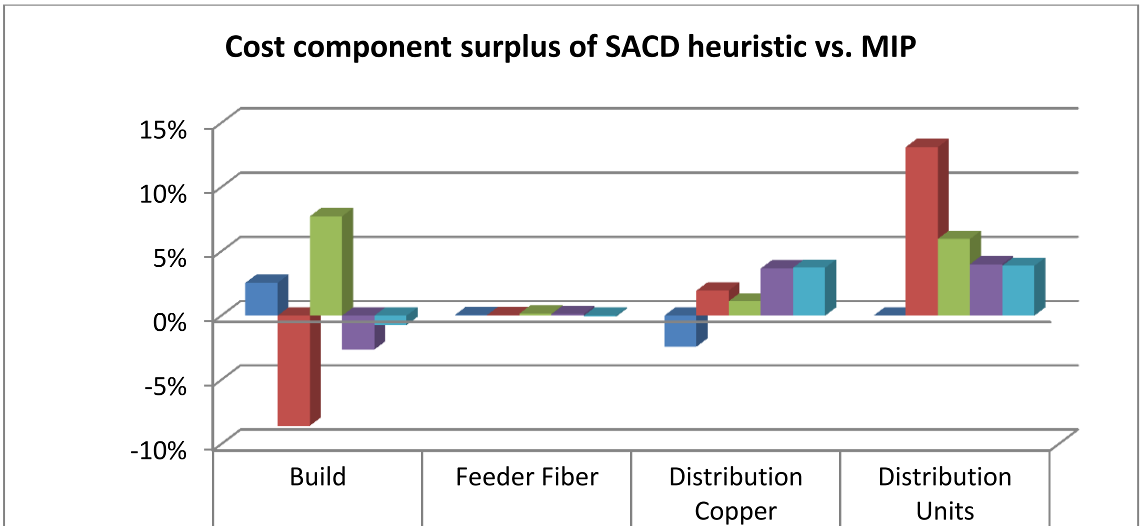


FIGURE 67 COST COMPONENT WEIGHTS FOR AETH NETWORKS

According to the cost component weights (Figure 67), build costs are more emphasized in “Kecel” than in any other case study, and that is the reason why the heuristic has just 6,5% total cost surplus compared to the MIP solution, despite the 20% higher equipment costs: savings in the build costs pay off. On the other hand, in the “Solymár” scenario, both the equipment and build costs are higher than that for the MIP solution, which leads to 15% difference in total. However, even these differences fall in a range which is still promising for practical applications, especially if we do not forget that we were not comparing the heuristic to the optimum itself, we were using a lower bound instead.



	Build	Feeder Fiber	Distribution Copper	Distribution Units
Kőszeg	2,6%	0,1%	-2,4%	0,0%
Kecel	-8,6%	0,0%	2,0%	13,1%
Solymár	7,7%	0,2%	1,1%	6,0%
Sashegy	-2,6%	0,1%	3,7%	4,0%
Újpalota	-0,7%	-0,1%	3,7%	3,9%

FIGURE 68 INCA VS. MIP: COST ADDITION BY DISTINCT COST COMPONENTS

7.2.2.5. SIMULATED ANNEALING

Simulated Annealing is a complex iterative heuristic. If the necessary amount of resources (time and memory) is available, it leads to solid results. The specific resource requirements will be investigated in the next section, here we concentrate on the results in “ideal conditions”, when computation time and memory needs are satisfied.

Figure 69 and the following two figures show the cost comparison for PON, AON and DSL networks respectively, now with Simulated Annealing added. In most cases, SA results are between MIP solutions and results of the highly efficient BCA/INCA/SACD heuristics, i.e. SA provides even better results than the above presented heuristics. In a few cases, SA results fall within the MIP “gap”.

The proposed SA scheme provides really high quality results, at least if problem dimensions are not prohibitive: as the next section will show, SA suffers from scalability problems for really large-scale scenarios. However, according to the related work (Section 1.3), a heuristic capable to handle network scenarios with 1.000^{+} demand points is a solid result in itself. And SA provides even better results than the above presented heuristics for the presented five scenarios, with up to 3.000 buildings or 7.000 demand points.

Flexibility of the developed Simulated Annealing scheme is demonstrated through the fact that SA performs well for all three different network types, even though both the cost component weights, and the strictness of physical constraints are completely different. This flexibility is the most important strength of the SA based heuristic, which makes it a promising candidate for any (yet) unknown, future fixed access network technology. As an element of the algorithm set, it serves as the "Jolly Joker", if the more specialized heuristics were not suitable for the new technology.

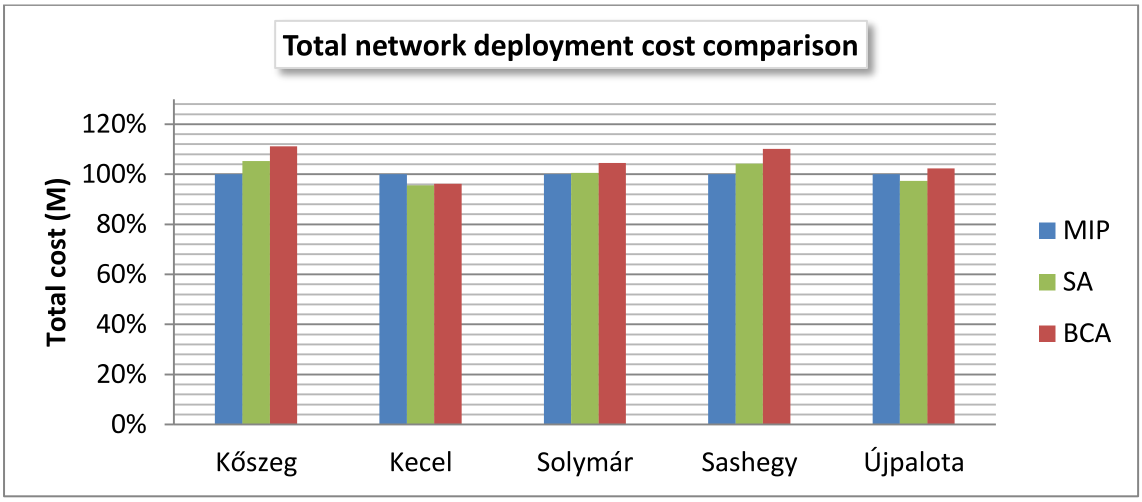


FIGURE 69 COST COMPARISON OF MIP, SA AND BCA HEURISTICS

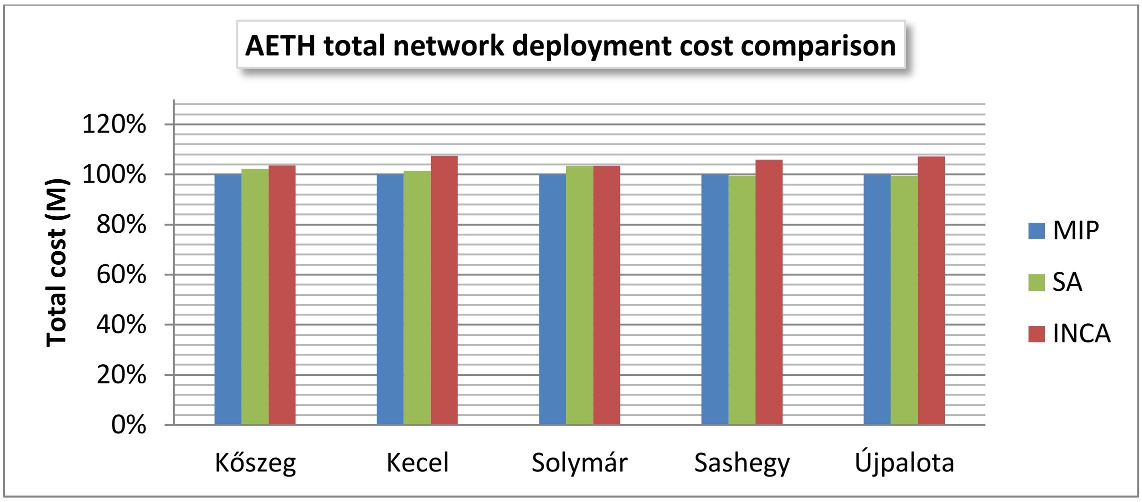


FIGURE 70 COST COMPARISON OF MIP, SA AND INCA HEURISTICS

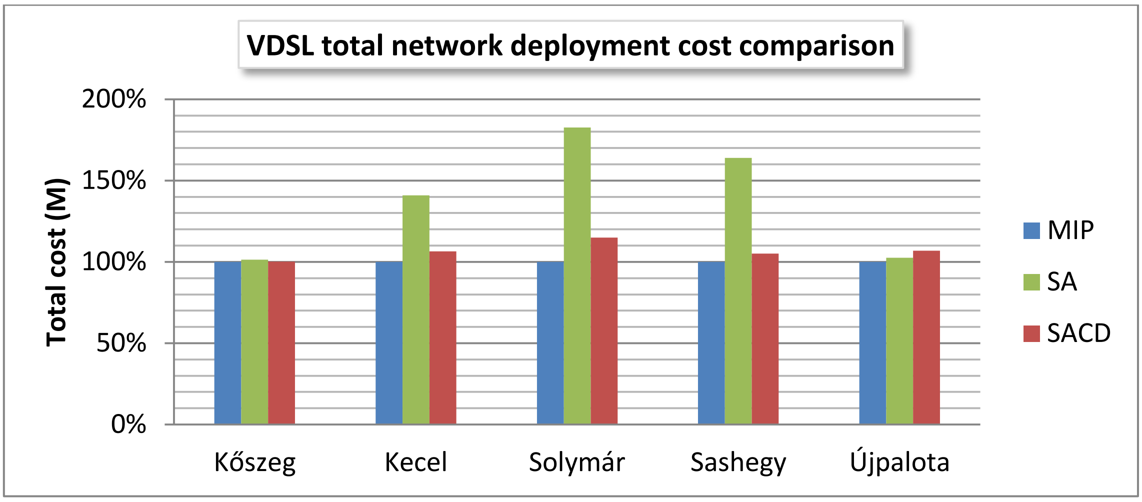


FIGURE 71 COST COMPARISON OF MIP, SA AND SACD HEURISTICS

The more detailed comparison, considering the distinct cost factors follows on the next page. Simulated Annealing provides GPON topologies with typically 5-10% higher build costs than the MIP results, but had almost identical fiber and equipment costs (Figure 75).

The difference is even lower for AETH networks: all cost components of all scenarios are around or under 2%, except one scenario (suburban "Sashegy"), where the distribution fiber costs are 7% higher, but at the same time we get a bit lower build costs, i.e. a bit higher concentration of fiber over the installed networks links. Moreover, the amount of DUs, which is the most important optimization criteria, is identical to the MIP optimum on all five case studies.

Figure 74 summarizes results for VDSL networks. As we have seen, DU costs are the dominating component of the cost function - SA approximates the MIP optimum within 6% even in the "worst" scenario in this sense.

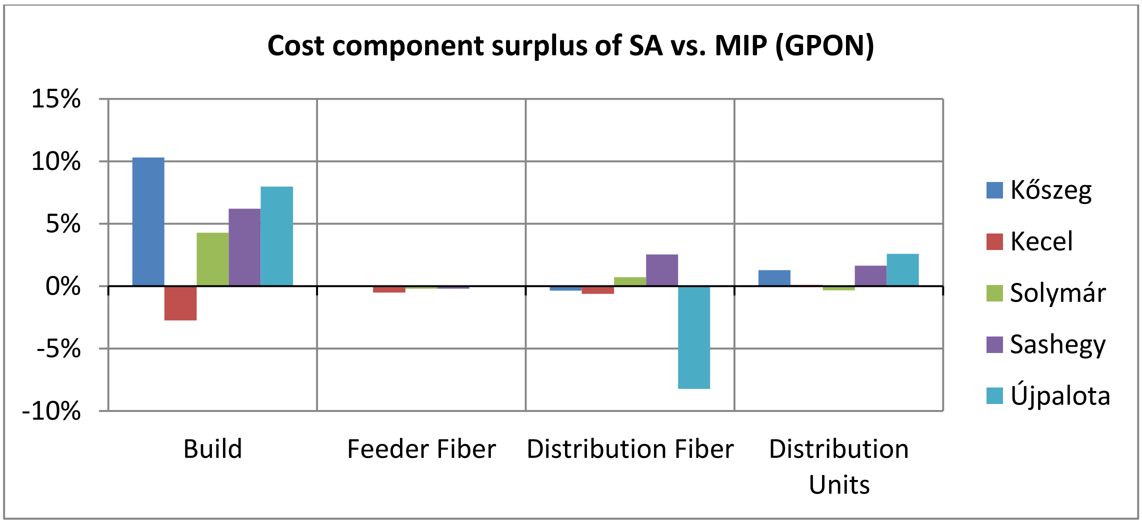


FIGURE 72 COST COMPONENT COMPARISON OF SA AND MIP (GPON)

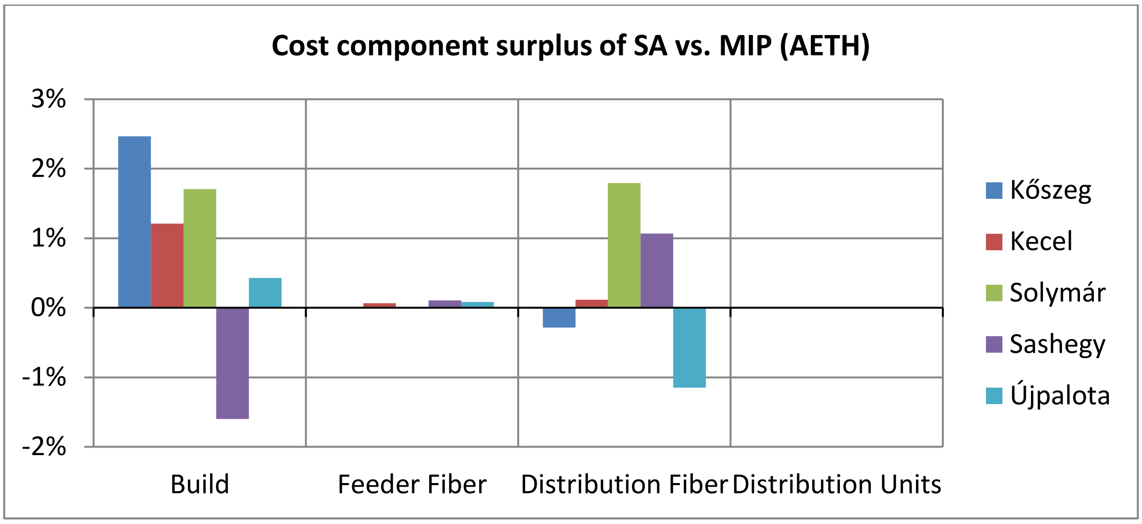


FIGURE 73 COST COMPONENT COMPARISON OF SA AND MIP (AETH)

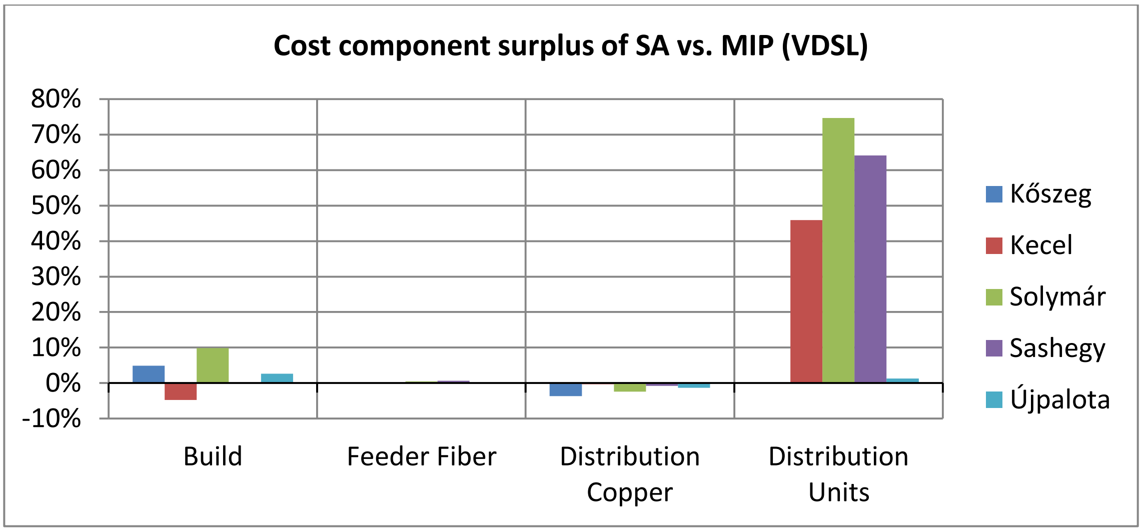


FIGURE 74 COST COMPONENT COMPARISON OF SA AND MIP (VDSL)

7.2.3. Resource requirements

Besides accuracy evaluation, scalability of the proposed algorithms adds another important aspect. Computational complexity, time consumption, and the limits of applicability regarding problem size have high importance. As we have mentioned in the introductory section, the literature is rich in methods which provide acceptable solutions for small problem instances, i.e. hundreds of demand points. What about realistic scenarios with thousands or tens of thousands of demand points? This will be the last part of evaluation.

Figure 75 presents a running time comparison of the evaluated methods for Sashegy scenario with 1.000^{+} buildings and 4.000^{+} demand points. The MIP solver still had 39% gap between the lower and upper bounds after 24 hours. The Simulated Annealing approach was converging to its final solution in approximately 1.5 hours, while the highly efficient BCA heuristic provided a solution in 2 minutes of computation. The difference is significant, even in a logarithmic timescale.

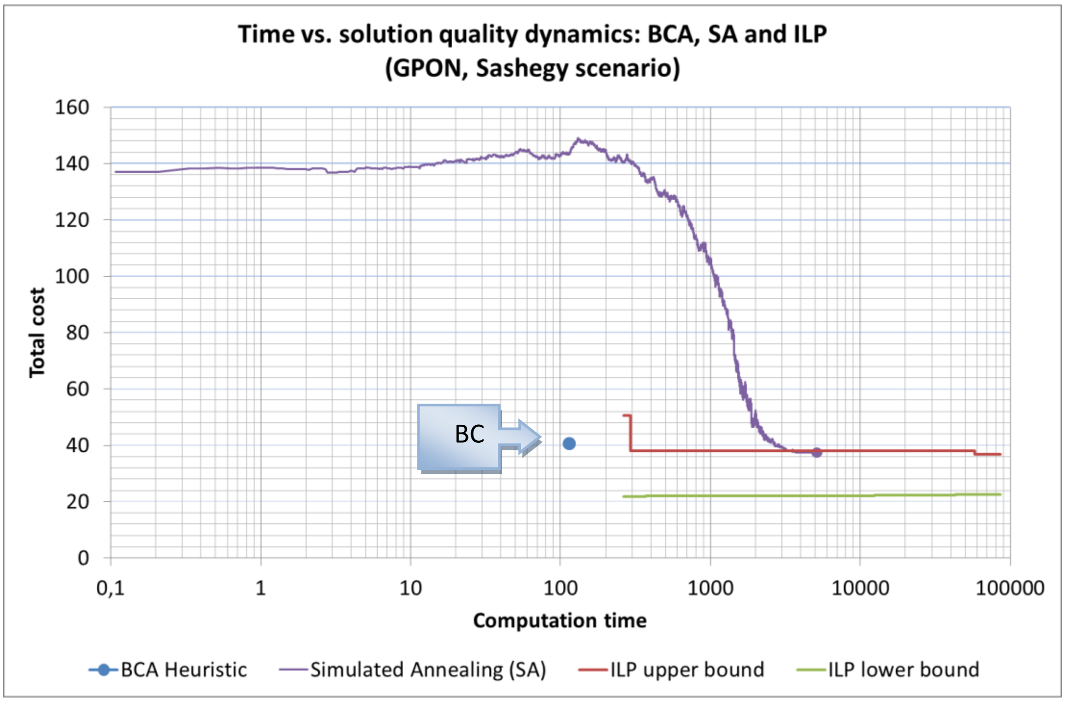
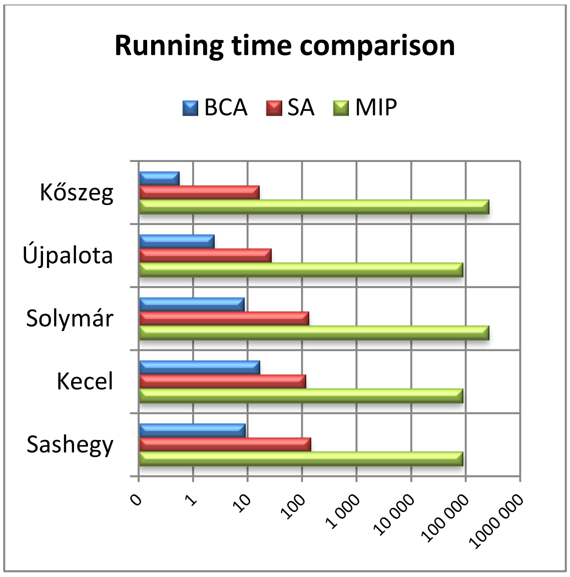


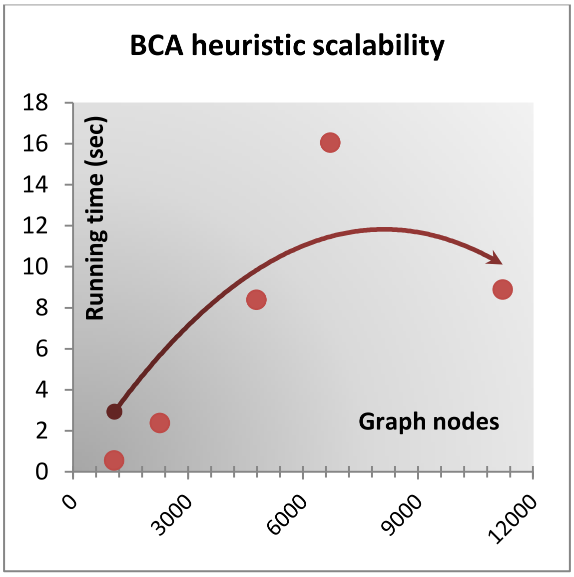
FIGURE 75 TIME CONSUMPTION OF BCA, SA AND MIP SOLUTIONS

The presented example explains the "dynamic behavior" behind the raw running time data, but a comprehensive time consumption analysis of the various methods was also carried out on the presented case studies.

The left side of Figure 76 depicts the measured complete running time on a logarithmic scale for MIP, SA and the specialized heuristics for GPON, AETH and VDSL networks, respectively. The specialized BCA/INCA/SACD heuristics are 2-3 orders of magnitude faster than Simulated Annealing, which is again faster than MIP by approximately 2 orders of magnitude.

On the right side, scalability of the fast heuristics is shown. The trends clearly show the polynomial running time dependence from the problem size, for all three fast heuristics, according to the analytic analysis (Sections 5.3.2, 5.5.2 and 5.4.2).





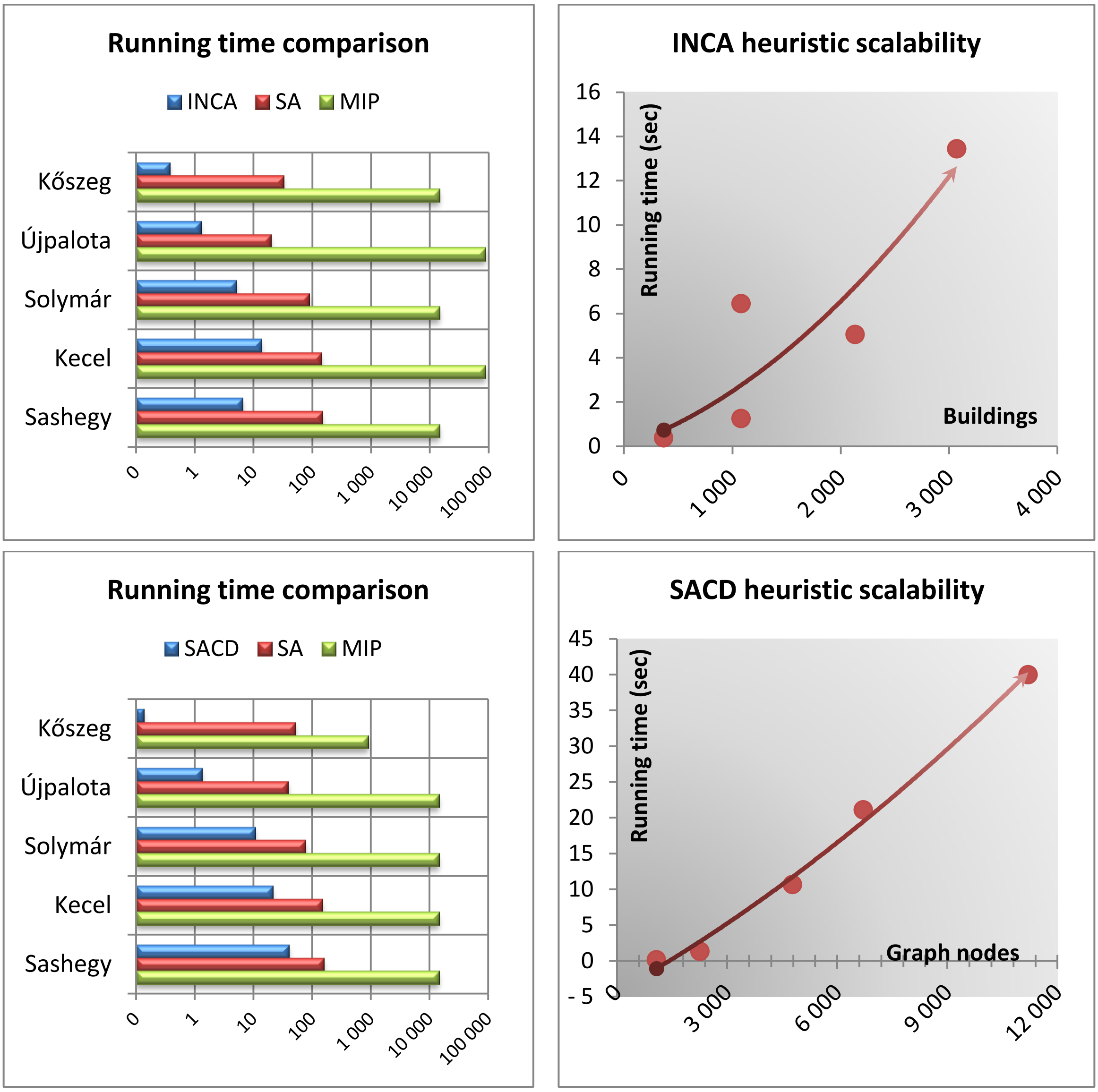


FIGURE 76 TIME CONSUMPTION ANALYSIS

The resource requirement difference between these methods is significant, even from a pure theoretical, algorithmic point of view, and it makes the fast BCA, INCA and SACD heuristics interesting from "l'art pour l'art" algorithmic aspects. Constructing fast, polynomial heuristics for the highly complex Network Topology Design (NTD) problem is not straightforward: as we have seen in the related work section, the earlier published methods had much higher resource requirements.

7.2.3.1. LARGE-SCALE SCENARIOS AND PRACTICAL APPLICATIONS

From a more practical point of view, network planning is an "offline" problem, therefore time constraints are not strict: a couple of hours, or even a few days of computation is still acceptable. However, the difference between the specialized fast heuristics and SA (or even MIP) implies a decisive difference for really large-scale scenarios. Areas with 10.000^{+} demand points are beyond possibilities of MIP, and also SA has trong restrictions, due to both memory and time resource needs.

For these, really large-scale scenarios, the fast and efficient BCA/INCA/SACD heuristics are viable solutions. These large-scale scenarios have practical interest: due to the network node consolidation trends, the Central Offices are serving larger and larger areas currently and in the near future. Our last case study is a complete district of Budapest ("District XII", Figure 77). This is still not the largest possible service area for a single Central Office, but with 4.000^{+} buildings and 25.000^{+} demand points, it represents a tough algorithmic challenge (Table 10).

TABLE 10 SPECIFICATIONS OF "DISTRICT XII" CASE STUDY

Scenario: District XII.

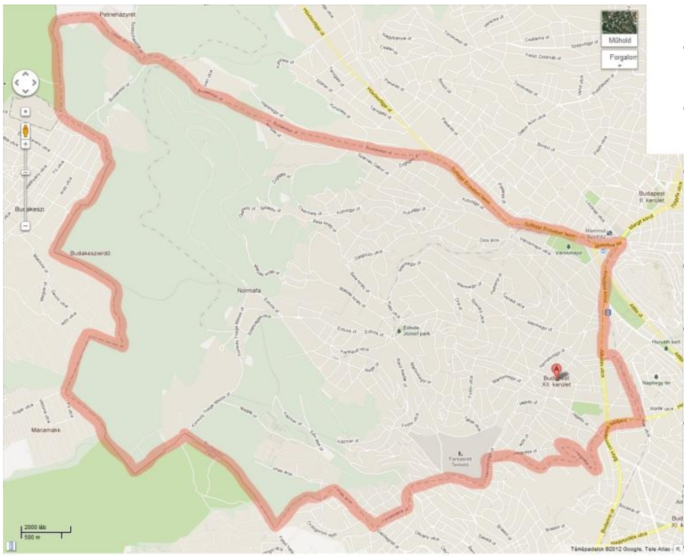


FIGURE 77 MAP OF DISTRICT XII.

Area (km2)	13,56
Buildings	4 151
Demand points	26 489
Demand points/building	6,38
Buildings / km2	310
Demand points / km2	1950
Graph nodes	19 331
Graph edges	20 136
Street system length (m)	303 927

Considering practical applications, the MIP approach itself cannot be used for topology design, since it does not result in a valid network topology, just gives an upper bound on the cost by solving a flow problem, but without solving the DU-demand point assignment, and also the length constraints are relaxed. It was used only for evaluation of the heuristics. For "District XII" scenario, MIP became practically useless: the GPON problem was not solved, AETH was solved, but it still had 38% gap after 24 hours, it provided acceptable results only for VDSL (13% relative gap on exit with memory limit exceeded).

Simulated Annealing, in its original version and implementation required more than 8 GB of memory, and almost endless running time. The memory usage was therefore reduced, however it increased the necessary time for calculations. Therefore a faster cooling strategy was required, in order achieve acceptable running times. As a result of these modifications, the simplified SA approach required approx. 2 GB memory, and two days (48 hours) of computation for GPON and VDSL, and still provided low quality results. For AETH networks SA achieved acceptable results in 18 hours.

In contrary, the fast BCA, INCA and SACD heuristics delivered good results without any modification or simplification, even if evaluating solution quality is difficult, since the reference methods were almost useless here. Except for VDSL, where the SACD heuristic total cost was only 3% higher than the best known solution of the MIP problem. However these heuristics were within 10-15% from the MIP optimum for all five earlier presented case studies, therefore these are supposed to have a similar solution quality in somewhat larger scenarios as well.

The total cost comparison of SA and the fast heuristics is depicted on Figure 78, while the respective running time comparison is given on Figure 79. The fast BCA, INCA and SACD heuristics were still under one hour (400-2500 seconds), but Simulated Annealing, even with significantly decreased iteration count (faster cooling process) needs 24-48 hours or even more.

The figures also illustrate why PON is treated as the most complex special case: Simulated Annealing, even after 45 hours still provided poor results: approximately 75% higher cost than the BCA heuristic. For AETH networks, SA was very time consuming, but at least provided acceptable results. For VDSL networks, SA computations lasted for 180 000 seconds, i.e. 50 hours, more than two days, and it still lead to approximately 100% higher cost than the SACD heuristic.

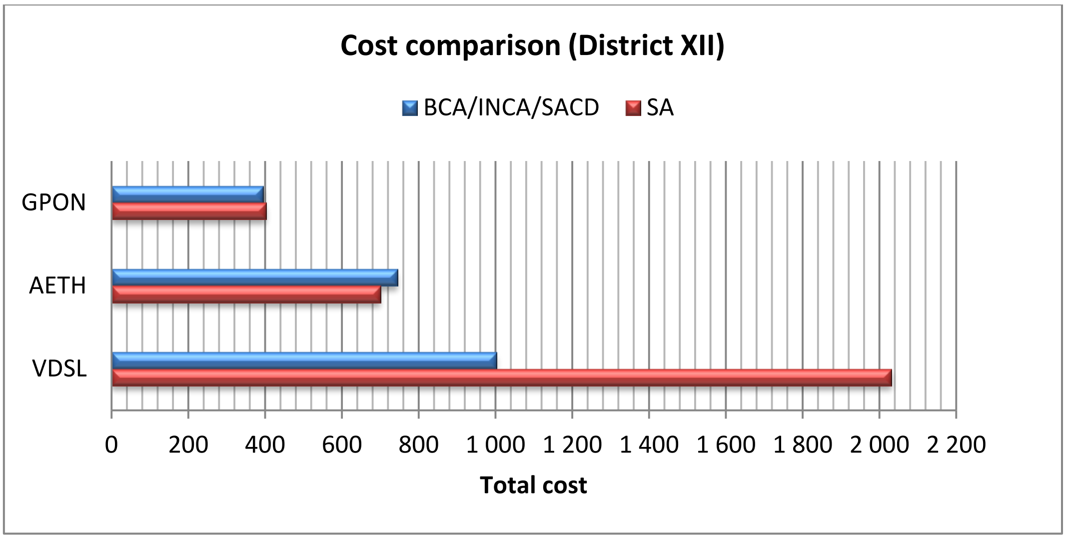


FIGURE 78 TOTAL COST COMPARISON ON A LARGE SCALE SCENARIO (DISTRICT XII)

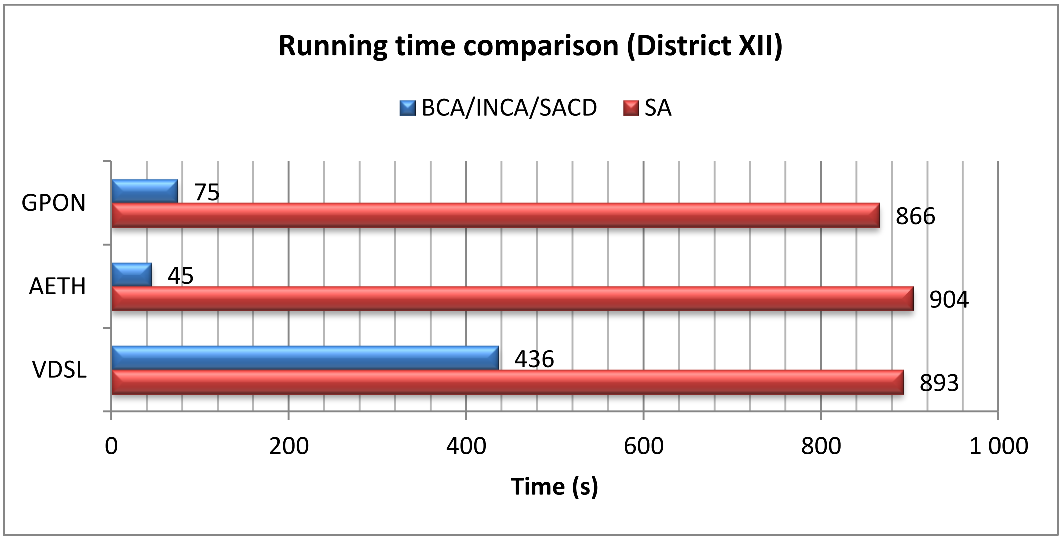


FIGURE 79 RUNNING TIME COMPARISON ON A LARGE SCALE SCENARIO (DISTRICT XII)

8. Conclusion

8.1. Contributions

Theoretic background

The first necessary step towards algorithmic topology design for Next Generation Access (NGA) networks was to clarify its theoretic background. Therefore I have defined a formal model for the NGA Topology Design (NTD) problem.

The ***parameterized graph model*** is technology agnostic, i.e. with the adequate parameter settings, it models all point-to-multipoint and also point-to-point NGA network technology types. Considering these different concepts or technology types, I have also identified the respective ***special cases*** of the NTD problem for Passive Optical Networks (PON), Active Optical Networks (AON) and Digital Subscriber Line (DSL) networks, and as a “degenerate” special case, point-to-point networks also fit in the scope of the model.

The optimization problem was formulated for the NTD problem in general, and also for its special cases. Based on the formal model, the mathematical problem of topology design for NGA networks was analyzed algorithmically. I have identified ***complexity*** and ***approximability*** of the addressed problems. It was fundamental for defining reasonable requirements for the proposed algorithms. The problem in general, and all of its special cases were proven to be NP-complete, and their approximability features were also clarified.

The in-depth investigation of the mathematical problem and its graph representation lead to recognition of key features of the modeling graph. The notion of ***criticality***, the most critical nodes and distribution unit locations highlighted the “pillars” of the topology.

Proposed methodology

I have analyzed the effect and significance of various physical constraints and cost components, and then I have proposed a set of highly specialized and ***effective heuristic algorithms*** for the identified special cases of the NTD problem, based on the underlying theoretic work.

A common feature of the heuristics is the ***decomposition*** of the topology design problem. The decomposed subproblems are then solved with respect to the strong cross-dependence among them. The presented BCA, INCA and SACD heuristics use different techniques to solve these underlying problems, considering the different features of the respective NGA technology types. The BCA heuristic for PON networks utilizes a tree-based segmentation technique, the INCA heuristic for AON networks is built on a bottom-up clustering concept, while the SACD heuristic for DSL networks is using a top-down clustering approach. However, the fact that the clustering is carried out on a graph, with a priori given cluster sizes makes these problems significantly different from the known clustering problems.

The proposed methods have proven their ability to handle large-scale scenarios even with 10.000^{+} demand points with moderate time consumption (within one hour even for the largest scenario), and at the same time, delivered typically 10% or sometimes even better approximation of the optimum. The ***scalability*** has key importance, it makes these highly efficient heuristics a valuable contribution to the existing literature, not to mention the practical consequences, i.e. their possible applications.

Reference methods

Obviously, there is "no free lunch", according to a principle of optimization. The fast heuristics deliver their peak performance for the respective special cases of the topology design problem, but may perform poor within significantly different circumstances. For the NTD problem in its general form, more general tools of optimization theory were used.

An impressive variety of ***metaheuristic concepts*** exist in the literature. Some of them were reviewed considering applicability for solving the addressed access network topology design problem [38]. The concept of ***Simulated Annealing*** turned out to be a promising optimization strategy. Its building blocks, i.e. the neighbor state generation routine, the cooling strategy and the decision functions were adapted to the problem. The careful adaptation and a highly effective implementation lead to a complex heuristic solution for the topology design problem in general. The numerical results have proven that it delivers high quality results with acceptable resource requirements for scenarios with up to thousands of end points.

Exact optimization is the ultimate goal for every optimization problem. Unfortunately, as for many interesting problems, exact optimization was beyond possibilities due to the NP-completeness and typical dimensions of the problem: scenarios of practical interest contain thousands of demand points and tens of thousands graph nodes. ***Mathematical programming*** as a universal tool for solving optimization problems was applied, however it was not trivial. The original formulation of the problem derived from its formal representation turned out to be quadratic. The linearization further increased complexity: the matrix describing the Integer Linear Programming problem had $V^3 \times V^3$ elements, completely excluding its application even for the smallest test networks. At this point, a nice transformation of the problem lead to a linear representation with $V \times V$ dimensions, and this step permitted exact optimization. It was a crucial step for evaluation of the heuristics: even if this MIP formulation does not lead to a complete network topology itself, solving the flow problem defines a lower bound for the minimal network deployment cost, hence it was used as a benchmark for the heuristic solutions.

Validation & evaluation

The proposed heuristics were validated against regular grid structures, where they approximated the analytically proven optimum within the 10% range. Finally, during the evaluation against real-world case the proposed methods have shown approximation performance and scalability that fulfills the requirements, and offers promising applications also in realistic conditions.

8.2. APPLICATIONS

Even if the work presented in this study is partly theoretical, the original questions, motivating the research are of practical interest. Next Generation Access (NGA) networks are just at the beginning of a rapid growth: the technology is now mature, but the huge investment needs delayed network deployments. Both optimized network planning and techno-economic evaluation supports the process: minimization of network deployment costs is key for profitable investments, while techno-economic evaluation is an integral part of the decision making about the service area and network technology for deployment.

In our recent publications we have proven the significant accuracy advantage of a network deployment cost estimation methodology based on our topology design heuristics over the existing geometric modeling approaches [51].

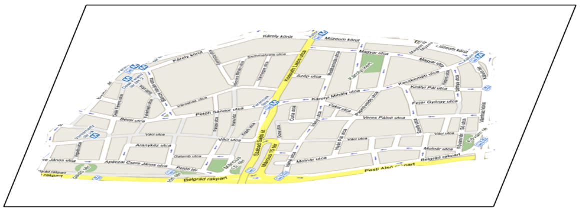
8.2.1. NGADESIGNER FRAMEWORK

A complex framework was developed during the last few years, driven by the theoretic results in the modeling, and the proposed heuristics and algorithms were implemented. The framework supports automatic strategic network design and techno-economic evaluation and comparison of NGA network technologies. The numerical results presented in the validation and evaluation section of the study were not possible to achieve without the framework.

Geospatial data, technology specific physical constraints, and cost values are brought together within the framework and then the presented heuristics are used for automatic topology design. The resulting network topologies are later analyzed not only for network deployment costs (CAPEX), but also a more detailed business case evaluation module is integrated into the framework.

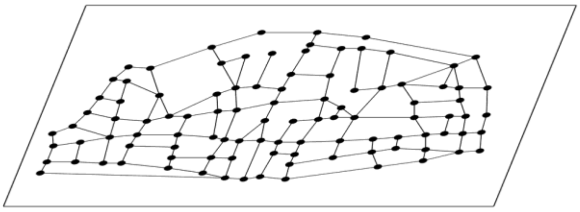
Data processing module

- GIS data (various map formats)
- Infrastructure information
- Demand point data
- Cost database
- Technology specifications



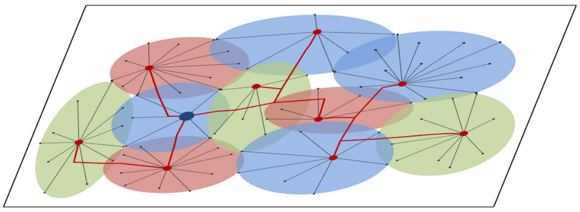
Network modeling module

- Graph model
- Parameters
- Initialization



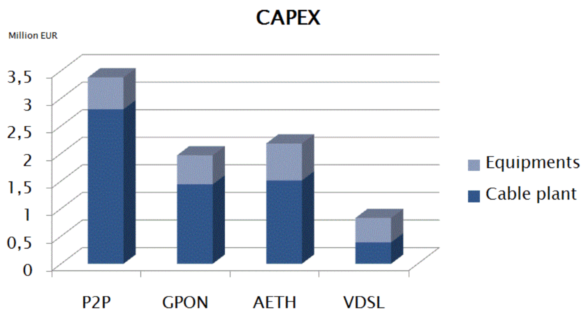
Optimization module

- Topology design (system design, cable plant and equipment)
- Clustering of subscribers
- Efficient, scalable heuristics
- Provides network data for analysis



Analysis module

- Visualization of network topology
- Statistics and reports
- Deployment cost: Bill of Material
- Business Case Analysis
- Techno-economic evaluation & comparison of NGA technologies
- Feasibility studies and decision support



8.3. Experience & reference

The methodology and the framework was used within several research and R&D projects in the recent years. We were involved in a Joint Activity of the EU FP7 BONE project, targeting techno-economic evaluation of optical access networks, and we were also collaborating with Magyar Telekom within OASE (Optical Access Seamless Evolution) EU FP7 IP project. Recently, we have been invited in COMBO, an EU FP7 Integrated Project consortium about convergence of fixed and mobile broadband access/aggregation networks, due to our map-based techno-economic evaluation methodology.

Part of the case study data is from joint R&D projects about optical access network evaluation and comparison with Magyar Telekom, the incumbent network operator in Hungary. Finally, once the methodology was mature, part of the framework development was done within a joint project with NETvisor, a telecommunication software development company.

9. References

[1] M. Tsubokawa, K. Kumozaki, "Evolution of Next Generation Access", IEEE Globecom 2008, New Orleans, USA

[2] S. Chamberland, "Global Access Network Evolution", IEEE/ACM Transactions on Networking, Vol. 18, No. 1, Feb 2010

[3] FTTH Council Europe, "FTTH Handbook", 4th Edition, 2011

[4] H. Mukai, T. Yokotani, T. Kida, "Rapid growth of FTTH by use of PON in Japan", IEEE Globecom Workshops 2007, Washington, USA

[5] J. O. Farmer, K. Bourg, "Practical deployment of Passive Optical Networks", IEEE Communications Magazine, Vol. 46, No. 7, pp.136-145 (2008)

[6] C-H. Lee, W. V. Sorin, B. Y. Kim, "Fiber to the Home using a PON infrastructure", IEEE Journal of Lightwave Technology, Vol. 24, No. 12, pp. 4568-4583 (2006)

[7] Rujian Lin, "Next Generation PON in Emerging Networks", OFC/NFOEC 2008

[8] Keiji Tanaka, Akira Agata, Yukio Horiuchi, "IEEE 802.3av 10G-EPON Standardization and Its Research and Development Status", Journal of Lightwave Technology, Vol. 28, No. 4, Feb 2010

[9] ITU-T Recommendation G.983.x family on ATM passive optical network (APON), 1998

[10] ITU-T Recommendation G.983.x family on Broadband passive optical network (BPON), 2005

[11] IEEE 802.3ah Ethernet Passive Optical Networks (EPON), 2004

[12] ITU-T Recommendation G.984.x family on Gigabit-capable passive optical networks (GPON), 2008

[13] IEEE 802.3av 10 Gbit/s Passive optical Networks (10G EPON), 2009

[14] ITU-T Recommendation G.987.x family in 10-Gigabit-capable passive optical networks (XG-PON), 2010

[15] IEEE 802.ah Ethernet in the First Mile, 2004

[16] ITU-T Recommendation G.985 on 100Mbit/s point-to-point Ethernet based optical access system, 2003

[17] ITU-T Recommendation G.986.1 on Gbit/s point-to-point Ethernet based optical access system, 2010

[18] ITU-T Recommendation G.992.1 Asymmetric digital subscriber line (ADSL), 1999

[19] ITU-T Recommendation G.992.3 Asymmetric digital subscriber line 2 (ADSL2), 2002

[20] ITU-T Recommendation G.992.1 Asymmetric digital subscriber line 2+ (ADSL2+), 2003

[21] ITU-T Recommendation G.993.1 Very high speed digital subscriber line (VDSL), 2004

[22] ITU-T Recommendation G.993.2 Very high speed digital subscriber line 2 (VDSL2), 2006

[23] Per-Erik Eriksson, Björn Odenhammar, "VDSL2: Next important broadband technology", Ericsson Review, No. 1., 2006

[24] M. G. C. Resende, P. M. Pardalos, ed. "Handbook of Optimization in Telecommunications", Springer, 2006

[25] M. Pióro, D. Medhi, "Routing, Flow, and Capacity Design in Communication and Computer Networks", Elsevier / Morgan Kaufmann, 2004

[26] Samee Ullah Khan, "Heuristics-Based PON Deployment", IEEE Communications Letters. Vol. 9, No. 9, pp. 847-849. (2005)

[27] Samee Ullah Khan, "Passive Optical Network Layout in Manhattan", IEEE Photonics Technology Letters, Vol. 15, No. 10, Oct 2003

[28] B. Lakic, M. Hajduczenia, “On optimized Passive Optical Network (PON) deployment”, AccessNets 2007, Ottawa, Canada
[29] E. Bonsma, N. Karunatillake, R. Shipman, M. Shackleton, D. Mortimore, “Evolving Greenfield Passive Optical Networks”, BT Technology Journal, Vol. 21, No. 4, pp. 44-49. (2003)
[30] A. Kokangul, A. Ari, “Optimization of passive optical network planning”, Applied Mathematical Modelling 35, p. 3345-3354. (2011)
[31] K. F. Poon, D. B. Mortimore, J. Mellis, “Designing optimal FTTH and PON networks using new automatic methods“. 2nd IET International Conference on Access Technologies, pp. 49-52., Cambridge, UK, 2006
[32] A. Haidine, R. Zhao et al, “Planning of hybrid fibre-VDSL access networks using particle swarm optimization”, WTC 2006, Budapest, Hungary
[33] S. Chamberland “Planning Multitechnology Access Networks with Performance Constraints”, AccessNets 2008, Vol. 6, pp. 410-426., Las Vegas, USA
[34] Ji Li, G. Shen, “Cost Minimization Planning for Passive Optical Networks”, OFC/NFOEC 2008, San Diego, USA
[35] J. Li, G. Shen, ”Cost Minimization Planning for Greenfield Passive Optical Networks”, Journal of Optical Communication Networks, Vol. 1, No. 1, pp. 17-29. (2009)
[36] A. Agata, Y. Horiuchi, “PON Network Designing Algorithm for Suboptimal Deployment of Optical Fiber Cables”, ACP 2009, Shanghai, China
[37] A. Mitcsenkov, G. Paksy, T. Cinkler, “Geography and Infrastructure Aware Topology Design Methodology for Broadband Access Networks (FTTx)”, J. Photonic Network Communications, Vol. 21, No 3, p. 253-266. (2011)
[38] P. Bakos, P. Katzenberger, A. Mitcsenkov, G. Paksy, T. Cinkler, “Topology Optimization for Next Generation Access Networks: Algorithmic Overview”, MACRo 2011, Marosvásárhely, Romania
[39] FTTH Council Europe, “FTTH Business Guide”, 2010
[40] M. Kantor, K. Wajda, B. Lannoo, K. Casier, S. Verbrugge, M. Pickavet, L. Wosinska, J. Chen, A. Mitcsenkov, “General framework for techno-economic analysis of next generation access networks”, ICTON 2010, Munich, Germany
[41] B. T. Olsen et al., “Technoeconomic evaluation of the major telecommunication investment options for European players”, IEEE Network Magazine, Vol. 20, No. 4, pp. 6-15 (2006)
[42] B. Lannoo, M. Kantor, L. Wosinska, K. Casier, J. Van Ooteghem, S. Verbrugge, Jiajia Chen, K. Wajda, M. Pickavet, “Economic analysis of future access network deployment and operation”, ICTON 2009, Island of São Miguel, Azores, Portugal
[43] K. Casier, B. Lannoo, J. Van Ooteghem, S. Verbrugge, D. Colle, M. Pickavet, P. Demeester, “Game-Theoretic Optimization of a Fiber-to-the-Home Municipality Network Rollout”, Journal of Optical Communication Networks, Vol. 1, No. 1, p. 30-42 (2009)
[44] B. Van Den Bossche, R. Meersman, J. Vanhaverbeke, A. Schoutteet, “Maximizing the return on investment for FTTX-rollout through the use of GIS street maps and geomarketing data”, CTTE 2010, Ghent, Belgium
[45] J. L. Riding, J. C. Ellershaw, A. V. Tran, L. J. Guan, Tim Smith, “Economics of broadband access technologies for rural areas”, OFC/NFOEC 2009, San Diego, USA
[46] B. Olsen, “OPTIMUM – techno-economic tool”, Telektronikk, Vol 95. No. 2/3, pp. 239-250. (1999)
[47] K. Casier, “Techno-Economic Evaluation of Next Generation Access Network Deployment in a Competitive Setting”, Ph. D. thesis, Faculty of Engineering, Ghent University, 2010

[48] Stefanus van der Merwe, Claus G. Gruber, Yelena Grigoreva, Thomas Kessler, “A Model-based Techno-Economic Comparison of Optical Access Technologies”, IEEE GlobeCom 2009, Hawaii, USA
[49] A. Mitcsenkov, G. Paksy, T. Cinkler, “Topology Design and Capex Estimation for Passive Optical Networks”, BroadNets 2009, Madrid, Spain
[50] A. Mitcsenkov, M. Kantor, K. Casier, B. Lannoo, K. Wajda, J. Chen, L. Wosinska, “Geographic Model for Cost Estimation of FTTH Deployment: Overcoming Inaccuracy in Uneven-populated Areas”, ACP 2010, Shanghai, China
[51] A. Mitcsenkov, M. Kantor, K. Casier, B. Lannoo, K. Wajda, J. Chen, L. Wosinska, “Geometric versus Geographic Models for the Estimation of an FTTH Deployment”, Telecommunication Systems Journal, Vol. 62, May 2016 (Online available)
[52] M. Garey, D. Johnson, “Computers and intractability – A guide to the Theory of NP-completeness”, W. H. Freeman, New York, USA, 1979
[53] Wolpert, D.H. Macready, W.G., “No free lunch theorems in optimization”, IEEE Transactions on Evolutionary Computation, Vol. 1, No. 1, pp. 67-82. (1997)
[54] Richard M. Karp, "Reducibility Among Combinatorial Problems". R. E. Miller and J. W. Thatcher (editors). Complexity of Computer Computations. New York: Plenum. pp. 85–103, 1972.
[55] C. H. Papadimitriou, M. Yannakakis, “Optimization, approximation, and complexity classes”, Journal of Computer and System Sciences, Vol. 43, No. 3, p.425–440. (1991)
[56] H. Takahashi, A. Matsuyama, “An approximate solution for the Steiner problem in graphs”, Math. Jap. 24 (1980), 573-577
[57] L. Kou, G. Markowsky, L. Berman, “A fast algorithm for Steiner trees”, Acta Informatica, Vol. 15, No. 2, pp. 141-145. (1981)
[58] M. Thimm, “On the approximability of the Steiner tree problem”, Theoretical Computer Science, Vol. 295, No. 1-3. (2003)
[59] G. Robins, A. Zelikovsky, “Improved Steiner tree approximation in graphs”, 10th Annual ACM-SIAM Symposium on Discrete Algorithms, ACM-SIAM 2000, p. 770-779. (2000)
[60] P. Narváez, K-Y. Siu, and H-Y. Tzeng, “New Dynamic Algorithms for Shortest Path Tree Computation”, IEEE/ACM Transactions on Networking, Vol. 8, No. 6, Dec. 2000
[61] D. Shmoys, É. Tardos, K. Aardal, “Approximation algorithms for facility location problems”, STOC’97, ACM Symposium on Theory of Computing, p. 265-274. (1997)
[62] S. Guha, S. Khuller, “Greedy strikes back: Improved facility location algorithms”, 9th ACM-SIAM Symposium on Discrete Algorithms, p. 649-657. (1998)
[63] M. Mahdian, Y. Ye, J. Zhang, “Approximation Algorithms for Metric Facility Location Problems”, SIAM Journal on Computing. 2006.
[64] G. Ghiani, F. Guerriero, R. Musmanno, “The capacitated plant location problem with multiple facilities in the same site”, Computers and Operations Research, Vol. 29, No. 13, pp. 1903-1912. (2002)
[65] V. Arya, N. Garg, R. Khandekar, A. Meyerson, K. Munagala, V. Pandit, “Local Search Heuristics for k-Median and Facility Location Problems”, 30th Annual ACM Symposium on Theory of Computing, 2001.
[66] A. Meyerson, L. O’Callaghan, S. Plotkin, “A k-median Algorithm with Running Time Independent of Data Size”, Journal of Machine Learning, Special Issue on Theoretical Advances in Data Clustering (MLJ), 2004.

[67] S. Guha, N. Mishra, R. Motwani, L. O'Callaghan, "Clustering Data Streams", 41st Annual IEEE Conference on Foundations of Computer Science, 2000.

[68] William Cook, André Rohe, "Computing Minimum-Weight Perfect Matchings", INFORMS Journal on Computing, Vol. 11, No. 2, pp. 138-148. (1999)

[69] Z. Gotthilf, M. Lewenstein, E. Rainshmidt, "A $(2 - c\frac{\log n}{n})$ Approximation Algorithm for the Minimum Maximal Matching Problem", LNCS Vol. 5426, Springer, Berlin, 2009.

[70] M. Yannakakis, F. Gavril, "Edge dominating sets in graphs", SIAM Journal of Applied Mathematics, Vol. 38, No. 3, p. 364-372. (1980)

[71] M. Minoux, "Mathematical Programming", Wiley, 1986.

[72] Laurence A. Wolsey, "Integer Programming", Wiley, 1988.

[73] Leon S. Lasdon, "Optimization Theory for Large Systems", MacMillan, 1970

[74] G. B. Dantzig: "Linear Programming and Extensions", Princeton University Press, 1963

[75] Matyas, J., "Random optimization", Journal of Automation and Remote Control, Vol. 26, pp. 246–253. (1965)

[76] J. H. Holland, "Adaptation in natural and artificial systems", University of Michigan Press, 1975.

[77] S. Kirkpatrick, C. D. Gelatt, M. P. Vecchi, "Optimization by Simulated Annealing", Science, Vol. 220, p. 671-680. (1983)

[78] Fred Glover, "Future paths for integer programming and links to artificial intelligence", Computers & Operations Research, Vol. 13, No. 5, p. 533-549. (1986)

[79] M. Dorigo, "Optimization, Learning and Natural Algorithms", PhD thesis, Politecnico di Milano, Italy, 1992

[80] Yang, X.-S., "Nature-Inspired Metaheuristic Algorithms", Luniver Press, 2008.

[81] OpenStreetMap (OSM) – http://www.openstreetmap.org/