Performance Evaluation of DPS based Bandwidth Sharing Schemes

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1 Introduction

The original discriminatory processor sharing (DPS) model has been presented and analyzed first in [1] and [2] for modeling purposes of time-sharing computer operation. In this model there are K number of classes of users, and the state of the system can be attributed by n_i denoting the number of class-i ($i = 1 \dots K$) users in the C capacity processor sharing system. There is also a set of weights ϕ_i, $i = 1, \dots, K$ which can be used to control the sharing of the processor capacity among the classes of customers. More formally the (instantaneous) service rate of a class$-i$ customer is

$$c_i = \frac{\phi_i}{\sum_{j=1}^{K} \phi_j n_j} C \quad . \tag{1}$$

In [3] Fayolle et al. proved the results for DPS with respect to the steady-state average response times. In [4] Rege and Sengupta showed how to obtain the moments of the queue length distributions as the solutions to linear equations in case of exponential service time requirements, and they also presented a heavy-traffic limit theorem for the joint queue length distribution. These results were extended to phase type distributions by van Kessel et al. [5]. A further remarkable milestone in DPS analysis is [6] in which the authors showed that the mean queue lengths of all classes are finite under reasonable stability conditions, regardless of the higher moments of the service requirements.

Introducing capacity limits for the customers is mainly motivated by involving access rate limitations of users (e.g. in DSL-type access systems) into the modeling framework. In [7] Lindberger analyzed the M/G/R-PS system, which is a single-class processor sharing model with access rate limit b on the users ($R := C/p$ is the "number of servers" in this system). Several improvements of this model were studied for dimensioning purposes of IP access networks, e.g. in [8] and [9] still remaining at the single-class models.

In case of multi-class discriminative processor sharing with limited access rates the question of bandwidth re-distribution is an important issue, which was not addressed in the literature. This means that if users in a class can not fully utilize their service capacity share (bandwidth share) due to their access rate limit, the problem is how this unused bandwidth is re-distributed among the other classes. In one of the extreme cases, there is no re-distribution at all meaning that the possible remaining unused bandwidth due to rate limits is wasted. One can also interpret this as the server capacity may not be fully utilized, even in those cases when there is "enough" customers in the system. This approach is followed for example in the papers [10], [11].

In this paper we present and analyze the capacity conserving case of access rate limited discriminatory processor sharing, in which all the unused bandwidth left by rate limited customers are fully utilized by the other (non-limited) customers. This is referred to as bandwidth economical discriminatory processor sharing with access rate limitations. We characterize the state space of this model, with identifying those traffic classes which are compressed (whose users are not able to utilize its access rates) and those which are not compressed (which can receive service with their access rates) and with feasible computations for their respective service rates. Two asymptotic regimes of this bandwidth economical DPS are shown and their equivalence is proven. We present that the asymptotic equilibrium point of the bandwidth efficient system is always in the non-compressed region and can simply be formulated (for every class of users), as opposed to the more complicated asymptotic equilibrium of the previously analyzed model [11].

The significance of the fluid limits lies in the following. There is still no solution in the literature for the multi-class access rate limited DPS system (in case Poisson arrivals and exponential service time requirements the the equilibrium of the underlying Markov chain, consequently, the expected response times are not known). Therefore, achieving high quality approximations of system parameters have an utmost importance, e.g. from viewpoint of dimensioning tasks of communication channels for elastic flows in aggregation part of access networks or of processing capacity in highly loaded computer systems like data centers [12]. One "extreme" type of access rate limited multi-class DPS is the limitless case (no compression imposed on the classes), for which Fayolle et al. have already given the solution [3] in terms of the steady-state average response times (by integro-differential equations), and also showed that in the special case of exponential distribution of the service time requirements, the steady-state average response times of classes can be obtained by solving a system of linear equations. The fluid limit is the other extreme case of this DPS system in the sense that some of (or all) classes are "infinitely" compressed (due to infinitely speed up the system), whilst the scaled down performance parameters remain (tend to) finite values. Operational systems to be modeled or dimensioned based on DPS models stand between these two extremes, surprisingly sometimes very close to the fluid limit.

2 DPS Extended by the Limits of Service Rates

In DPS for every pair of classes i, j the ratio of the service rates allocated to class-i and class-j users is equal to the ratio of the class weights (see formula (1)), that is

$$\frac{c_i}{c_j} = \frac{\phi_i}{\phi_j} \ , \ \forall i, j \in 1, \ldots, K \ . \tag{2}$$

The total amount of capacity (in a non-empty system) used by the users of classes is evidently C, i.e.

$$\sum_{i=1}^{K} n_i c_i = C \ . \tag{3}$$

Regarding the incorporation of access rate (customer service capacity share) limits into the DPS model, in [10] and [11] a very simple approach is followed. Namely, first computing the bandwidth shares of class-i users according to (1) and then cutting at the access rate limits p_i, i.e.

$$c_i = \min\left(\frac{\phi_i}{\sum_{j=1}^{K} \phi_j n_j} C, p_i\right) \ . \tag{4}$$

The benefit of this bandwidth share calculation is its simplicity. Nevertheless the price for simplicity is that this approach is not a bandwidth saving one, because it may happen that the total amount of capacity used by the customers is smaller then the server capacity (the server capacity is not completely shared among the users), i.e.

$$\sum_{i=1}^{K} n_i c_i < C \tag{5}$$

even in those cases when there are "enough" users in the system, that is

$$\sum_{i=1}^{K} n_i p_i > C \quad . \tag{6}$$

In this paper we follow the other "extreme" approach, in which all the unused parts of capacity shares due to access rate limits are redistributed among users which are not imposed by these limits on. Because redistribution and sharing the whole capacity C is possible when $\sum_{i=1}^{K} n_i p_i > C$, hereafter we assume the system is in this regime. Otherwise, when $\sum_{i=1}^{K} n_i p_i \leq C$, the bandwidth shares are trivially $c_i = p_i$. In what follows we define our bandwidth economical DPS. The bandwidth economical DPS is such a discriminatory processor sharing system in which the bandwidth shares c_i of the users of K classes at a given state $\mathbf{n} = \{n_1, \ldots, n_K\}$ are determined by the following equations:

$$c_i = \min\left\{p_i, \frac{\phi_i}{\phi_j} c_j\right\} \ \forall i, j \in \{1, \ldots, K\} \ , \ c_j < p_j \tag{7}$$

and

$$\sum_{i=1}^{K} n_i c_i = C \tag{8}$$

where p_i is the service rate limit of class$-i$ users, $0 < p_i \leq C$.

For the next lemma without loss of generality let us assume that

$$\frac{\phi_K}{p_K} \leq \frac{\phi_i}{p_i} \ , \forall i = 1, \ldots, K \ . \tag{9}$$

Lemma 1 *For class$-K$ users $c_K < p_K$ always holds.*

The proof is based on contradiction. Assume that $c_K = p_K$. Due to (7) and the assumption (9) above it follows that

$$c_i = \min\left\{p_i, \frac{\phi_i}{\phi_K} c_K\right\} = p_i \ , \ \forall i = 1, \ldots, K \ . \tag{10}$$

But in this case $\sum_{i=1}^{K} n_i c_i = \sum_{i=1}^{K} n_i p_i > C$ which contradicts to equation (8).

In the next corollary we show the following statement:

Corollary 1 *There is a unique solution of equations (7) and (8) with respect to c_i, $i = 1, \ldots, K$.*

class index	1	2	3	4	5
ϕ_i/p_i	5	4.5	3.33	2	1
p_i	2	2	1.5	2	10
orig. DPS	2.5974	2.3377	1.2987	1.0389	0.2597
equ (4)	2	2	1.2987	1.0389	0.2597
bw eco. DPS	2	2	1.5	1.3714	0.343

Table 1: Example of bandwidth shares of different DPS systems

Because of Lemma 1 and (8) and the monotone increasing property of $\min\{p_i, \frac{\phi_i}{\phi_K}x\}$ w.r.t. x, a class$-K$ user bandwidth share is a unique solution of the equation

$$\sum_{i=1}^{K} n_i \min\left\{p_i, \frac{\phi_i}{\phi_K}x\right\} = C \tag{11}$$

with respect to x. Therefore, every other bandwidth share is also unique and can be calculated by using c_K and the equality $c_i = \min\{p_i, \frac{\phi_i}{\phi_K}c_K\}$.

Let a numerical example be presented for this calculation. Let $C = 100$ [Mbit/s] and five classes (with index 1 to 5 in sequence) are set up with the following paramaters: $\mathbf{n} = (8, 15, 20, 10, 30)$, $\mathbf{p} = (2, 2, 1.5, 2, 10)$ [Mbit/s], $\phi = (10, 9, 5, 4, 1)$. The following table shows the ϕ_i/p_i ratios, the access rate limits p_i, the bandwidth shares in case of original DPS (without access rate limit), of DPS with access rate limit with simple cutting at the limits using formula (4), and the new bandwidth economical DPS according to equations (7) and (8).

The fifth line of the table clearly shows that in case of simple cutting DPS (using equation (4), or simple comparing the third and fourth lines of the table), the class-1 and class-2 users can utilize their access rates (they are uncompressed), while classes 3, 4 and 5 are compressed (they can not reach their access rates). It can also be observed that $\sum_{i=1}^{5} n_i c_i = 90.16$ Mbit/s, that is from the total capacity 100 Mbit/s almost ten percent is wasted.

On the contrary, the last row presenting the bandwidth share of the new DPS system shows, that not only class-1 and class-2 can achieve their access rate limits, but also class-3 became uncompressed, thanks to the redistribution[1] of the unused bandwidth left by class-1 and class-2 customers. Furthermore, class-4 and class-5 bandwidth shares are also higher than in the previous case, because they can also gain from bandwidth reuse. In this case, of course $\sum_{i=1}^{5} n_i c_i = 100$ Mbit/s , hence this is attributed as bandwidth economical.

Although the computational approach above is straightforward, it is worth exploring further the structure

[1] The term 'redistribution' is used because it can be shown that the following process results exactly the same solution: start with the original DPS bandwidth share, cut at the access rate limits, and redistribute the residual banwidths among the still compressed classes, which may result some classes become uncompressed. Repeat this until the bandwidth shares no longer change.

of the system. For this, let us assume again without restriction that

$$\frac{\phi_1}{p_1} \geq \frac{\phi_2}{p_2} \geq \ldots \geq \frac{\phi_K}{p_K} \ . \tag{12}$$

Lemma 2 *If $\sum_{i=1}^{K} n_i p_i > C$ there exists an i^*, $1 \leq i^* \leq K-1$ such that*

$$\sum_{k=1}^{i^*-1} n_k p_k + \sum_{k=i^*}^{K} n_k \phi_k \frac{p_{i^*}}{\phi_{i^*}} \leq C \ \textit{and} \tag{13}$$

$$\sum_{k=1}^{i^*} n_k p_k + \sum_{k=i^*+1}^{K} n_k \phi_k \frac{p_{i^*+1}}{\phi_{i^*+1}} > C \ . \tag{14}$$

Note that the function

$$f(i) = \sum_{k=1}^{i-1} n_k p_k + \sum_{k=i}^{K} n_k \phi_k \frac{p_i}{\phi_i}$$

is increasing w.r.t. i due to (12) and exceeds C for some $i^* + 1 \leq K$, otherwise $f(K) = \sum_{i=1}^{K} n_i p_i \leq C$ would hold which is not true. As an important consequence of this lemma it is also worth noting that

$$\{1, \ldots, i\} \subset \mathcal{U}(\underline{n}) \text{ iff } \sum_{k=1}^{i-1} n_k p_k + \sum_{k=i}^{K} n_k \phi_k \frac{p_i}{\phi_i} \leq C \tag{15}$$

where $\mathcal{U}(\underline{n}) := \{1, \ldots, i^*\}$ is the set of uncompressed classes in the state $\underline{n}$.

Now the main theorem of this section is the following:

Theorem 1 *The unique solution of (7) and (8) can be expressed through i^* in the following way:*

$$c_k = p_k, \ \textit{if } k \leq i^* \ \textit{and} \tag{16}$$

$$c_k = \frac{\phi_k}{\sum_{i=i^*+1}^{K} \phi_i n_i} \left(C - \sum_{j=1}^{i^*} n_j p_j \right), \textit{if } i^* < k. \tag{17}$$

The validity of (8) can easily be checked. Next we show that (7) is fulfilled by c_k, c_l for which $k, l \in \mathcal{Z}(\underline{n}) := \{1, \ldots, K\} \setminus \mathcal{U}(\underline{n})$. In this case due to (14) and (12) $c_k < p_k$ and $c_l < p_l$. Moreover $c_k/p_k = c_l/p_l$ holds, therefore (7) is satisfied, that is $c_k = \min\{p_k, \frac{\phi_k}{\phi_l} c_l\}$.

Now assume that $l \in \mathcal{U}(\underline{n})$ and $k \in \mathcal{Z}(\underline{n})$. In this case $c_k < p_k$, therefore

$$\frac{\phi_l}{\phi_k} c_k = \frac{\phi_l}{\sum_{i=i^*+1}^{K} \phi_i n_i} \left(C - \sum_{j=1}^{i^*} n_j p_j \right) \tag{18}$$

which is not less than p_l due to (13) and (12). Hence, $c_l = \min\{p_l, \frac{\phi_l}{\phi_k} c_k\} = p_l$ that is (7) is again fulfilled.

3 Asymptotic Behaviors of the Bandwidth Economical DPS

In this section we first show that the so-called fluid limit of the processor sharing model investigated in this paper exists. Then we find the equilibrium of the fluid limit. Further, we show that the equilibrium is stable. Finally, central limit like theorem will be proved, that is, we show that the equilibrium of the diffusion limit.

Assume that the service times are exponentially distributed and the arrival processes follow Poisson processes. Then in this case the number of jobs (of customers) in the system can be modeled by a Markov chain. The equilibrium of the Markov chain, consequently, the expected response times are not known. Fluid scaling is a possible asymptotic regime in which one may expect computing the equilibrium at least for the limiting structure. In fluid limit the arrival processes are accelerated by a common factor and the capacity of the server is speed up by the same factor. If the accelerating factor goes to infinity then in limit one gets the fluid limit of the number of waiting jobs. The limiting process of the number of waiting jobs is deterministic, it is a solution of a differential equation. The equilibrium of this differential equation can be found using analytical considerations. We remark that the fluid limit of many processor sharing model, as well as the one investigated in this paper, can be determined by using classical results presented in e.g. [13, Chapter 11].

For finding the fluid limit of our model first the transition rates are to be determined $q(\underline{n}, \underline{n}+\underline{l})$ from state $\underline{n}$ to $\underline{n}+\underline{l}$. Let $\underline{e}_k$ be a vector such that in $\underline{e}_j$ 1 stands at coordinate j and except this coordinate each coordinate is 0. For any $j = 1, \ldots, K$

$$\begin{aligned} q(\underline{n}, \underline{n}+\underline{e}_j) &= \lambda_j \\ q(\underline{n}, \underline{n}-\underline{e}_j) &= \mu_j n_j p_j && \text{if } j \in \mathcal{U}(\underline{n}) \\ q(\underline{n}, \underline{n}-\underline{e}_j) &= \mu_j n_j \phi_j \frac{C - \sum_{i \in \mathcal{U}(\underline{n})} p_i n_i}{\sum_{i \in \mathcal{Z}(\underline{n})} \phi_i n_i} && \text{if } j \in \mathcal{Z}(\underline{n}) \\ q(\underline{n}, \underline{n}+\underline{l}) &= 0 && \text{if } \underline{l} \neq \pm \underline{e}_k \\ & && \text{for some } k = 1, \ldots, K. \end{aligned} \tag{19}$$

Let $r_j(\underline{n})$ denote the bandwidth that a stream of class j obtains. We have

$$r_j(\underline{n}) = p_j \mathbf{I}\{j \in \mathcal{U}(\underline{n})\} + \phi_j \frac{C - \sum_{i \in \mathcal{U}(\underline{n})} p_i n_i}{\sum_{i \in \mathcal{Z}(\underline{n})} \phi_i n_i} \mathbf{I}\{j \in \mathcal{Z}(\underline{n})\}. \tag{20}$$

We remark that using (15), $r_j(\underline{n})$ can be given as an explicit function of $\underline{n}$ as follows:

$$\begin{aligned} r_j(\underline{n}) = p_j \mathbf{I} &\left\{ \sum_{k=1}^{j-1} n_k p_k + \sum_{k=j}^{K} n_k \phi_k \frac{p_j}{\phi_j} \leq C \right\} \\ &+ \phi_j \frac{C - \sum_{i \in \mathcal{U}(\underline{n})} p_i n_i}{\sum_{i \in \mathcal{Z}(\underline{n})} \phi_i n_i} \mathbf{I} \left\{ \sum_{k=1}^{j-1} n_k p_k + \sum_{k=j}^{K} n_k \phi_k \frac{p_j}{\phi_j} > C \right\}. \end{aligned} \tag{21}$$

Of course, this definition makes sense for $\underline{n} \in \mathbb{R}_+^K$.

Let $\Pi_j^a(t), t \geq 0$ and $\Pi_j^d(t), t \geq 0$ for $j = 1, \ldots, K$ be $2K$ independent Poisson processes with rate 1. Let $N_j(t)$ be the number of flows from class j in the system at time t. Then by the rates in (19) we have

$$N_j(t) = N_j(0) + \Pi_j^a(\lambda_j t) - \Pi_j^d \left(\int_0^t \mu_j N_j(s) r_j\left(\underline{N}(s)\right) \mathrm{d}s \right). \tag{22}$$

Let

$$\begin{aligned} \lambda_j^L &= \lambda_j L, \quad j = 1, \dots, K \\ C^L &= CL \end{aligned}$$

Let $N_j^L(t)$ be the number of flows from class j in the system at time t if the arrival intensities to the classes are $\lambda_1^L, \dots, \lambda_K^L$ respectively and the capacity is C^L. Simply rewriting the equation (22) for $N^L(t), t \geq 0$ and dividing by L we get

$$\frac{N_j^L(t)}{L} = \frac{N_j^L(0)}{L} + \frac{1}{L}\Pi_j^a(L\lambda_j t) - \frac{1}{L}\Pi_j^d\left(L\int_0^t \mu_j \frac{N_j^L(s)}{L} r_j\left(\frac{\underline{N}^L(s)}{L}\right) \mathrm{d}s\right) \quad j = 1, \dots, K.$$

For the ease of notations we rewrite this equation. Introducing

$$n_j^L(t) = \frac{N_j^L(t)}{L} \quad j = 1, \dots, K$$

we have

$$n_j^L(t) = n_j^L(0) + \frac{1}{L}\Pi_j^a(\lambda_j L t) - \frac{1}{L}\Pi_j^d\left(L\int_0^t \mu_j n_j^L(s) r_j\left(\underline{n}^L(s)\right) \mathrm{d}s\right) \quad j = 1, \dots, K \tag{23}$$

The theory presented in [13, Ch 6.4 and Ch 11.2], see also the Appendix, can be applied to the process $\underline{n}^L(t), t \geq 0$ for obtaining convergence to $\underline{n}(t), t \geq 0$ the solution of the system of equations

$$n_j(t) = n_j(0) + \lambda_j t - \int_0^t \mu_j n_j(s) r_j\left(\underline{n}(s)\right) \mathrm{d}s, \quad j = 1, \dots, K \tag{24}$$

as it is stated in the following theorem.

Theorem 2 *Assume that* $\lim_{L\to\infty} n_j^L(0) = n(0) \in [0, \infty)$ *for any* $j = 1, \dots, K$*. Then for every* $t \geq 0$*,*

$$\lim_{L\to\infty} \sup_{s\leq t} \left|\underline{n}^L(s) - \underline{n}(s)\right| = 0 \quad a.s. \tag{25}$$

We will apply Theorem 2.1 of [13, p 456]. We have to check three conditions. First, for any compact set $B \subset [0, \infty)^K$ the following bound holds

$$\sup_{\underline{n}\in B} n_j r_j(\underline{n}) < \infty \quad j = 1, \dots, K, \tag{26}$$

second, there exist M_B such that for any $j = 1, \dots, K$

$$|n_j r_j(\underline{n}) - m_j r_j(\underline{m})| \leq M_B\, |\underline{n} - \underline{m}| \quad \underline{n}, \underline{m} \in B. \tag{27}$$

Third,

$$\lim_{L\to\infty} n_j^L(0) = n(0) \in [0,\infty) \quad j = 1,\ldots,K. \tag{28}$$

Using (21) Simple calculations show that (26) and (27) hold. The condition (28) is the same as the assumption of Theorem 2. Therefore, the convergence (25) holds.

The main results of this section is the following.

Theorem 3 *If the function $\underline{n}(t), t \geq 0$ satisfies the equations (24) then in the stationary state n_j^*, $j = 1,\ldots,K$ each class is uncompressed and the the following holds:*

$$n_j^* = \frac{\lambda_j}{\mu_j p_j} \quad j = 1,\ldots,K.$$

For finding the stationary state $\underline{n}^*$ of the fluid limit differentiate $n_j(t)$, $j = 1,\ldots,K$ with respect to t and find the solution of the system $n_j'(t) = 0$, $j = 1,\ldots,K$. Using (24) and (20) one gets

$$0 = n_j'(t) = \lambda_j - \mu_j n_j(t) \cdot \left(p_j \mathbf{I}\{j \in \mathcal{U}(\underline{n}(t))\} + \phi_j \frac{C - \sum_{i\in\mathcal{U}(\underline{n})} p_i n_i(t)}{\sum_{i\in\mathcal{Z}(\underline{n})} \phi_i n_i(t)} \mathbf{I}\{j \in \mathcal{Z}(\underline{n}(t))\} \right)$$

This means that in the stable state we have

$$\begin{aligned} \lambda_j &= \mu_j p_j n_j^* \text{ if } j \in \mathcal{U}(\underline{n}^*), \\ \lambda_j &= \mu_j n_j^* \frac{\phi_j}{\sum_{i\in\mathcal{Z}} \phi_i n_i^*} \left(C - \sum_{i\in\mathcal{U}} p_i n_i^* \right) \text{ if } j \in \mathcal{Z}(\underline{n}^*). \end{aligned}$$

If there is at least one compressed class, that is, $\mathcal{Z}(\underline{n}^*) \neq \emptyset$ then we have for $j \in \mathcal{Z}(\underline{n}^*)$

$$\begin{aligned} \lambda_j &= \mu_j n_j^* \frac{\phi_j}{\sum_{i\in\mathcal{Z}(\underline{n}^*)} \phi_i n_i^*} \left(C - \sum_{i\in\mathcal{U}(\underline{n}^*)} p_i n_i^* \right) \\ &= \mu_j n_j^* \frac{\phi_j}{\sum_{i\in\mathcal{Z}(\underline{n}^*)} \phi_i n_i^*} \left(C - \sum_{i\in\mathcal{U}(\underline{n}^*)} \frac{\lambda_i}{\mu_i} \right) \\ &= \mu_j n_j^* \frac{\phi_j}{\sum_{i\in\mathcal{Z}(\underline{n}^*)} \phi_i n_i^*} \left(C - \sum_{i\in\mathcal{U}(\underline{n}^*)} C\varrho_i \right) \end{aligned}$$

since the definition $\varrho_i = \frac{\lambda_i}{\mu_i C}$. Dividing by $\mu_j C$ and using $\varrho_j = \frac{\lambda_j}{\mu_j C}$ one gets

$$\varrho_j = \frac{\phi_j n_j^*}{\sum_{i\in\mathcal{Z}(\underline{n}^*)} \phi_i n_i^*} \left(1 - \sum_{i\in\mathcal{U}(\underline{n}^*)} \varrho_i \right),$$

rearranging the terms on the right we have

$$\frac{\varrho_j}{1 - \sum_{i\in\mathcal{U}(\underline{n}^*)} \varrho_i} = \frac{\phi_j n_j^*}{\sum_{i\in\mathcal{Z}(\underline{n}^*)} \phi_i n_i^*},$$

then summing both sides over $j \in \mathcal{Z}(\underline{n}^*)$ one has

$$\frac{\sum_{j\in\mathcal{Z}(\underline{n}^*)} \varrho_j}{1-\sum_{i\in\mathcal{U}(\underline{n}^*)} \varrho_i} = 1,$$

this is equivalent to $\sum_{j=1}^{K} \varrho_j = 1$ which is contradiction. Consequently, $\mathcal{Z}(\underline{n}^*) = \emptyset$ and

$$\text{for any } j = 1, \dots, K \quad n_j^* = \frac{\lambda_j}{\mu_j p_j}$$

It can also be shown that the equilibrium $\underline{n}^*$ is stable, nevertheless, due to the lack of space it is not performed here. It has been elaborated following the argumentation in [11, pp 48-49] and also using [14, Lemma 3].

Here we note that in this bandwidth economical DPS the fluid limit lies completely in the uncompressed region (every classes in the limit are uncompressed), and the closed form expression of the fluid limit of a class depends only on the class parameters (λ_j, μ_j, p_j), and is quite simple.

On the contrary, in case of the previously analyzed DPS [11] (based on equation (4)) the fluid limit has no closed form solution, an algorithm is needed to determine the compressed and uncompressed classes and the corresponding limits in the asymptotics. Furthermore, the limit of a class may depend on the parameters of other classes (see Proposition 1.3. in [11]).

4 Fluid limit as the number of servers goes to infinity

In the concept of fluid limit the intensity of the arrival processes and the capacity of the server increase in the same pace by a multiplier L. Consequently, the number of packets under service increases and the number of served packets in unit time increases as well. The first property can be rephrased as the number of servers increases. It is natural to ask whether one can take an asymptotic regime in which the number of servers increases but the intensities of the arrivals and the capacity are fixed. If so, then what can be said about the limit process. A possible way of considering such an asymptotic is that we decrease the access rates by L and take $p_j^L = p_j/L$. This is not enough to obtain fluid scaling like set up because the number of served packets per unit time does not increase. One can get over this problem and obtain limit of similar kind as the fluid limit if the time of the system is accelerated too. This regime will be described in this section.

Let us fix C and λ_j and decrease the access rate limits p_j, $j = 1, \dots, K$ such that

$$p_j^L = \frac{p_j}{L}, \quad j = 1, \dots, K$$

for $L > 0$. Let $M_j^L(t)$ be the number of flows from class j in the system at time t if the bandwidths are p_j^L, $j = 1, \dots, K$. It can be proved that the rescaled and time accelerated process has fluid limit.

Theorem 4 *Assume that* $\lim_{L\to\infty} \frac{M_j^L(Lt)}{L} = m(0) \in [0,\infty)$ *for any* $j = 1,\ldots,K$*. For the processes* $M_j^L(t)$*,* $j = 1,\ldots,K$ *defined above we have the following fluid limit*

$$\lim_{L\to\infty} \sup_{s\le t} \left| \frac{M_j^L(Lt)}{L} - \underline{n}(s) \right| = 0 \quad a.s. \tag{29}$$

where $\underline{n}(s)$ *is the solution of the differential equation (24). Consequently,*

$$\lim_{t\to\infty} \lim_{L\to\infty} \frac{M_j^L(Lt)}{L} = n_j^* \quad j = 1,\ldots,K, \tag{30}$$

where n_j^* *is defined in Theorem 3. urther,*

$$\lim_{L\to\infty} \lim_{T\to\infty} \frac{1}{T} \int_0^T \frac{M_j^L(t)}{L}\, \mathrm{d}t = n_j^* \quad j = 1,\ldots,K. \tag{31}$$

We will prove that the process $\frac{\underline{M}^L(Lt)}{L}, t \ge 0$ satisfies equation (23). Consequently, Theorem 2 can be applied for $\frac{\underline{M}^L(Lt)}{L}, t \ge 0$ yielding the same convergence (29). Then one can conclude that Theorem 3 holds for the limit process (30) without any further modification.

Proving the process $\frac{\underline{M}^L(Lt)}{L}, t \ge 0$ satisfies equation (23), we first rewrite the equation (22) for $M^L(t), t \ge 0$. We have

$$M_j^L(t) = M_j^L(0) + \Pi_j^a(\lambda_j t)$$
$$-\Pi_j^d\left(\int_0^t \mu_j M_j^L(s) r_j^{L*}\left(\underline{M}^L(s)\right)\, \mathrm{d}s\right),$$

where for any $\underline{m} \in [0,\infty)^K$ we define

$$r_j^{L*}(\underline{m}) = p_j \mathbf{I}\left\{\sum_{k=1}^{j} m_k \frac{p_k}{L} + \sum_{k=j+1}^{K} m_k \phi_k \frac{p_j}{\phi_j L} \le C\right\} +$$
$$\mu_j \phi_j \frac{C - \sum_{i\in\mathcal{U}(\underline{m})} p_i m_i}{\sum_{i\in\mathcal{Z}(\underline{m})} \phi_i n_i} \mathbf{I}\left\{\sum_{k=1}^{j} m_k \frac{p_k}{L} + \sum_{k=j+1}^{K} m_k \phi_k \frac{p_j L}{\phi_j} > C\right\}.$$

As previously we devide by L and for having fluid limit we speed up the time by L:

$$\frac{M_j^L(Lt)}{L} = \frac{M_j^L(0)}{L} + \frac{1}{L}\Pi_j^a(\lambda_j Lt)$$
$$-\frac{1}{L}\Pi_j^d\left(\int_0^{Lt} \mu_j M_j^L(s) r_j^{L*}\left(\underline{M}^L(s)\right)\, \mathrm{d}s\right).$$

Using the fact that $\int_0^{Lt} f(s)\,\mathrm{d}s = \int_0^t Lf(Ls)\,\mathrm{d}s$ we have

$$\frac{M_j^L(Lt)}{L} = \frac{M_j^L(0)}{L} + \frac{1}{L}\Pi_j^a(\lambda_j Lt)$$
$$-\frac{1}{L}\Pi_j^d\left(L\int_0^t \mu_j M_j^L(Ls) r_j^{L*}\left(\underline{M}^L(Ls)\right) \frac{1}{L}\, \mathrm{d}s\right). \tag{32}$$

From the definition of r_j^{L*} and r_j it follows that

$$r_j^{L*}\left(M_j^L(Lt)\right) = \frac{1}{L} r_j \left(\frac{M_j^L(Lt)}{L}\right).$$

This equation and (32) implies that

$$\frac{M_j^L(Lt)}{L} = \frac{M_j^L(0)}{L} + \frac{1}{L}\Pi_j^a(\lambda_j Lt)$$
$$-\frac{1}{L}\Pi_j^d\left(\int_0^t \mu_j M_j^L(Ls) r_j \left(\frac{\underline{M}^L(Ls)}{M}\right) \mathrm{d}s\right). \qquad (33)$$

Introducing

$$m_j^L(t) = \frac{M_j^L(Lt)}{L}$$

(33) can be written as

$$m_j^L(t) = m_j^L(0) + \frac{1}{L}\Pi_j^a(\lambda_j Lt)$$
$$-\frac{1}{L}\Pi_j^d\left(L\int_0^t \mu_j m_j^L(s) r_j\left(\underline{m}^L(s)\right) \mathrm{d}s\right).$$

which is the same as equation (23) and for the processes $m_j^L(t), t \geq 0$ we have fluid limit.

5 Performance Evaluation by Wolfram Mathematica

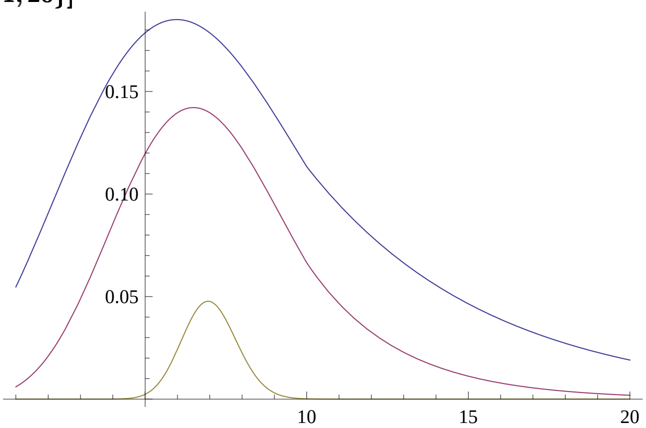
speed up factor should be introduced somehow*)

probmgr[ρ_, r_, k_]:=If $\left[k \leq r, \frac{\frac{(r\rho)^k}{k!}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}, \frac{\frac{(r\rho)^k}{r!}\frac{1}{r^{k-r}}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}\right]$

here ρisλ/size/C*)

Plot[{probmgr[.7, 5, *k*/2], probmgr[.7, 10, *k*], probmgr[.7, 100, 10*k*]},

{*k*, 1, 20}]



Now we define the expected number of customers in this MGR-PS system, extended and scaled down by the speed factor s*) emgr[ρ_, r_, s_]:=$\frac{1}{s}\sum_{k=0}^{\infty}$ probmgr[ρ, sr, k]k*)

emgr[.7, 10, 100]

$$\frac{1}{100}\sum_{k=0}^{\infty} k\mathrm{If}\left[k \le 1000, \frac{(10000.7)^k}{k!\left(\sum_{l=0}^{1000-1}\frac{(10000.7)^l}{l!}+\frac{(10000.7)^{1000}}{1000!(1-0.7)}\right)}, \frac{(10000.7)^k}{1000!1000^{k-1000}\left(\sum_{l=0}^{1000-1}\frac{(10000.7)^l}{l!}+\frac{(10000.7)^{1000}}{1000!(1-0.7)}\right)}\right]$$

nagyobb s ertekekre sokaig tart a szamolas nem is tudja vegigcsinalni, valszeg nem igy kell definialni emgr*)

probmgr[ρ, r, k]

$$\mathrm{If}\left[k \le r, \frac{(r\rho)^k}{k!\left(\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!(1-\rho)}\right)}, \frac{(r\rho)^k}{r!r^{k-r}\left(\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!(1-\rho)}\right)}\right]$$

we should replace the If function above by*)

$$\mathbf{FullSimplify}\left[\sum_{k=0}^{r}\frac{\frac{(r\rho)^k}{k!}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}k+\sum_{k=r+1}^{\infty}\frac{\frac{(r\rho)^k}{r!}\frac{1}{r^{k-r}}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}k\right]$$

$((-1+r(-1+\rho))\rho(r\rho)^r + \left(r(-1+\rho)^2\left((r\rho)^r\left(-r\rho(r\rho)^r + r^r\rho^r(1+r+r\rho) - e^{r\rho}(1+r)\left(-r^{1+r}\rho^r + (r\rho)^r(-2+(2+r)\rho\right.\right.\right.$

it takes several minutes to simplify the expression above*)

a previous version of Math could make a simpler form$\rho\left(r^r\rho^r(1+r-r\rho) - r(-1+\rho)^2\left((r\rho)^r - e^{r\rho}\mathrm{Gamma}[1+r, r\rho]\right)\right)$

$\{\rho = .5, r = 100\}$

$\{0.5, 100\}$

$\{((-1+r(-1+\rho))\rho(r\rho)^r+$

$(r(-1+\rho)^2$

$((r\rho)^r\left(-r\rho(r\rho)^r + r^r\rho^r(1+r+r\rho)-\right.$

$e^{r\rho}(1+r)\left(-r^{1+r}\rho^r + (r\rho)^r(-2+(2+r)\rho)\right)\mathrm{ExpIntegralE}[1-r, r\rho])$

$\mathrm{Gamma}[1+r]^2 - e^{r\rho}(1+r)\left(r^r\rho^r - (r\rho)^r\right)\mathrm{Gamma}[1+r]^3-$

$r(r\rho)^{2r}\mathrm{Gamma}[r]\mathrm{Gamma}[2+r]+$

$e^{r\rho}\mathrm{Gamma}[1+r]\mathrm{Gamma}[3+r]\mathrm{Gamma}[r, r\rho]$

$(-(r\rho)^r + e^{r\rho}r(-1+\rho)\rho\mathrm{Gamma}[r, r\rho]))))/$

$(\mathrm{Gamma}[3+r]\left((r\rho)^r\mathrm{Gamma}[r] - e^{r\rho}(-1+\rho)r!\mathrm{Gamma}[r, r\rho]\right)))/$

$((-1+\rho)\left((r\rho)^r - e^{r\rho}r(-1+\rho)\mathrm{Gamma}[r, r\rho]\right)),$

$\frac{\rho\left(r^r\rho^r(1+r-r\rho)-r(-1+\rho)^2\left((r\rho)^r-e^{r\rho}\mathrm{Gamma}[1+r,r\rho]\right)\right)}{(-1+\rho)\left(-(r\rho)^r+e^{r\rho}r(-1+\rho)\mathrm{Gamma}[r,r\rho]\right)}\}$

$\{50., 50.\}$

Clear[ρ, r]

we use the simpler form to define the scaled expectation value*)

$\frac{\rho\left(r^r\rho^r(1+r-r\rho)-r(-1+\rho)^2\left((r\rho)^r-e^{r\rho}\mathrm{Gamma}[1+r,r\rho]\right)\right)}{(-1+\rho)\left(-(r\rho)^r+e^{r\rho}r(-1+\rho)\mathrm{Gamma}[r,r\rho]\right)}$/.{r->sr}

$\left(\rho\left((rs)^{rs}\rho^{rs}(1+rs-rs\rho) - rs(-1+\rho)^2\left((rs\rho)^{rs} - e^{rs\rho}\mathrm{Gamma}[1+rs, rs\rho]\right)\right)\right)/\left((-1+\rho)\left(-(rs\rho)^{rs} + e^{rs\rho}rs(-1+\rho)\mathrm{Ga}\right.\right.$

emgr[ρ_, r_, s_]:=

$\frac{1}{s}$

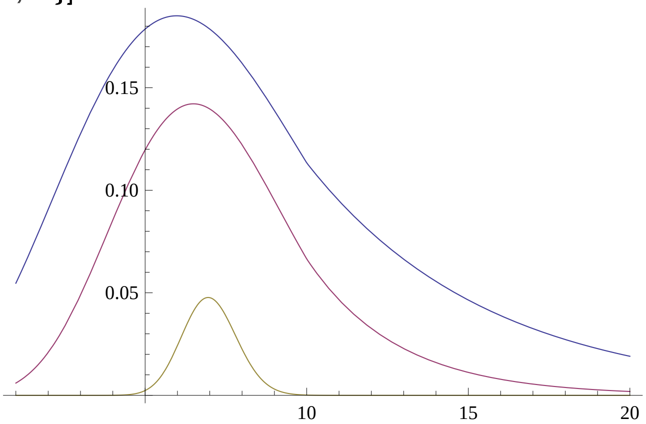
$(\rho((rs)^{rs}\rho^{rs}(1+rs-rs\rho)-$

$rs(-1+\rho)^2\left((rs\rho)^{rs}-e^{rs\rho}\text{Gamma}[1+rs,rs\rho]\right)\big)\big)/$

$((-1+\rho)\left(-(rs\rho)^{rs}+e^{rs\rho}rs(-1+\rho)\text{Gamma}[rs,rs\rho]\right))$

Plot[{probmgr[.7, 5, k/2], probmgr[.7, 10, k], probmgr[.7, 100, 10k]},

{k, 1, 20}]



emgr[.7, 10, 100]

7.00000000000059

emgr[.7, 10, 200]

7.00000000000180

Limit[emgr[.7, 10, s], $s\to\infty$]

$\text{Limit}\left[-\left(2.33333\left(7.^{10s}s^{10s}(1+3.s)-0.9s\left(7.^{10s}s^{10s}-e^{7.s}\text{Gamma}[1+10s,7.s]\right)\right)\right)/\left(s\left(-7.^{10s}s^{10s}-3.e^{7.s}s\text{Gamma}[10s,\right.\right.\right.$

there is some numerical imprecision, it should converge exactly to rρ, that is seven in the above example*)

We try to use the less simpler formula*)

$((-1+r(-1+\rho))\rho(r\rho)^r+$

$\big(r(-1+\rho)^2$

$((r\rho)^r\left(-r\rho(r\rho)^r+r^r\rho^r(1+r+r\rho)-\right.$

$\left.e^{r\rho}(1+r)\left(-r^{1+r}\rho^r+(r\rho)^r(-2+(2+r)\rho)\right)\text{ExpIntegralE}[1-r,r\rho]\right)$

$\text{Gamma}[1+r]^2-e^{r\rho}(1+r)\left(r^r\rho^r-(r\rho)^r\right)\text{Gamma}[1+r]^3-$

$r(r\rho)^{2r}\text{Gamma}[r]\text{Gamma}[2+r]+$

$e^{r\rho}\text{Gamma}[1+r]\text{Gamma}[3+r]\text{Gamma}[r,r\rho]$

$(-(r\rho)^r+e^{r\rho}r(-1+\rho)\rho\text{Gamma}[r,r\rho]))))/$

$(\text{Gamma}[3+r]\left((r\rho)^r\text{Gamma}[r]-e^{r\rho}(-1+\rho)r!\text{Gamma}[r,r\rho]\right)))/$

$((-1+\rho)\left((r\rho)^r-e^{r\rho}r(-1+\rho)\text{Gamma}[r,r\rho]\right))/.\{r\text{->}sr\}$

$\left((-1+rs(-1+\rho))\rho(rs\rho)^{rs}+\left(rs(-1+\rho)^2\left((rs\rho)^{rs}\left(-rs\rho(rs\rho)^{rs}+(rs)^{rs}\rho^{rs}(1+rs+rs\rho)-e^{rs\rho}(1+rs)\left(-(rs)^{1+rs}\rho^{rs}\right.\right.\right.\right.\right.$

emgr2[ρ_, r_, s_]:=

$\frac{1}{s}$

$((-1+rs(-1+\rho))\rho(rs\rho)^{rs}+$

$\left(rs(-1+\rho)^2\right.$

$((rs\rho)^{rs}\left(-rs\rho(rs\rho)^{rs}+(rs)^{rs}\rho^{rs}(1+rs+rs\rho)-\right.$

$e^{rs\rho}(1+rs)\left(-(rs)^{1+rs}\rho^{rs}+(rs\rho)^{rs}(-2+(2+rs)\rho)\right)$

ExpIntegralE$[1-rs,rs\rho])$**Gamma**$[1+rs]^2-$

$e^{rs\rho}(1+rs)\left((rs)^{rs}\rho^{rs}-(rs\rho)^{rs}\right)$ **Gamma**$[1+rs]^3-$

$rs(rs\rho)^{2rs}$**Gamma**$[rs]$**Gamma**$[2+rs]+$

$e^{rs\rho}$**Gamma**$[1+rs]$**Gamma**$[3+rs]$**Gamma**$[rs,rs\rho]$

$(-(rs\rho)^{rs}+e^{rs\rho}rs(-1+\rho)\rho$**Gamma**$[rs,rs\rho])))/$

(**Gamma**$[3+rs]\left((rs\rho)^{rs}\right.$**Gamma**$[rs]-e^{rs\rho}(-1+\rho)(rs)!$**Gamma**$[rs,rs\rho])))/$

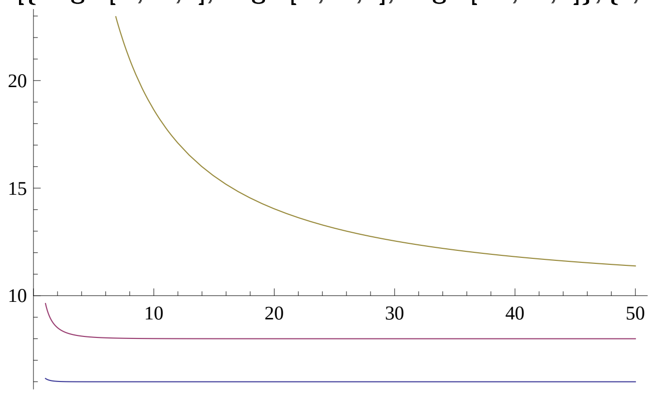
$((-1+\rho)\left((rs\rho)^{rs}-e^{rs\rho}rs(-1+\rho)\right.$**Gamma**$[rs,rs\rho]))$

emgr2[.7, 10, 5]

7.00515

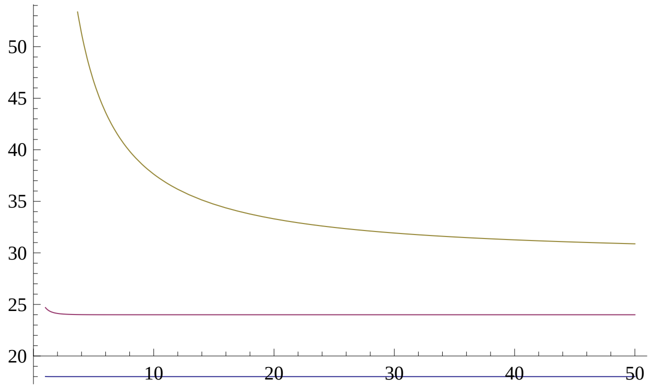
Now this is better, so it is worth using the less simpler (generated by Math 7 version) for the expectation value computation*)

Plot[{emgr2[.6, 10, s], emgr2[.8, 10, s], emgr2[.99, 10, s]}, {s, 1, 50}]



for lower ρ, thespeedupsystemsaveragenumberareconvergingfasterforthefluidlimit*)

Plot[{emgr2[.6, 30, s], emgr2[.8, 30, s], emgr2[.99, 30, s]}, {s, 1, 50}]



Fluid-limit for the elastic model:*)

Plotting the relative difference*)

reldiff[ρ_, r_, s_]:=

$\frac{1}{s}$

$((-1+rs(-1+\rho))\rho(rs\rho)^{rs}+$

$\left(rs(-1+\rho)^2\right.$

$((rs\rho)^{rs}\left(-rs\rho(rs\rho)^{rs}+(rs)^{rs}\rho^{rs}(1+rs+rs\rho)-e^{rs\rho}\right.$

$(1+rs)\left(-(rs)^{1+rs}\rho^{rs}+(rs\rho)^{rs}(-2+(2+rs)\rho)\right)$

ExpIntegralE$[1-rs,rs\rho])$**Gamma**$[1+rs]^2-$

$e^{rs\rho}(1+rs)\left((rs)^{rs}\rho^{rs}-(rs\rho)^{rs}\right)$**Gamma**$[1+rs]^3-$

$rs(rs\rho)^{2rs}$**Gamma**$[rs]$**Gamma**$[2+rs]+$

$e^{rs\rho}$**Gamma**$[1+rs]$**Gamma**$[3+rs]$**Gamma**$[rs,rs\rho]$

$(-(rs\rho)^{rs}+e^{rs\rho}rs(-1+\rho)\rho$**Gamma**$[rs,rs\rho])))/$

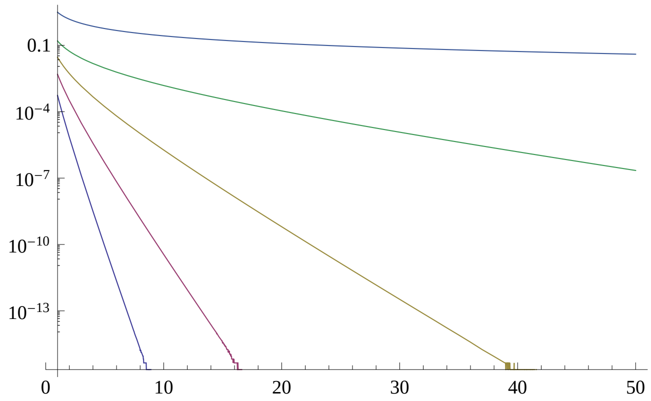
(**Gamma**$[3+rs]$

$((rs\rho)^{rs}$**Gamma**$[rs]-e^{rs\rho}(-1+\rho)(rs)!$**Gamma**$[rs,rs\rho])))/$

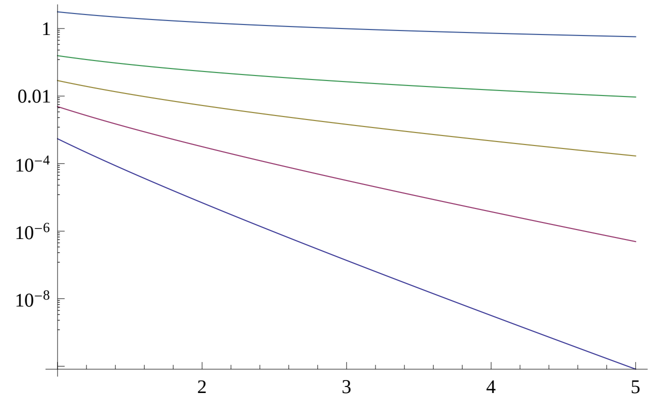
$((-1+\rho)\left((rs\rho)^{rs}-e^{rs\rho}rs(-1+\rho)\right.$**Gamma**$[rs,rs\rho]))/(r\rho)-1$

LogPlot[{reldiff[.6, 30, s], reldiff[.7, 30, s], reldiff[.8, 30, s],

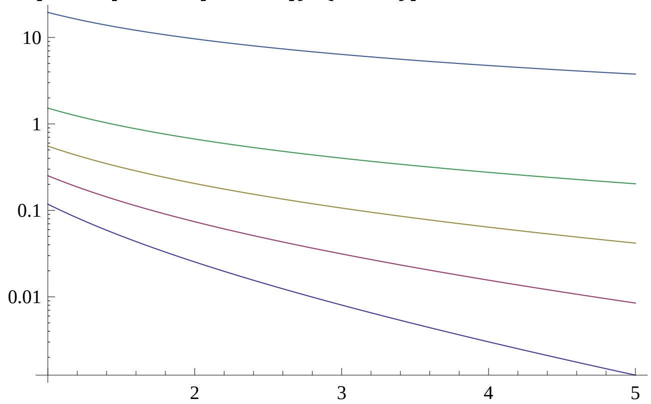
reldiff[.9, 30, s], reldiff[.99, 30, s]}, {s, 1, 50}]



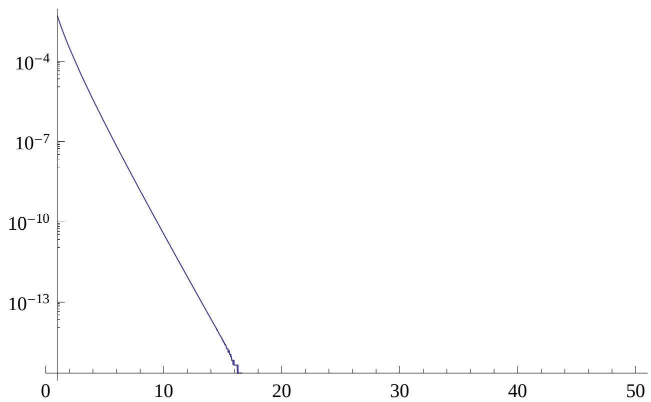
LogPlot[{reldiff[.6, 30, s], reldiff[.7, 30, s], reldiff[.8, 30, s], reldiff[.9, 30, s], reldiff[.99, 30, s]}, {s, 1, 5}]



LogPlot[{reldiff[.6, 5, s], reldiff[.7, 5, s], reldiff[.8, 5, s], reldiff[.9, 5, s], reldiff[.99, 5, s]}, {s, 1, 5}]



LogPlot[{reldiff[.7, 30, s]}, {s, 1, 50}]



Clear[ρ, r]

c = 20

20

ρ = {.1, .2, .4}

{0.1, 0.2, 0.4}

r = {20, 20, 10}

{20, 20, 10}

equ1 = n1 == Max $\left[\frac{\rho[[1]]}{c/r[[1]]}(\text{n1} + \text{n2} + 2\text{n3}), \rho[[1]]r[[1]]\right]$

n1 == Max[2., 0.1(n1 + n2 + 2n3)]

equ2 = n2 == Max $\left[\frac{\rho[[2]]}{c/r[[2]]}(\text{n1} + \text{n2} + 2\text{n3}), \rho[[2]]r[[2]]\right]$

n2 == Max[4., 0.2(n1 + n2 + 2n3)]

equ3 = n3 == Max $\left[\frac{\rho[[3]]}{c/r[[3]]}(\text{n1} + \text{n2} + 2\text{n3}), \rho[[3]]r[[3]]\right]$

n3 == Max[4., 0.2(n1 + n2 + 2n3)]

Solve[{equ1, equ2, equ3}, {n1, n2, n3}]

Solve::ifun : Inverse functions are being used by \!\(*StyleBox[Solve, "MT"]\), so some solutions may not be found; us

{{n1 → 2., n2 → 4., n3 → 4.}}

from excel*)

1.927912 3.85354 3.80454 *)

ρ = {.15, .3, .5}

{0.15, 0.3, 0.5}

r = {50, 25, 50}

{50, 25, 50}

equ1 = n1 == Max $\left[\frac{\rho[[1]]}{c/r[[1]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[1]]r[[1]]\right]$

n1 == Max $\left[7.5, 0.375\left(\frac{2\text{n1}}{5} + \frac{4\text{n2}}{5} + \frac{2\text{n3}}{5}\right)\right]$

equ2 = n2 == Max $\left[\frac{\rho[[2]]}{c/r[[2]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[2]]r[[2]]\right]$

n2 == Max $\left[7.5, 0.375\left(\frac{2\text{n1}}{5} + \frac{4\text{n2}}{5} + \frac{2\text{n3}}{5}\right)\right]$

equ3 = n3 == Max $\left[\frac{\rho[[3]]}{c/r[[3]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[3]]r[[3]]\right]$

n3 == Max $\left[25., 1.25\left(\frac{2\text{n1}}{5} + \frac{4\text{n2}}{5} + \frac{2\text{n3}}{5}\right)\right]$

Solve[{equ1, equ2, equ3}, {n1, n2, n3}]

Solve::ifun : Inverse functions are being used by \!\(*StyleBox[Solve, "MT"]\), so some solutions may not be found; us

{{n1 → 7.5, n2 → 7.5, n3 → 25.}}

from excel*)

6.783443 6.663038 22.54243

it is interesting, that this multidiim case the fluid limit is being approched from below, as opposed to the single dim mgr − pscase*)

the relative error here is higher, appr 10%, meybe due to the higher load of traffic, $\rho = .95$*)

belattam, hogy a max masodikargumentumaanagyobbafluidlimiteseten*)

===*)

c **= 5**

5

ρ **= {.1, .2, .4}**

{0.1, 0.2, 0.4}

r **= {5, 2.5, 5}**

{5, 2.5, 5}

equ1 = n1 == Max $\left[\frac{\rho[[1]]}{c/r[[1]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[1]]r[[1]]\right]$

n1 == Max[0.5, 0.1(n1 + 2.n2 + n3)]

equ2 = n2 == Max $\left[\frac{\rho[[2]]}{c/r[[2]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[2]]r[[2]]\right]$

n2 == Max[0.5, 0.1(n1 + 2.n2 + n3)]

equ3 = n3 == Max $\left[\frac{\rho[[3]]}{c/r[[3]]}(c/r[[1]]\text{n1} + c/r[[2]]\text{n2} + c/r[[3]]\text{n3}), \rho[[3]]r[[3]]\right]$

n3 == Max[2., 0.4(n1 + 2.n2 + n3)]

Solve[{equ1, equ2, equ3}, {n1, n2, n3}]

Solve::ifun : Inverse functions are being used by \!\(*StyleBox[Solve, "MT"]\), so some solutions may not be found; us

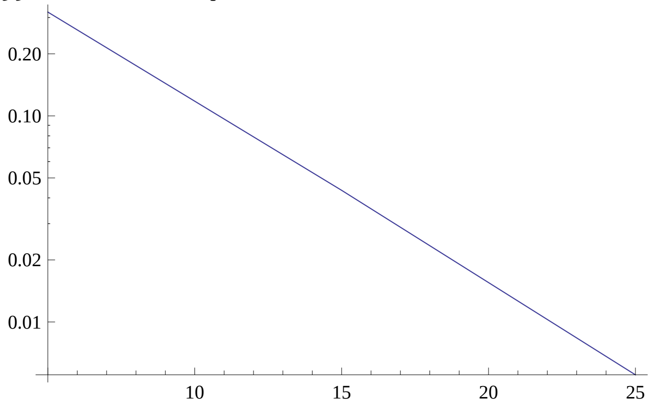
{{n1 → 0.5, n2 → 0.5, n3 → 2.}}

ListLogPlot[{{5, 2/1.516428

−1}, {15, 2/1.916578

−1}, {25, 2/1.98898

−1}}, Joined → True]

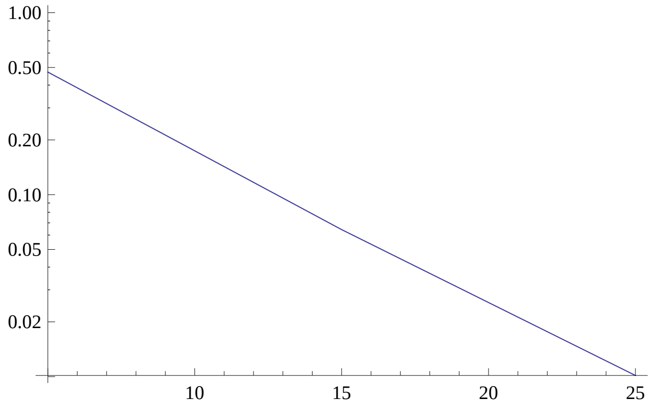


ListLogPlot[{{5, .5/0.339725

−1}, {15, .5/0.4698025

−1}, {25, .5/0.494975

−1}}, Joined → True]



in the logarithmic scale the rel difference between the scaled avergae values and the fluid limit is appr linear.

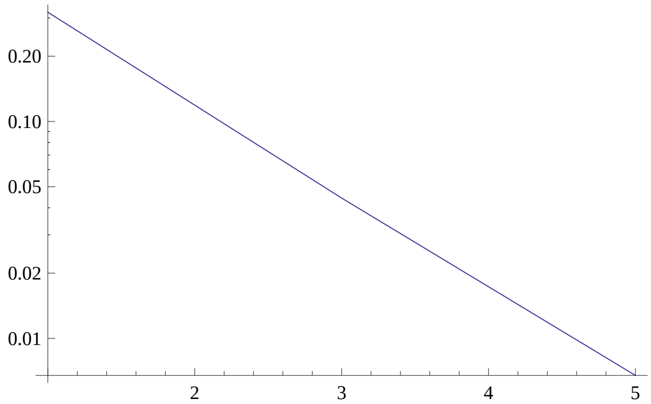
*)

ListLogPlot[{{5, .5/0.379107

−1}, {15, .5/0.4787635

−1}, {25, .5/0.4966475

−1}}, Joined → True]



the three figures above are from the Fluid_limit_elastic.xls speed_up_comparison worksheet*)

we could obtain the same figure in the function of the speed up factor*)

——————————————-*)

Now it is time to prepare the estimation function*)

n1: the average number of cust in the first system, n2 the scaled down av numb of cust in the sped up system, nf is the fluid limit, nc is the required scaled down av numb of cust with capacity c*)

Solve $\left[\frac{\mathrm{Log[nf/n1-1]-Log[nf/n2-1]}}{\mathrm{c1-c2}}(c-\mathrm{c1})+\mathrm{Log[nf/n1-1]}==\mathrm{Log[nf/nc-1]},\right.$

nc]

$$\left\{\left\{\mathrm{nc}\to\frac{\mathrm{nf}}{1+e^{\frac{c\mathrm{Log}\left[-1+\frac{\mathrm{nf}}{\mathrm{n1}}\right]-\mathrm{c2Log}\left[-1+\frac{\mathrm{nf}}{\mathrm{n1}}\right]-c\mathrm{Log}\left[-1+\frac{\mathrm{nf}}{\mathrm{n2}}\right]+\mathrm{c1Log}\left[-1+\frac{\mathrm{nf}}{\mathrm{n2}}\right]}{\mathrm{c1-c2}}}}\right\}\right\}$$

FullSimplify[%1]

$$\left\{\left\{\mathrm{nc}\to\frac{\mathrm{nf}}{1+\left(-1+\frac{\mathrm{nf}}{\mathrm{n1}}\right)^{\frac{c-\mathrm{c2}}{\mathrm{c1-c2}}}\left(-1+\frac{\mathrm{nf}}{\mathrm{n2}}\right)^{\frac{-c+\mathrm{c1}}{\mathrm{c1-c2}}}}\right\}\right\}$$

first we test this estimation for the above figure presented results, speed_up_comparison*)

n1 = {0.379107, 0.339725, 1.516428}; (*thiswasfor*c **= 5*)

n2 = {0.4787635, 0.4698025, 1.916578}; (*thiswasfor*c **= 15*)

n3 = {0.4966475, 0.494975, 1.98898}; (*thiswasfor*c **= 25*)

n3 are to be estimated from n1 and n2*)

nf = {.5, .5, 2.}; (*this is the fluid limit*)

c1 = 5; c2 = 15; c **= 25;**

$$\frac{\mathrm{nf[[1]]}}{1+\left(-1+\frac{\mathrm{nf[[1]]}}{\mathrm{n1[[1]]}}\right)^{\frac{c-\mathrm{c2}}{\mathrm{c1-c2}}}\left(-1+\frac{\mathrm{nf[[1]]}}{\mathrm{n2[[1]]}}\right)^{\frac{-c+\mathrm{c1}}{\mathrm{c1-c2}}}}$$

0.49937

%15/n3[[1]] − 1

0.00548219

$$\frac{\mathrm{nf[[2]]}}{1+\left(-1+\frac{\mathrm{nf[[2]]}}{\mathrm{n1[[2]]}}\right)^{\frac{c-\mathrm{c2}}{\mathrm{c1-c2}}}\left(-1+\frac{\mathrm{nf[[2]]}}{\mathrm{n2[[2]]}}\right)^{\frac{-c+\mathrm{c1}}{\mathrm{c1-c2}}}}$$

0.495659

%19/n3[[2]] − 1

0.00138256

$$\frac{\mathbf{nf[[3]]}}{1+\left(-1+\frac{\mathbf{nf[[3]]}}{\mathbf{n1[[3]]}}\right)^{\frac{c-c2}{c1-c2}}\left(-1+\frac{\mathbf{nf[[3]]}}{\mathbf{n2[[3]]}}\right)^{\frac{-c+c1}{c1-c2}}}$$

1.98819

%21/n3[[3]] − 1

−0.000398233

estimations seems good, the relative error far below one percent*)

==*)

c=50, $\rho = \{.15, .3, .5\}$esetenafluidlimittermeszetesen7.5, 7.5, es25voltak, migazexcel6.783443 6.663038 22.54243adott*)

*feed the excel with two scaled down system,

and from the results try to estimate the values 6.783443 6.663038 22.54243)

le the scaled down system capacity is c=5 and c=10*)

n2 = {0.4550685, 0.41477625, 1.5019775};

n1 = {0.28089975, 0.240985875, 0.89977};

{c1 = 5, c2 = 10, *c* = 50};

nf = {0.75, .75, 2.5};

n3 = {0.67834425, 0.66630375, 2.2542425};

$$\frac{\mathbf{nf[[1]]}}{1+\left(-1+\frac{\mathbf{nf[[1]]}}{\mathbf{n1[[1]]}}\right)^{\frac{c-c2}{c1-c2}}\left(-1+\frac{\mathbf{nf[[1]]}}{\mathbf{n2[[1]]}}\right)^{\frac{-c+c1}{c1-c2}}}$$

0.749925

$$\frac{\mathbf{nf[[2]]}}{1+\left(-1+\frac{\mathbf{nf[[2]]}}{\mathbf{n1[[2]]}}\right)^{\frac{c-c2}{c1-c2}}\left(-1+\frac{\mathbf{nf[[2]]}}{\mathbf{n2[[2]]}}\right)^{\frac{-c+c1}{c1-c2}}}$$

0.749982

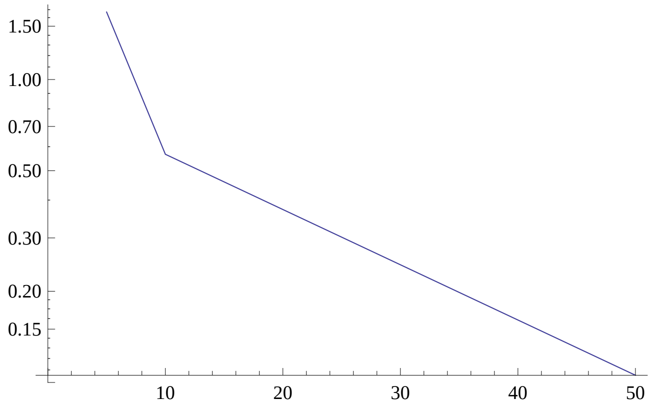
$$\frac{\mathbf{nf[[3]]}}{1+\left(-1+\frac{\mathbf{nf[[3]]}}{\mathbf{n1[[3]]}}\right)^{\frac{c-c2}{c1-c2}}\left(-1+\frac{\mathbf{nf[[3]]}}{\mathbf{n2[[3]]}}\right)^{\frac{-c+c1}{c1-c2}}}$$

2.5

$$\mathbf{list} = \left\{\left\{5, -1+\frac{\mathbf{nf[[1]]}}{\mathbf{n1[[1]]}}\right\}, \left\{10, -1+\frac{\mathbf{nf[[1]]}}{\mathbf{n2[[1]]}}\right\}, \left\{50, -1+\frac{\mathbf{nf[[1]]}}{\mathbf{n3[[1]]}}\right\}\right\}$$

{{5, 1.66999}, {10, 0.566535}, {50, 0.105633}}

ListLogPlot[list, Joined → True]



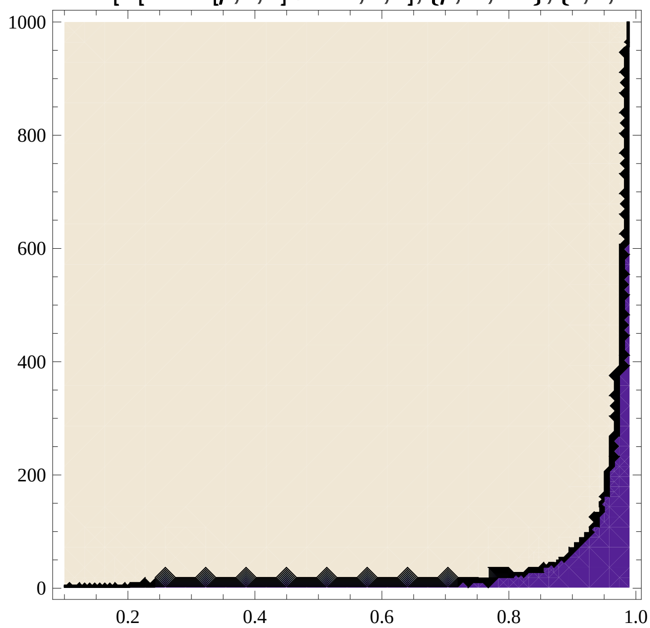
valszeg abbol van nagyobb elteres, hogy a mx flow number nem stimmelt*)

===*)

reldiff[.7, 1000, .1]

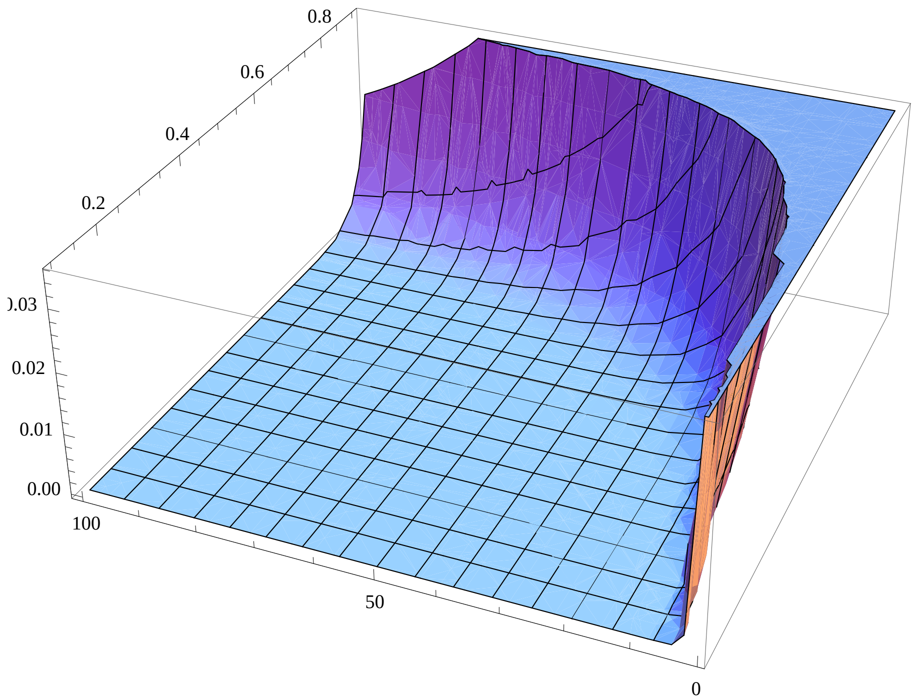
0.0000153085

ContourPlot[If[reldiff[ρ, r, 1] > .05, 0, 1], {ρ, .1, .99}, {r, 1, 1000}]

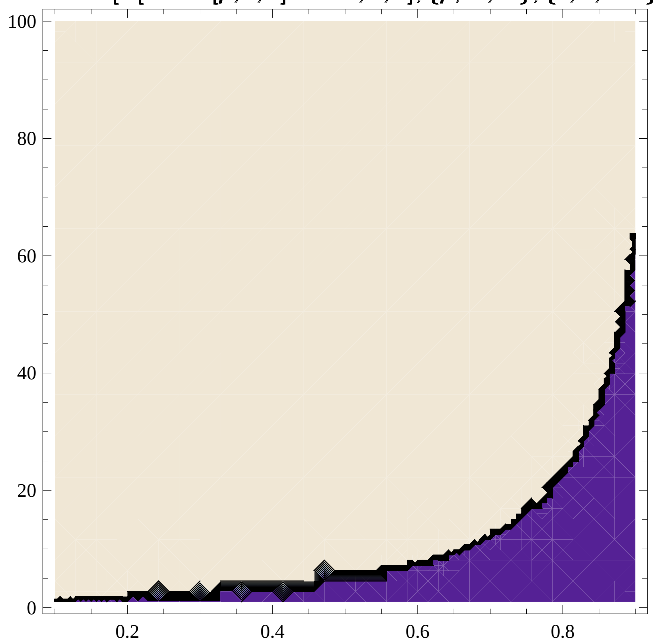


Plot3D[If[reldiff[ρ, r, 1] < .05, reldiff[ρ, r, 1], .05], {ρ, .1, .9},

{r, 1, 100}]



ContourPlot[If[reldiff[ρ, r, 1] > .05, 0, 1], {ρ, .1, .9}, {r, 1, 100}]



FullSimplify[reldiff[ρ, r, s] − reldiff[ρ, sr, 1]]

0

reldiff[.6, 30, 1]

0.000544148

FindRoot[reldiff[.8, r, 1] == .05, {r, 1, 10}]

$\{r \to 22.8105\}$

In the one dimensional case, reldiff$[\rho, 1, 1]$representsthelimitlesscase, whichis$\rho/\{1-\rho\}$*)

emgr2*)

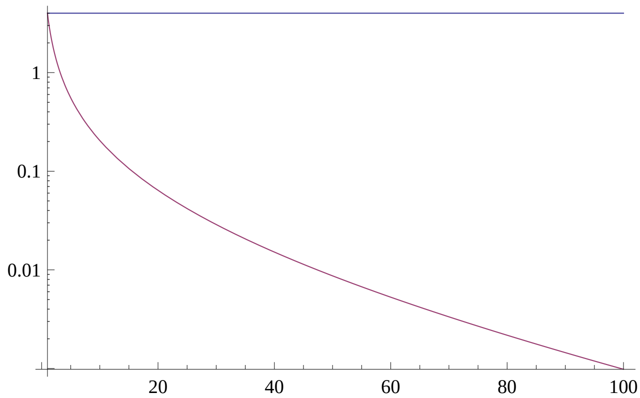
emgr2[.8, 100, 1]/100

0.800786

ρ = .8

0.8

LogPlot[{ρ/(1 − ρ), emgr2[ρ, r, 1]/($r\rho$) − 1}, {r, 1, 100}]



FullSimplify[reldiff[ρ, r, 1] − (emgr2[ρ, r, 1]/($r\rho$) − 1)]

0.

in one dim case, thelimitlesscaseisrepresentedby$\rho/(1-\rho)$andthefluidlimitisrepresentedbyρ.Asthenumberofserversincreas

Clear[ρ]

∂_rLog[emgr2[ρ, r, 1]/($r\rho$) − 1];

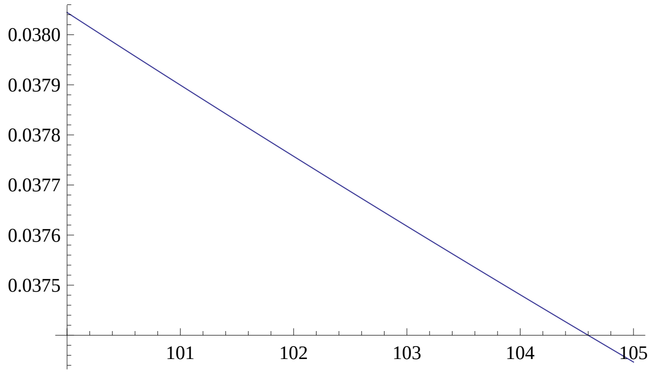
Clear[ρ]

deremgr2[ρ_, r_] = %70;

deremgr2[.5, 100.]

−0.208247

Plot[−deremgr2[.8, r], {r, 100, 105}]



—————————————————*)

approximation issues based on the fluid limit and the one dimensional case*)

some examples should be generated by excel...*)

Fayolle's formula for two dim case, fortheexpectednumberofcustomers*)

$$\mathbf{w1[\lambda1\_, \lambda2\_, \mu1\_, \mu2\_, g1\_, g2\_]:=\frac{\lambda1}{\mu1\left(1-\frac{\lambda1}{\mu1}-\frac{\lambda2}{\mu2}\right)}\left(1+\frac{\mu1\frac{\lambda2}{\mu2}(g2-g1)}{\mu1g1\left(1-\frac{\lambda1}{\mu1}\right)+\mu2g2\left(1-\frac{\lambda2}{\mu2}\right)}\right)}$$

$$\mathbf{w2[\lambda1\_, \lambda2\_, \mu1\_, \mu2\_, g1\_, g2\_]:=\frac{\lambda2}{\mu2\left(1-\frac{\lambda1}{\mu1}-\frac{\lambda2}{\mu2}\right)}\left(1+\frac{\mu2\frac{\lambda1}{\mu1}(g1-g2)}{\mu1g1\left(1-\frac{\lambda1}{\mu1}\right)+\mu2g2\left(1-\frac{\lambda2}{\mu2}\right)}\right)}$$

at kene irni olyan alakba, hogy $\rho1$ $\rho2$ legyen benne, meg a $\mu1$ $\mu2$ *)

$$\mathbf{n1[\rho1\_, \rho2\_, \mu1\_, \mu2\_, g1\_, g2\_]:=\frac{\rho1}{(1-\rho1-\rho2)}\left(1+\frac{\mu1\rho2(g2-g1)}{\mu1g1(1-\rho1)+\mu2g2(1-\rho2)}\right)}$$

$$\mathbf{n2[\rho1\_, \rho2\_, \mu1\_, \mu2\_, g1\_, g2\_]:=\frac{\rho2}{(1-\rho1-\rho2)}\left(1+\frac{\mu2\rho1(g1-g2)}{\mu1g1(1-\rho1)+\mu2g2(1-\rho2)}\right)}$$

*eloszor leteszteljuk, hogy stimmel-e az excellel)

μmosta1/size $* C - t$jeloli, tehat$a$servicetimerequirementreciprokat...*)

n1[.8, 0., .05, 0, 1, 1]

4.

1 dimenziora stimmel*)

n1[.5, .3, .05, .1, 1, 2]

2.72727

n2[.5, .3, .05, .1, 1, 2]

1.04545

stimmel, az excelben is ez jon ki*)

l1 = {2.727255372,

2.753440424,

2.780511948,

2.808554858,

2.837644188,

2.867819339,

2.899138956,

2.931657207,

2.965467135,

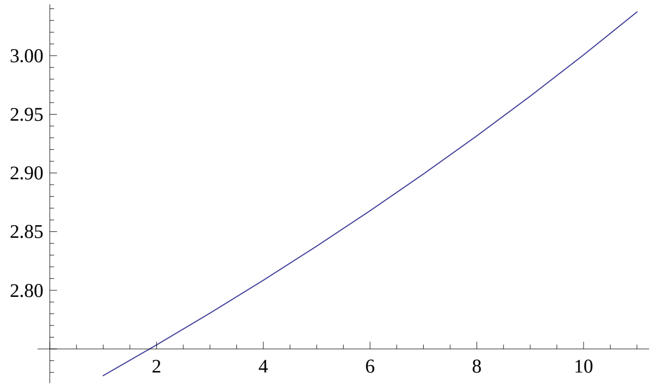
3.000642137,

3.037251894

}

{2.72726, 2.75344, 2.78051, 2.80855, 2.83764, 2.86782, 2.89914, 2.93166, 2.96547, 3.00064, 3.03725}

ListPlot[l1, PlotJoined → True]



ld. approx_to_limit.xls fajl*)

(a fenti abran a szerverek szama az elso osztaly eseten felemelkedett 1-**

rol 2-re, tehat meg mindig nagyon alacsony szerver szam mellett dolgozunk,

a masodik osztalye maradt 1.)

l2 = {1.04544916,

1.050690566,

1.056100034,

1.061708253,

1.067528281,

1.073562276,

1.079824205,

1.086330701,

1.093091299,

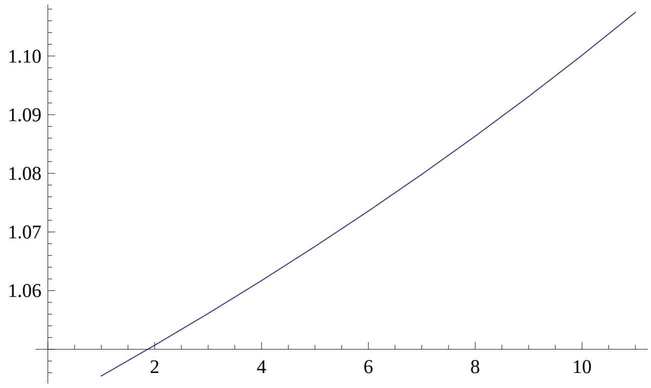
1.100126148,

1.107448328

}

{1.04545, 1.05069, 1.0561, 1.06171, 1.06753, 1.07356, 1.07982, 1.08633, 1.09309, 1.10013, 1.10745}

ListPlot[l2, PlotJoined → True]



aux = ReadList["c:\docs\papers\Dim_for_elastic_traffic\aux_for_math.txt",

{Number, Number, Number}]

{{2.72726, 1.04545, 1}, {2.75344, 1.05069, 1.05263}, {2.78051, 1.0561, 1.11111}, {2.80855, 1.06171, 1.17647}, {2.83764, 1.067

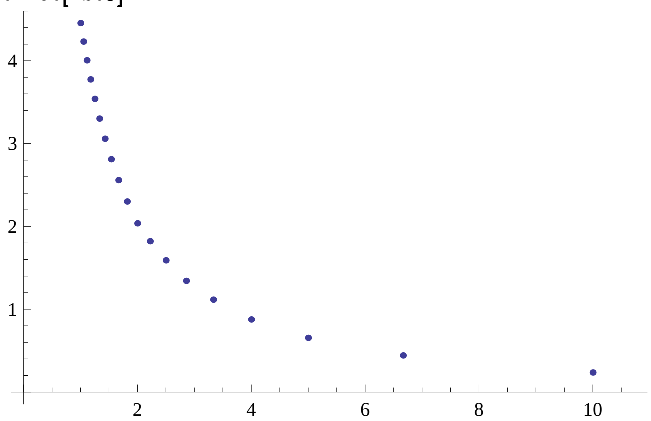
For[i = 1; list3 = {}; list4 = {}, i < 21, i = i + 1,

list3 = Append[list3, {aux[[i]][[3]], aux[[i]][[1]]/aux[[i]][[3]]/.5 − 1}]]

list3

{{1, 4.45451}, {1.05263, 4.23154}, {1.11111, 4.00492}, {1.17647, 3.77454}, {1.25, 3.54023}, {1.33333, 3.30173}, {1.42857, 3.0

ListPlot[list3]



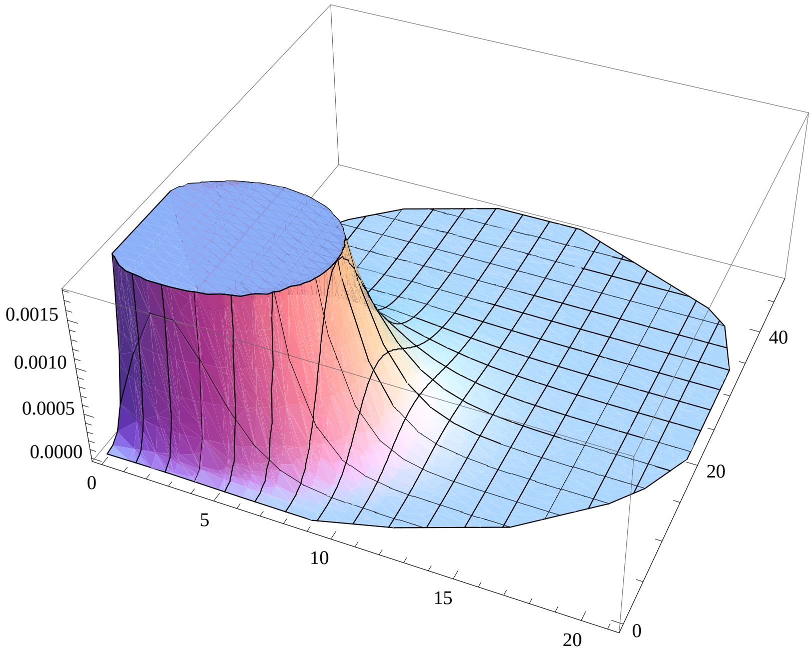
pdflist = ReadList["c:\docs\papers\Dim_for_elastic_traffic\aux2_for_math.txt",

{Number, Number, Number}];

pdflist[[200]][[3]]

2.6288*^-7

ListPlot3D[pdflist]



——————————————————*)

megprobalunk az egy dim alapjan eloallitani egy kozelitest*)

$$\mathbf{Solve}\left[\left\{\mathbf{k1}\frac{\rho}{1-\rho}+\mathbf{k2}==\mathbf{nfay},\mathbf{k1}\rho+\mathbf{k2}==\mathbf{nfluid}\right\},\{\mathbf{k1},\mathbf{k2}\}\right]$$

$$\left\{\left\{\mathrm{k1}\to-\frac{-\mathrm{nfay}+\mathrm{nfluid}+\mathrm{nfay}\rho-\mathrm{nfluid}\rho}{\rho^2},\mathrm{k2}\to-\frac{\mathrm{nfay}-\mathrm{nfluid}-\mathrm{nfay}\rho}{\rho}\right\}\right\}$$

$\mathbf{emgr2}[\rho,r,1]/r$

$((-1+r(-1+\rho))\rho(r\rho)^r+\left(r(-1+\rho)^2\left((r\rho)^r\left(-r\rho(r\rho)^r+r^r\rho^r(1+r+r\rho)-e^{r\rho}(1+r)\left(-r^{1+r}\rho^r+(r\rho)^r(-2+(2+r)\rho\right.\right.\right.\right.$

$\mathbf{estim1}[\rho\_,\mathbf{r}\_,\mathbf{nfay}\_,\mathbf{nfluid}\_]:=$

$\left(-\frac{\mathbf{-nfay+nfluid+nfay}\rho\mathbf{-nfluid}\rho}{\rho^2}\right.$

$(((-1+r(-1+\rho))\rho(r\rho)^r+$

$\left(r(-1+\rho)^2\left((r\rho)^r\left(-r\rho(r\rho)^r+r^r\rho^r(1+r+r\rho)-\right.\right.\right.$

$\left.e^{r\rho}(1+r)\left(-r^{1+r}\rho^r+(r\rho)^r(-2+(2+r)\rho)\right)\right.$

$\mathbf{ExpIntegralE}[1-r,r\rho])\mathbf{Gamma}[1+r]^2-$

$e^{r\rho}(1+r)\left(r^r\rho^r-(r\rho)^r\right)\mathbf{Gamma}[1+r]^3-$

$r(r\rho)^{2r}\mathbf{Gamma}[r]\mathbf{Gamma}[2+r]+e^{r\rho}\mathbf{Gamma}[1+r]\mathbf{Gamma}[$

$3+r]\mathbf{Gamma}[r,r\rho]\left(-(r\rho)^r+e^{r\rho}r(-1+\rho)\rho\mathbf{Gamma}[r,r\rho]\right)))/$

$(\mathbf{Gamma}[3+r]\left((r\rho)^r\mathbf{Gamma}[r]-e^{r\rho}(-1+\rho)r!\mathbf{Gamma}[r,r\rho]\right)))/$

$\left.(r(-1+\rho)\left((r\rho)^r-e^{r\rho}r(-1+\rho)\mathbf{Gamma}[r,r\rho]\right)))-\frac{\mathbf{nfay-nfluid-nfay}\rho}{\rho}\right)r$

ellenorzes arra az esetre, amikor nincs suly es egy osztaly van*)

emgr2[.8, 5, 1]/5

1.24329

estim1[.8, 5, 4, .8]

1.24329

estim1[.5, 20/3, 2.727255372, .5]

3.70351

az approx_to_limit.xls-ben aloallitott ertekekhez kepest nem tunik teljesen rossznak. mindazonaltal alulrol kozelit es jelentos hibaval...*)

a kovetkezokben a fluid_limit_elastic.xls fajlbol kellene peldat venni, c=50, b=1,2,1, g/b konstans*)

ehhez eloszor Fayolle-t kellene szamolni, 3 dimenziora*)

σ[i_, K_]:= $\sum_{l=1}^{K} \frac{\lambda[l]g[l]}{g[i]\mu[i]+g[l]\mu[l]}$

b[k_, j_, K_]:=If $\left[j{=}{=}0, \frac{1}{\mu[k](1-\sigma[k,K])}, \frac{g[j]}{(g[k]\mu[k]+g[j]\mu[j])(1-\sigma[k,K])}\right]$

setoflinequs[K_]:=

{For[aux = {}; $k = 1, k \leq K, k = k + 1$,

aux = Append $\left[\text{aux}, t[k] == b[k,0,K] + \sum_{j=1}^{K} t[j]\lambda[j]b[k,j,K]\right]$], aux}

ez egy lista, amelynek masodik eleme lesz az egyenletek listaja, az aux segitsegevel*)

az alabbi eloallitja harom osztalyra az egyenletrendszert*)

$\{\lambda[1] = .075, \lambda[2] = 1.5, \lambda[3] = 25, \mu[1] = 1/2., \mu[2] = 25/5., \mu[3] = 50.,$

$g[1] = 1, g[2] = 2, g[3] = 1\}$

$\{0.075, 1.5, 25, 0.5, 5., 50., 1, 2, 1\}$

Solve[setoflinequs[3][[2]], $\{t[1], t[2], t[3]\}$]

$\{\{t[1] \to 42.7003, t[2] \to 2.53545, t[3] \to 0.468632\}\}$

{42.7003.075, 2.53541.5, .46863225}

$\{3.20252, 3.8031, 11.7158\}$

estim1[.15, 50, 3.20252, .15]

7.5

estim1[.3, 25, "3.8031", .3]

7.5

estim1[.5, 50, "11.7158", .5]

25.0002

nem rossz, de kb a fluid limitet hozta ki ez a kozelites, amitol azert kb 10%os elteresre van az xls-ben kapott eredmenytol*)

az alabbi approx_to_limit.xls munka3 fulben levo pelda*)

$\{\lambda[1] = .05, \lambda[2] = .05, \lambda[3] = .025, \mu[1] = 1/5., \mu[2] = .1, \mu[3] = 1/4.,$

$g[1] = 2, g[2] = 5, g[3] = 4\}$

$\{0.05, 0.05, 0.025, 0.2, 0.1, 0.25, 2, 5, 4\}$

Solve[setoflinequs[3][[2]], {t[1], t[2], t[3]}]

$\{\{t[1] \to 55.6683, t[2] \to 55.6213, t[3] \to 26.0559\}\}$

{55.6683.05, 55.6213.05, 26.0559.025}

$\{2.78342, 2.78107, 0.651398\}$

{estim1[.25, 10, "2.78342", .25], estim1[.5, 4, 2.78107, .5],

estim1[.1, 5, .651398, .1]}

$\{2.50291, 2.79342, 0.500968\}$

sajnos ez nem jo kozelites*)

emgr2[.45, 10, 1]

4.51548

a Fluid_Limit_MGR_PS.nb fajlban mar megcsinaltam a varhato ertek szamitast, amelyet alabb reprodukalok*)

$$\textbf{FullSimplify}\left[\sum_{k=0}^{r}\frac{\frac{(r\rho)^k}{k!}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}k+\sum_{k=r+1}^{\infty}\frac{\frac{(r\rho)^k}{r!}\frac{1}{r^{k-r}}}{\sum_{l=0}^{r-1}\frac{(r\rho)^l}{l!}+\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}k\right]$$

$\left((-1+r(-1+\rho))\rho(r\rho)^r+\left(r(-1+\rho)^2\left((r\rho)^r\left(-r\rho(r\rho)^r+r^r\rho^r(1+r+r\rho)-e^{r\rho}(1+r)\left(-r^{1+r}\rho^r+(r\rho)^r(-2+(2+r)\rho\right.\right.\right.\right.$

$\text{Gamma}[1+r]^2-e^{r\rho}(1+r)\left(r^r\rho^r-(r\rho)^r\right)\text{Gamma}[1+r]^3-r(r\rho)^{2r}\text{Gamma}[r]\text{Gamma}[2+r]+e^{r\rho}\text{Gamma}[1+r]\text{Gamma}[3+r]$

$\text{Gamma}[r,r\rho]\left(-(r\rho)^r+e^{r\rho}r(-1+\rho)\rho\text{Gamma}[r,r\rho]\right)/\left(\text{Gamma}[3+r]\left((r\right.\right.$

$\rho)^r\text{Gamma}[r]-e^{r\rho}(-1+\rho)r!\text{Gamma}[r,r\rho]/\left((-1+\rho)\left((r\rho)^r-e^{r\rho}r(-1+\rho)\text{Gamma}[r,r\rho]\right)\right)$

Based on the papers Lindberger and Riedl it can be formulated as follows*)

,

erlangc[r_, ρ_]:= $\dfrac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}$

most megnezzuk, hogy a korabban szamolt keplet es a erlangc-es keplet koztt mi a kulonbseg.*)

diff[r_, ρ_]:= $r\rho\left(1+\dfrac{\frac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}}{r(1-\rho)}\right)-$

$((-1+r(-1+\rho))\rho(r\rho)^r+$

$\big(r(-1+\rho)^2$

$((r\rho)^r\,(-r\rho(r\rho)^r + r^r\rho^r(1+r+r\rho)-$

$e^{r\rho}(1+r)\left(-r^{1+r}\rho^r + (r\rho)^r(-2+(2+r)\rho)\right)\,\text{ExpIntegralE}[1-r, r\rho]\big)$

$\text{Gamma}[1+r]^2 - e^{r\rho}(1+r)\,(r^r\rho^r - (r\rho)^r)\,\text{Gamma}[1+r]^3-$

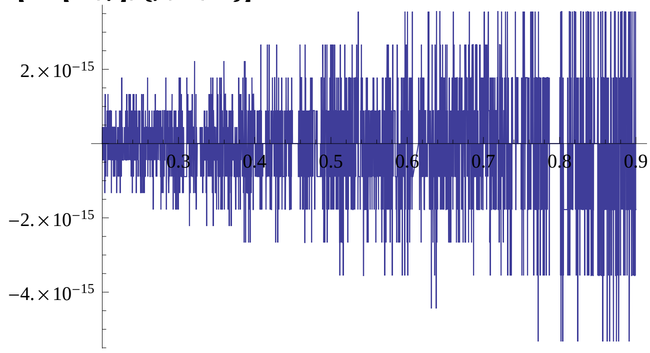
$r(r\rho)^{2r}\text{Gamma}[r]\text{Gamma}[2+r] + e^{r\rho}\text{Gamma}[1+r]\text{Gamma}[3+r]$

$\text{Gamma}[r, r\rho]\,(-(r\rho)^r + e^{r\rho}r(-1+\rho)\rho\text{Gamma}[r, r\rho]))))/$

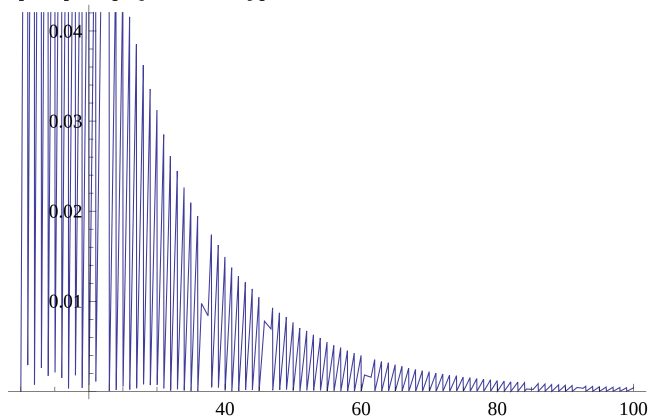
$(\text{Gamma}[3+r]\,((r\rho)^r\text{Gamma}[r] - e^{r\rho}(-1+\rho)r!\text{Gamma}[r, r\rho])))/$

$((-1+\rho)\,((r\rho)^r - e^{r\rho}r(-1+\rho)\text{Gamma}[r, r\rho]))$

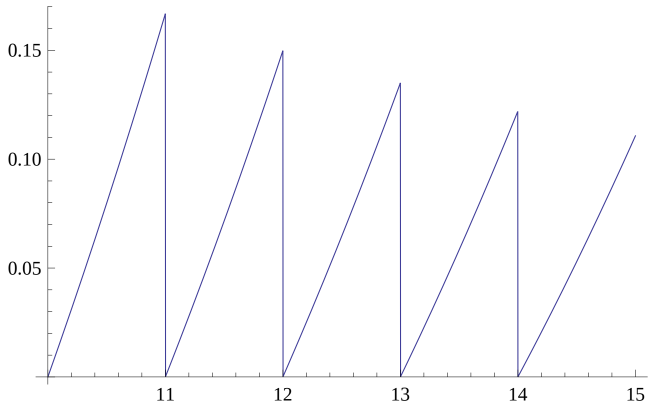
Plot[diff[10, ρ], {ρ, .2, .9}]



Plot[diff[r, .8], {r, 10, 100}]



Plot[diff[r, .8], {r, 10, 15}]



ugy tunik, hogy egesz szerver ertekekre nagy pontossaggal megegyeznek, de az egesz szamok kozott jelentos elteresek mutatkoznak. *)

————*)

kulonbozo egyszerusitesek az erlangc-t tartalmazo formulakra*)

$$\textbf{FullSimplify}\left[\frac{(r\rho)^r}{r!}\right]$$

$$\frac{(r\rho)^r}{r!}$$

$$\textbf{FullSimplify}\left[\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}\right]$$

$$\frac{e^{r\rho}\text{Gamma}[r,r\rho]}{\text{Gamma}[r]}$$

$$\textbf{FullSimplify}\left[r\rho\left(1+\frac{\frac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}}{r(1-\rho)}\right)\right]$$

$$\rho\left(r+\frac{1}{1-\rho+e^{r\rho}r(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}\right)$$

$$\textbf{FullSimplify}\left[1+\frac{1/r}{1-\rho+e^{r\rho}r(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}\right]$$

$$1+\frac{1}{r(1-\rho+e^{r\rho}r(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho])}$$

$$\textbf{FullSimplify}\left[1+\frac{\frac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}}{r(1-\rho)}\right]$$

$$1+\frac{1}{-r(-1+\rho)+e^{r\rho}r^2(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}$$

$$\textbf{FullSimplify}\left[\frac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}\right]$$

$$\frac{1}{1-e^{r\rho}r(-1+\rho)\text{ExpIntegralE}[1-r,r\rho]}$$

Az egyszerusitesbol adodo osszehasonlitasa az eredeti keplettel*)

tehat a nem egesz szerverszamokra a szummat tartalmazo kepletek nem tul jok, mert azokat lepcsosen kozeliti, viszont a matematika altal adott expintegralos formula nem egesz szamokra is jol mukodik es mintha folytonosan kozelitene a varhato erteket*)

Korabbi eredmenyek reprodukalasa az ujabb expintegralos formulaval*)

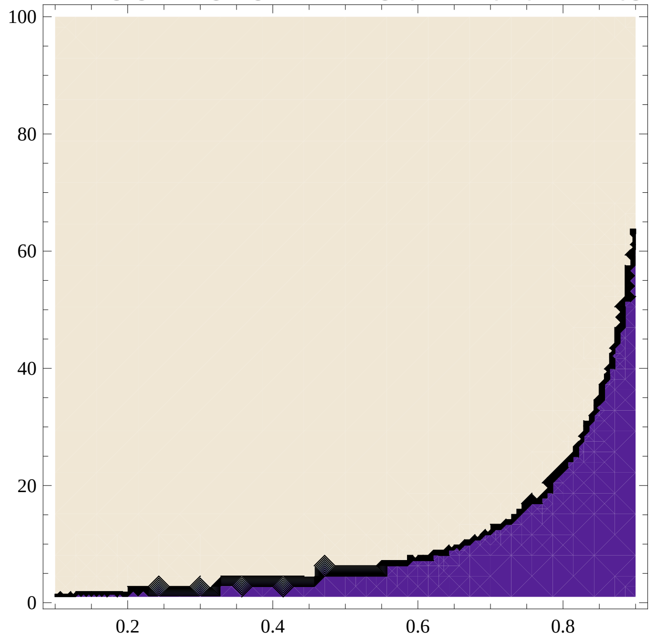
Relativ hiba szamitasa, a fluid limithez kepest*)

$$\textbf{FullSimplify}\left[\frac{\rho\left(r+\frac{1}{1-\rho+e^{r\rho}r(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}\right)-r\rho}{r\rho}\right]$$

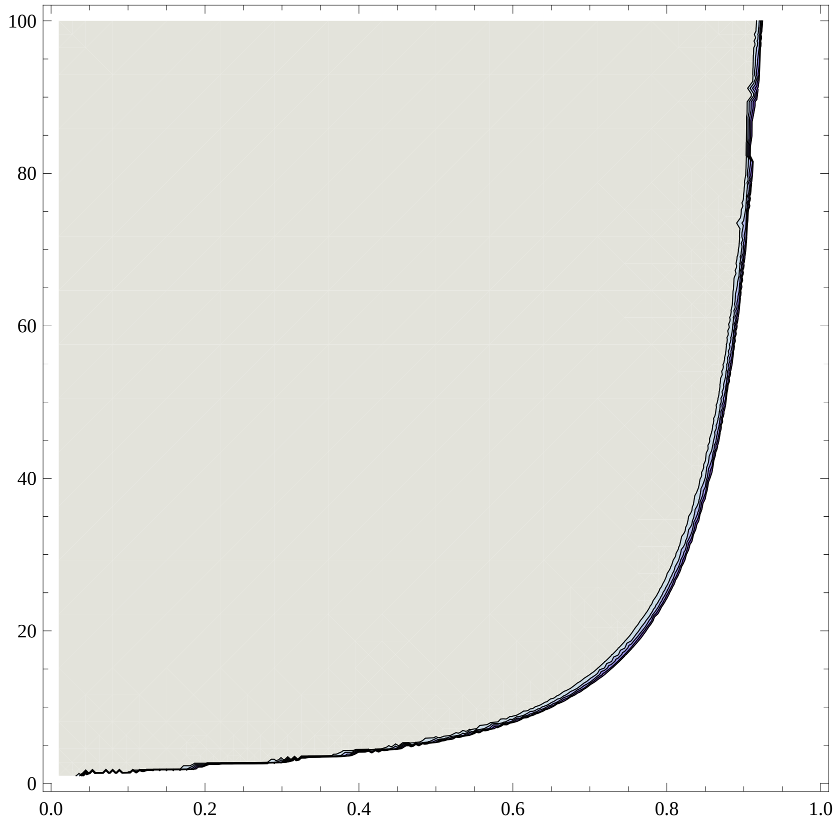
$$\frac{1}{-r(-1+\rho)+e^{r\rho}r^2(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}$$

$$\textbf{reldiff}[\mathrm{r}\_,\rho\_]:=\frac{1}{-r(-1+\rho)+e^{r\rho}r^2(-1+\rho)^2\text{ExpIntegralE}[1-r,r\rho]}$$

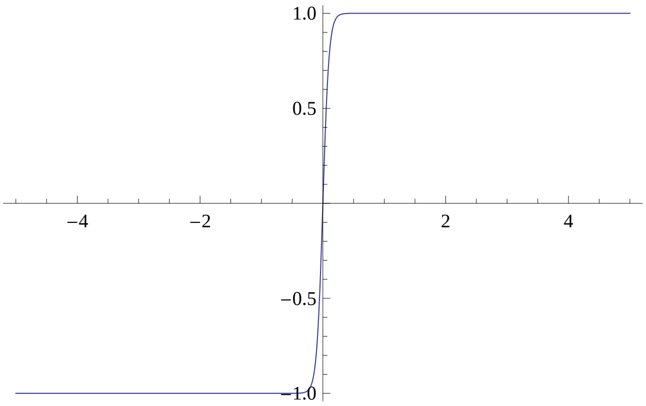
ContourPlot[If[reldiff[r, ρ] > .05, 0, 1], {ρ, .1, .9}, {r, 1, 100}]



$$\textbf{ContourPlot}\left[-1-\text{Tanh}\left[\frac{\text{reldiff}[r,\rho]-.05}{.01}\right],\{\rho,.01,.99\},\{r,1,100\}\right]$$



Plot[Tanh[x/.1], {x, −5, 5}]



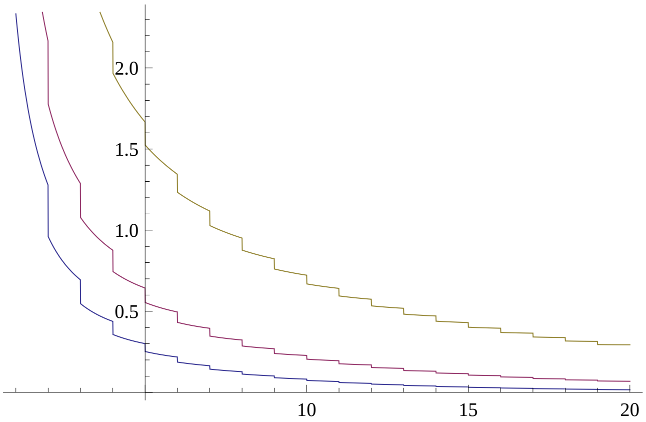
*)

emgr[r_, ρ_]:= $\left(\frac{\frac{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}}{\frac{(r\rho)^r}{r!}\frac{1}{1-\rho}+\sum_{i=0}^{r-1}\frac{(r\rho)^i}{i!}}}{r(1-\rho)}\right)$

emgr[10, 0.7]

0.0739104

Plot[{emgr[r, .7], emgr[r, .8], emgr[r, .9]}, {r, 1, 20}]



===*)

reldiff[r_, ρ_]:=

$(((-1 + r(-1 + \rho))\rho(r\rho)^r +$

$\left(r(-1 + \rho)^2\right.$

$((r\rho)^r\,(-r\rho(r\rho)^r + r^r\rho^r(1 + r + r\rho) - e^{r\rho}(1 + r)$

$\left(-r^{1+r}\rho^r + (r\rho)^r(-2 + (2 + r)\rho)\right)$ ExpIntegralE$[1 - r, r\rho]\big)$

Gamma$[1 + r]^2 - e^{r\rho}(1 + r)\,(r^r\rho^r - (r\rho)^r)$ Gamma$[1 + r]^3-$

$r(r\rho)^{2r}$Gamma$[r]$Gamma$[2 + r] + e^{r\rho}$Gamma$[1 + r]$Gamma$[3 + r]$

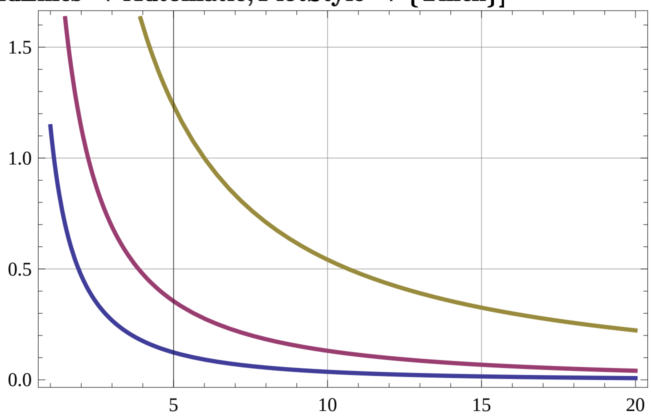
Gamma$[r, r\rho]\,(-(r\rho)^r + e^{r\rho}r(-1 + \rho)\rho$Gamma$[r, r\rho])))/$

(Gamma$[3 + r]\,((r\rho)^r$Gamma$[r] - e^{r\rho}(-1 + \rho)r!$Gamma$[r, r\rho])))/$

$((-1 + \rho)\,((r\rho)^r - e^{r\rho}r(-1 + \rho)$Gamma$[r, r\rho])) - r\rho)/\,r\rho$

Plot[{reldiff[r, .7], reldiff[r, .8], reldiff[r, .9]}, {r, 1, 20}, Frame → True,

GridLines → Automatic, PlotStyle → {Thick}]



FullSimplify[(ρ/(1 − ρ) − ρ)/ρ]

$\frac{\rho}{1-\rho}$

reldiff[10, .7]

0.0362161

$$\mathbf{w1}[\lambda 1\_,\lambda 2\_,\mu 1\_,\mu 2\_,\mathrm{g1}\_,\mathrm{g2}\_]:=\frac{1}{\mu 1\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}\left(1+\frac{\mu 1\frac{\lambda 2}{\mu 2}(\mathrm{g2}-\mathrm{g1})}{\mu 1\mathrm{g1}\left(1-\frac{\lambda 1}{\mu 1}\right)+\mu 2\mathrm{g2}\left(1-\frac{\lambda 2}{\mu 2}\right)}\right)$$

$$\mathrm{FullSimplify}\left[\mathrm{Solve}\left[\frac{1}{\mu 1\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}\left(1+\frac{\mu 1\frac{\lambda 2}{\mu 2}(\mathrm{g2}-\mathrm{g1})}{\mu 1\mathrm{g1}\left(1-\frac{\lambda 1}{\mu 1}\right)+\mu 2\mathrm{g2}\left(1-\frac{\lambda 2}{\mu 2}\right)}\right)==\frac{1}{\mu 1\left(1-\frac{\lambda 1}{\mu 1\mathrm{c1}}\right)},\mathrm{c1}\right]\right]$$

$$\left\{\left\{\mathrm{c1}\to\frac{\lambda 1\left(\mathrm{g1}(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)-\mathrm{g2}\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)\right)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}\right\}\right\}$$

$$\mathrm{FullSimplify}\left[\mathrm{Solve}\left[\frac{1}{\mu 2\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}\left(1+\frac{\mu 2\frac{\lambda 1}{\mu 1}(\mathrm{g1}-\mathrm{g2})}{\mu 1\mathrm{g1}\left(1-\frac{\lambda 1}{\mu 1}\right)+\mu 2\mathrm{g2}\left(1-\frac{\lambda 2}{\mu 2}\right)}\right)==\frac{1}{\mu 2\left(1-\frac{\lambda 2}{\mu 2\mathrm{c2}}\right)},\mathrm{c2}\right]\right]$$

$$\left\{\left\{\mathrm{c2}\to\frac{\mathrm{g1}\lambda 2((\lambda 1-\mu 1)\mu 1-\lambda 1\mu 2)+\mathrm{g2}\lambda 2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}{\lambda 2(\mathrm{g2}\lambda 2+\mathrm{g1}(\lambda 1-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g1}\lambda 1\mu 2^2}\right\}\right\}$$

$$\mathrm{FullSimplify}\left[\frac{\lambda 1\left(\mathrm{g1}(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)-\mathrm{g2}\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)\right)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}+\frac{\mathrm{g1}\lambda 2((\lambda 1-\mu 1)\mu 1-\lambda 1\mu 2)+\mathrm{g2}\lambda 2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}{\lambda 2(\mathrm{g2}\lambda 2+\mathrm{g1}(\lambda 1-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g1}\lambda 1\mu 2^2}\right]$$

$$1+\frac{\mathrm{g2}\lambda 2\mu 1(-\lambda 1-\lambda 2+\mu 1+\mu 2)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}+\frac{\mathrm{g1}\lambda 2((\lambda 1-\mu 1)\mu 1-\lambda 1\mu 2)+\mathrm{g2}\lambda 2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}{\lambda 2(\mathrm{g2}\lambda 2+\mathrm{g1}(\lambda 1-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g1}\lambda 1\mu 2^2}$$

$$f[\mathrm{g1}\_,\mathrm{g2}\_]:=1+\frac{\mathrm{g2}\lambda 2\mu 1(-\lambda 1-\lambda 2+\mu 1+\mu 2)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}+\frac{\mathrm{g1}\lambda 2((\lambda 1-\mu 1)\mu 1-\lambda 1\mu 2)+\mathrm{g2}\lambda 2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}{\lambda 2(\mathrm{g2}\lambda 2+\mathrm{g1}(\lambda 1-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g1}\lambda 1\mu 2^2}$$

$$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.4,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2},\frac{\lambda 1}{\mu 1},\frac{\lambda 2}{\mu 2}\right\}$$

$\{0.1,0.2,0.5,0.4,0.7,0.2,0.5\}$

$f[1,4]$

1.04528

c1[1, 4]

0.245283

c2[1, 4]

0.8

c1[1, 1]

0.285714

c2[1, 1]

0.714286

$$\mathrm{c1}[\mathrm{g1}\_,\mathrm{g2}\_]:=\frac{\lambda 1\left(\mathrm{g1}(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)-\mathrm{g2}\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)\right)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}$$

$$\mathrm{c2}[\mathrm{g1}\_,\mathrm{g2}\_]:=\frac{\mathrm{g1}\lambda 2((\lambda 1-\mu 1)\mu 1-\lambda 1\mu 2)+\mathrm{g2}\lambda 2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}{\lambda 2(\mathrm{g2}\lambda 2+\mathrm{g1}(\lambda 1-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g1}\lambda 1\mu 2^2}$$

FullSimplify[%35]

$$1+\frac{\lambda 2\mu 1(2\lambda 2-\mu 1-2\mu 2)+\lambda 1\lambda 2(\mu 1+\mu 2)}{\lambda 2(\lambda 1+2\lambda 2-\mu 1)\mu 1+(\lambda 1+2\lambda 2)(\lambda 1-\mu 1)\mu 2-\lambda 1\mu 2^2}+\frac{2\lambda 2\mu 1(-\lambda 1-\lambda 2+\mu 1+\mu 2)}{2\lambda 2\mu 1(\lambda 2-\mu 1-\mu 2)+\lambda 1^2\mu 2+\lambda 1(\lambda 2-\mu 2)(\mu 1+2\mu 2)}$$

FullSimplify[%17]

$$\left\{\left\{\mathrm{c1}\to\frac{\lambda 1\left(\mathrm{g1}(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)-\mathrm{g2}\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)\right)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}\right\}\right\}$$

$$\left\{\left\{\mathbf{c1}\to\frac{\lambda 1\left(\mathrm{g1}(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)-\mathrm{g2}\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)\right)}{\lambda 2(\mathrm{g1}\lambda 1+\mathrm{g2}(\lambda 2-\mu 1))\mu 1+(\mathrm{g1}\lambda 1+\mathrm{g2}\lambda 2)(\lambda 1-\mu 1)\mu 2-\mathrm{g2}\lambda 1\mu 2^2}\right\}\right\}$$

$$\left\{\left\{\mathrm{c1}\to\frac{\lambda1\left(\lambda2+(-1+\lambda1)\mu2-\mathrm{g2}\left(\lambda2(1-\mu2)+\mu2^2\right)\right)}{(\lambda1+\mathrm{g2}(-1+\lambda2))\lambda2+(-1+\lambda1)(\lambda1+\mathrm{g2}\lambda2)\mu2-\mathrm{g2}\lambda1\mu2^2}\right\}\right\}$$

FullSimplify[%23]

$$\left\{\left\{\mathrm{c1}\to\frac{\lambda1(\lambda2(1+\mathrm{g2}(-1+\mu2))+\mu2(-1+\lambda1-\mathrm{g2}\mu2))}{(\lambda1+\mathrm{g2}(-1+\lambda2))\lambda2+(-1+\lambda1)(\lambda1+\mathrm{g2}\lambda2)\mu2-\mathrm{g2}\lambda1\mu2^2}\right\}\right\}$$

$\mathbf{g1=1}$

1

$\mathbf{\mu1=1}$

1

$$\mathbf{w2[\lambda1\_,\lambda2\_,\mu1\_,\mu2\_,g1\_,g2\_]:=\frac{1}{\mu2\left(1-\frac{\lambda1}{\mu1}-\frac{\lambda2}{\mu2}\right)}\left(1+\frac{\mu2\frac{\lambda1}{\mu1}(g1-g2)}{\mu1g1\left(1-\frac{\lambda1}{\mu1}\right)+\mu2g2\left(1-\frac{\lambda2}{\mu2}\right)}\right)}$$

$$\left\{\mathbf{\lambda1=.1,\lambda2=.2,\mu1=.5,\mu2=.4,g1=1,g2=2,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}}\right\}$$

Clear[λ1, λ2, μ1, μ2, g1, g2]

{w1[λ1, λ2, μ1, μ2, g1, g2], w2[λ1, λ2, μ1, μ2, g1, g2]}

$\{8.75, 7.5\}$

$$\left\{\mathbf{\lambda1=.1,\lambda2=.2,\mu1=.5,\mu2=.4,g1=1,g2=1,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}}\right\}$$

{w1[λ1, λ2, μ1, μ2, g1, g2], w2[λ1, λ2, μ1, μ2, g1, g2]}

$\{0.1, 0.2, 0.5, 0.4, 1, 1, 0.7\}$

$\{6.66667, 8.33333\}$

most le kellene ellenorizni, hogy elvileg jo-e a fixpontot megalapozo osszefugges, azaz ha kiszamitjuk EN2-t es rogzitjuk,majd Cohen eredmenye alapjan megolduk a PS rendszert az elso esetre, akkor meghatarozzuk EN1 etm ajd ebbol W1et. Ekkor vissza kellene kapnunk az eredetileg kapott W1-et*)

$$\frac{\sum_{n=0}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda1}{\mu1}\frac{g1i+g2\lambda2\text{"}8.33333\text{"}}{g1i}}{\sum_{m=0}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda1}{\mu1}\frac{g1i+g2\lambda2\text{"}8.33333\text{"}}{g1i}\right)}}{\lambda1}$$

6.66667

ez nagyon jol kijott*)

$$\left\{\mathbf{\lambda1=.1,\lambda2=.2,\mu1=.5,\mu2=.4,g1=1,g2=3,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}}\right\}$$

{w1[λ1, λ2, μ1, μ2, g1, g2], w2[λ1, λ2, μ1, μ2, g1, g2]}

$\{0.1, 0.2, 0.5, 0.4, 1, 3, 0.7\}$

$\{10., 7.\}$

$$\frac{\sum_{n=0}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda1}{\mu1}\frac{g1i+g2\lambda27}{g1i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda1}{\mu1}\frac{g1i+g2\lambda27}{g1i}\right)}}{\lambda1}$$

13.

$$\prod_{i=1}^{0} k$$

1

$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.4,\mathrm{g1}=1,\mathrm{g2}=2,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$

$\{\mathrm{w1}[\lambda 1,\lambda 2,\mu 1,\mu 2,\mathrm{g1},\mathrm{g2}],\mathrm{w2}[\lambda 1,\lambda 2,\mu 1,\mu 2,\mathrm{g1},\mathrm{g2}]\}$

$\{0.1,0.2,0.5,0.4,1,2,0.7\}$

$\{8.75,7.5\}$

$$\frac{\sum_{n=0}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2}\lambda 27.5}{\mathrm{g1}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2}\lambda 27.5}{\mathrm{g1}i}\right)}}{\lambda 1}$$

10.

$\frac{8.19+4.39}{2.}$

6.29

$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.5,\mathrm{g1}=2,\mathrm{g2}=1,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$

$\{\mathrm{w1}[\lambda 1,\lambda 2,\mu 1,\mu 2,\mathrm{g1},\mathrm{g2}],\mathrm{w2}[\lambda 1,\lambda 2,\mu 1,\mu 2,\mathrm{g1},\mathrm{g2}]\}$

$\{0.1,0.2,0.5,0.5,2,1,0.6\}$

$\{4.09091,5.45455\}$

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2}\lambda 25.45}{\mathrm{g1}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2}\lambda 25.45}{\mathrm{g1}i}\right)}}{\lambda 1}$$

3.8625

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1}\lambda 14.09}{\mathrm{g2}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1}\lambda 14.09}{\mathrm{g2}i}\right)}}{\lambda 2}$$

6.06

4/5.7.07875

5.663

$\mathrm{Clear}[\lambda 1,\lambda 2,\mu 1,\mu 2,\mathrm{g1},\mathrm{g2}]$

$r[\mathrm{n1},\mathrm{n2}]:=\frac{\mathrm{g1n1}}{\mathrm{g1n1}+\mathrm{g2n2}}$

$\phi[\lambda\_,\mu\_,\mathrm{n}\_]:=\frac{\lambda}{\mu}$

$\mathrm{p1}[\mathrm{n}\_,\mathrm{n2}\_]:=\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2n2}}{\mathrm{g1}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2n2}}{\mathrm{g1}i}\right)}$

$\mathrm{p2}[\mathrm{n1}\_,\mathrm{n}\_]:=\frac{\prod_{i=1}^{n}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}\right)}$

$$\frac{\prod_{i=1}^{n}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 2}{\mu 2}\frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}\right)}$$

$$\frac{\left(\frac{1}{\mathrm{g2}}\right)^n\mathrm{g2}^n\lambda 2^n\left(\frac{1}{\mu 2}\right)^n\mathrm{Pochhammer}\left[1+\frac{\mathrm{g1n1}}{\mathrm{g2}},n\right]}{\left(1-\frac{\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{-\frac{\mathrm{g1n1}}{\mathrm{g2}}}\left(\mu 2+\lambda 2\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}}-\mu 2\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}}\right)}{\lambda 2-\mu 2}\right)n!}$$

$\mathrm{Assuming}[\{i>=0,\mathrm{n1}>=0,\mathrm{n2}>=0,n\geq 0,\mathrm{n\_Integer}\},\mathrm{FullSimplify}[\%1]]$

$$\frac{\left(\frac{1}{\mathrm{g2}}\right)^n\mathrm{g2}^n\lambda 2^n\left(1-\frac{\lambda 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}}\left(\frac{1}{\mu 2}\right)^{1+n}(-\lambda 2+\mu 2)\mathrm{Pochhammer}\left[1+\frac{\mathrm{g1n1}}{\mathrm{g2}},n\right]}{n!}$$

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2n2}}{\mathrm{g1}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{\mathrm{g1}i+\mathrm{g2n2}}{\mathrm{g1}i}\right)}}{\lambda 1}$$

$$\frac{(g1+g2n2)\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}}}{g1(\lambda 1-\mu 1)^2\left(1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}-\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}\right)}{\lambda 1-\mu 1}\right)}$$

FullSimplify[%6]

$$\frac{g1+g2n2}{-g1\lambda 1+g1\mu 1}$$

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{g1i+g2n2+g3n3}{g1i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{g1i+g2n2+g3n3}{g1i}\right)}}{\lambda 1}$$

∞::indet : Indeterminate expression 0DirectedInfinity[g1] encountered. More...

∞::indet : Indeterminate expression 0Abs[g1]DirectedInfinity $\left[-\frac{\text{Sign[g1]}}{\text{Sign}[\langle\langle 1\rangle\rangle]}\right]$ encountered. More...

∞::indet : Indeterminate expression 0DirectedInfinity[g1] encountered. More...

General::stop : Further output of ∞::indet will be suppressed during this calculation. More...

$$\left((g1+g2n2+g3n3)\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}-\frac{g3n3}{g1}}\right)\Big/\left(g1(\lambda 1-\mu 1)^2\left(1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}-\frac{g3n3}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}+\frac{g3n3}{g1}}-\mu 1\left(\frac{-\lambda 1+}{\mu 1}\right.\ldots\right.}{\lambda 1-\mu 1}\right.\right.$$

FullSimplify[%85]

$$\frac{g1+g2n2+g3n3}{-g1\lambda 1+g1\mu 1}$$

ez nagyszeru, az jott ki amire szamitottam*)

$$\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{g1i+g2n2+g3n3}{g1i}$$

$$\frac{\left(\frac{1}{g1}\right)^n g1^n\lambda 1^n\left(\frac{1}{\mu 1}\right)^n\text{Pochhammer}\left[\frac{g1+g2n2+g3n3}{g1},n\right]}{n!}$$

$$1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{g1i+g2n2+g3n3}{g1i}\right)$$

$$1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}-\frac{g3n3}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}+\frac{g3n3}{g1}}-\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}+\frac{g3n3}{g1}}\right)}{\lambda 1-\mu 1}$$

$$\text{w1}[\lambda 1\_,\lambda 2\_,\mu 1\_,\mu 2\_,g1\_,g2\_]:=\frac{1}{\mu 1\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}\left(1+\frac{\mu 1\frac{\lambda 2}{\mu 2}(g2-g1)}{\mu 1 g1\left(1-\frac{\lambda 1}{\mu 1}\right)+\mu 2 g2\left(1-\frac{\lambda 2}{\mu 2}\right)}\right)$$

$$\text{w2}[\lambda 1\_,\lambda 2\_,\mu 1\_,\mu 2\_,g1\_,g2\_]:=\frac{1}{\mu 2\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}\left(1+\frac{\mu 2\frac{\lambda 1}{\mu 1}(g1-g2)}{\mu 1 g1\left(1-\frac{\lambda 1}{\mu 1}\right)+\mu 2 g2\left(1-\frac{\lambda 2}{\mu 2}\right)}\right)$$

$$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.4,g1=1,g2=2,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$$

$$\{\text{w1}[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2],\text{w2}[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]\}$$

$\{0.1, 0.2, 0.5, 0.4, 1, 2, 0.7\}$

$\{8.75, 7.5\}$

$$\frac{\sum_{n=1}^{\infty} n\,\text{p1}[n,n2]}{\lambda 1}$$

$$\frac{(g1+g2n2)\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}}}{g1(\lambda 1-\mu 1)^2\left(1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}-\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}\right)}{\lambda 1-\mu 1}\right)}$$

FullSimplify[%65]

$$\frac{g1+g2n2}{g1(-\lambda 1+\mu 1)}$$

$$\frac{g1+g27.5.2}{g1(-\lambda 1+\mu 1)}$$

10.

p1[n, n2]

$$\frac{\left(\frac{1}{g1}\right)^n g1^n \lambda 1^n \left(\frac{1}{\mu 1}\right)^n \text{Pochhammer}\left[1+\frac{g2n2}{g1}, n\right]}{\left(1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}-\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2}{g1}}\right)}{\lambda 1-\mu 1}\right) n!}$$

FullSimplify[%74]

$$\frac{\left(\frac{1}{g1}\right)^n g1^n \lambda 1^n \left(1-\frac{\lambda 1}{\mu 1}\right)^{\frac{g2n2}{g1}} \left(\frac{1}{\mu 1}\right)^{1+n} (-\lambda 1+\mu 1) \text{Pochhammer}\left[1+\frac{g2n2}{g1}, n\right]}{n!}$$

==============================*)

$$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, g1 = 1, g2 = 100000, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$$

{w1[λ1, λ2, μ1, μ2, g1, g2], w2[λ1, λ2, μ1, μ2, g1, g2]}

{0.1, 0.2, 0.5, 0.4, 1, 100000, 0.7}

{14.9998, 5.0001}

$$\frac{1}{\lambda 1}\frac{\lambda 1/\mu 1}{1-\lambda 1/\mu 1}\frac{g1+g2\lambda 2 w2[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]}{g1}$$

250007.

$$\frac{1}{\lambda 2}\frac{\lambda 2/\mu 2}{1-\lambda 2/\mu 2}\frac{g2+g1\lambda 1 w1[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]}{g2}$$

5.00007

lehet, hogy rossz a fayolle cikk keplete*)

Clear[λ1, λ2, μ1, μ2, g1, g2]

$$\text{equ1} = \frac{1}{\lambda 1}\frac{\lambda 1/\mu 1}{1-\lambda 1/\mu 1}\frac{g1+g2\lambda 2 w2}{g1} == w1$$

$$\frac{g1+g2w2\lambda 2}{g1\left(1-\frac{\lambda 1}{\mu 1}\right)\mu 1} == w1$$

$$\text{equ2} = \frac{1}{\lambda 2}\frac{\lambda 2/\mu 2}{1-\lambda 2/\mu 2}\frac{g2+g1\lambda 1 w1}{g2} == w2$$

$$\frac{g2+g1w1\lambda 1}{g2\left(1-\frac{\lambda 2}{\mu 2}\right)\mu 2} == w2$$

Solve[{equ1, equ2}, {w1, w2}]

$$\left\{\left\{w1 \to -\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, w2 \to -\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$$

w1[λ1, λ2, μ1, μ2, g1, g2]

$$\frac{1+\frac{(-g1+g2)\lambda 2\mu 1}{\mu 2\left(g1\left(1-\frac{\lambda 1}{\mu 1}\right)\mu 1+g2\left(1-\frac{\lambda 2}{\mu 2}\right)\mu 2\right)}}{\mu 1\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}$$

FullSimplify[%39]

$$\frac{-g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g2\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}$$

$$\frac{-g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g2\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)} + \frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}$$

$$\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)} + \frac{-g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g2\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}$$

FullSimplify[%41]

$$\frac{(g1-g2)\lambda 2(g1\lambda 1+g2(\lambda 2-\mu 2))}{g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)(g1(-\lambda 1+\mu 1)+g2(-\lambda 2+\mu 2))}$$

$$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, g1 = 1, g2 = 10, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$$

{w1[λ1, λ2, μ1, μ2, g1, g2], w2[λ1, λ2, μ1, μ2, g1, g2]}

$\{0.1, 0.2, 0.5, 0.4, 1, 10, 0.7\}$

$\{12.9167, 5.83333\}$

$$\frac{(g1-g2)\lambda 2(g1\lambda 1+g2(\lambda 2-\mu 2))}{g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)(g1(-\lambda 1+\mu 1)+g2(-\lambda 2+\mu 2))}$$

-23.75

a kulonbseg g1=g2 eseten 0 lesz, ha g2 sokkal nagyobb mint g1, akkor az en eredmenyemnel elso osztaly atlagos kesleltetese vegtelenhez tart, mig a fayolle eseten nem*)

most leellenorizzuk, hogy fayolle 4.12. keplete jo-e, azaz kijon-e belole a 4.13. as keplet*)

fayolle 4.12. keplete M=2 esetere*)

$$\mathbf{equ3} = w1\left(1-\frac{\lambda 1g1}{\mu 1g1+\mu 1g1}-\frac{\lambda 2g2}{\mu 2g2+\mu 1g1}\right)-\frac{\lambda 1g1w1}{\mu 1g1+\mu 1g1}-\frac{\lambda 2g2w2}{\mu 2g2+\mu 1g1}==\frac{1}{\mu 1}$$

$$-\frac{w1\lambda 1}{2\mu 1}-\frac{g2w2\lambda 2}{g1\mu 1+g2\mu 2}+w1\left(1-\frac{\lambda 1}{2\mu 1}-\frac{g2\lambda 2}{g1\mu 1+g2\mu 2}\right)==\frac{1}{\mu 1}$$

$$\mathbf{equ4} = w2\left(1-\frac{\lambda 1g1}{\mu 1g1+\mu 2g2}-\frac{\lambda 2g2}{\mu 2g2+\mu 2g2}\right)-\frac{\lambda 1g1w1}{\mu 1g1+\mu 2g2}-\frac{\lambda 2g2w2}{\mu 2g2+\mu 2g2}==\frac{1}{\mu 2}$$

$$-\frac{w2\lambda 2}{2\mu 2}-\frac{g1w1\lambda 1}{g1\mu 1+g2\mu 2}+w2\left(1-\frac{\lambda 2}{2\mu 2}-\frac{g1\lambda 1}{g1\mu 1+g2\mu 2}\right)==\frac{1}{\mu 2}$$

Solve[{equ3, equ4}, {w1, w2}]

$$\left\{\left\{w1\to -\frac{g1\lambda 2\mu 1-g2\lambda 2\mu 1+g1\lambda 1\mu 2+g2\lambda 2\mu 2-g1\mu 1\mu 2-g2\mu 2^2}{(g1\lambda 1+g2\lambda 2-g1\mu 1-g2\mu 2)(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, w2\to -\frac{g1\lambda 1\mu 1+g2\lambda 2\mu 1-g1\mu 1^2-g1\lambda 1\mu 2+g2\lambda 1\mu 2-g2\mu 1\mu 2}{(g1\lambda 1+g2\lambda 2-g1\mu 1-g2\mu 2)(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$$

FullSimplify[%80]

$$\left\{\left\{w1\to \frac{-g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g2\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}, w2\to \frac{-g2(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g1\left(\mu 1^2+\lambda 1(-\mu 1+\mu 2)\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}\right\}\right\}$$

$$\mathbf{FullSimplify}\left[\frac{-g1(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)+g2\left(\lambda 2(\mu 1-\mu 2)+\mu 2^2\right)}{(g1(\lambda 1-\mu 1)+g2(\lambda 2-\mu 2))(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}-w1[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]\right]$$

0

meg kellene nezni, hogy konvergens-e az en altalam eloallitott rekurzio*)

a rekurziot az equ1 es equ2 egyenletek hatarozzak meg*)

$$\mathbf{equ1} = \frac{1}{\lambda 1}\frac{\lambda 1/\mu 1}{1-\lambda 1/\mu 1}\frac{g1+g2\lambda 2w2}{g1}==w1$$

$$\frac{g1+g2w2\lambda 2}{g1\left(1-\frac{\lambda 1}{\mu 1}\right)\mu 1}==w1$$

$$\mathbf{equ2} = \frac{1}{\lambda 2}\frac{\lambda 2/\mu 2}{1-\lambda 2/\mu 2}\frac{g2+g1\lambda 1w1}{g2}==w2$$

$$\frac{g2+g1w1\lambda 1}{g2\left(1-\frac{\lambda 2}{\mu 2}\right)\mu 2}==w2$$

Solve[{equ1, equ2}, {w1, w2}]

$$\left\{\left\{w1\to -\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, w2\to -\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$$

$$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.4,g1=1,g2=5,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$$

$$\left\{\left\{w1\to -\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, w2\to -\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$$

$\{0.1, 0.2, 0.5, 0.4, 1, 5, 0.7\}$

$\{\{w1\to 20., w2\to 7.\}\}$

$$\frac{1}{\lambda 1}\frac{\lambda 1/\mu 1}{1-\lambda 1/\mu 1}\frac{g1+g2\lambda 26.99997}{g1}$$

19.9999

$\frac{1}{\lambda 2}\frac{\lambda 2/\mu 2}{1-\lambda 2/\mu 2}\frac{g2+g1\lambda 119.9999}{g2}$

6.99999

a fenti iteracio kb 5-10 lepes alatt nagyon gyorsan konvergalt a helyere*)

persze linearis egyenletrendszerrol van szo, de mindenkeppen varhato ez alapjan, hogy a peak rate limites eset is konvergalni fog*)

Clear[λ1, λ2, μ1, μ2, g1, g2, p1]

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\mathrm{If}\left[\frac{1}{p1}>\frac{g1i+g2n2+g3n3}{g1i},p1,\frac{g1i+g2n2+g3n3}{g1i}\right]}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\mathrm{If}\left[\frac{1}{p1}>\frac{g1i+g2n2+g3n3}{g1i},p1,\frac{g1i+g2n2+g3n3}{g1i}\right]\right)}}{\lambda 1}$$

$$\frac{\sum_{n=1}^{\infty}\frac{n\prod_{i=1}^{n}\frac{\lambda 1\mathrm{If}\left[\frac{1}{p1}>\frac{g1i+g2n2+g3n3}{g1i},p1,\frac{g1i+g2n2+g3n3}{g1i}\right]}{\mu 1}}{1+\sum_{m=1}^{\infty}\prod_{i=1}^{m}\frac{\lambda 1\mathrm{If}\left[\frac{1}{p1}>\frac{g1i+g2n2+g3n3}{g1i},p1,\frac{g1i+g2n2+g3n3}{g1i}\right]}{\mu 1}}}{\lambda 1}$$

szimbolikusan ugy tunk ez nem ertekelheto ki*)

$$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.4,g1=1,g2=5,p1=.01,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$$

$\{0.1, 0.2, 0.5, 0.4, 1, 5, 0.01, 0.7\}$

$$\mathrm{en1}[n\_,n2\_]:=\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\mathrm{Max}\left[\frac{1}{p1},\frac{g1i+g2n2}{g1i}\right]$$

en1[5., 5.]

3.2×10^{6}

$$\sum_{n=1}^{\infty} n\frac{\mathrm{en1}[n,5.]}{1+\sum_{m=1}^{\infty}\mathrm{en1}[m,5.]}$$

$$\sum_{n=1}^{\infty}\frac{n\prod_{i=1}^{n}0.2\mathrm{Max}\left[100.,\frac{25.+i}{i}\right]}{\left(1+\sum_{m=1}^{\infty}\prod_{i=1}^{m}0.2\mathrm{Max}\left[100.,\frac{25.+i}{i}\right]\right)^{2}}$$

FullSimplify[%132]

$$\sum_{n=1}^{\infty}\frac{n\prod_{i=1}^{n}0.2\mathrm{Max}\left[100.,\frac{25.+i}{i}\right]}{\left(1+\sum_{m=1}^{\infty}\prod_{i=1}^{m}0.2\mathrm{Max}\left[100.,\frac{25.+i}{i}\right]\right)^{2}}$$

$$\sum_{n=1}^{\infty}\frac{n\prod_{i=1}^{n}\text{"0.2"}\mathrm{Max}\left[\text{"100."},\frac{\text{"25."}+i}{i}\right]}{\left(1+\sum_{m=1}^{\infty}\prod_{i=1}^{m}\text{"0.2"}\mathrm{Max}\left[\text{"100."},\frac{\text{"25."}+i}{i}\right]\right)^{2}}$$

Ebbenazesetbenpl.mindig*a*100 − atkellvalasztani

$$\frac{\sum_{n=1}^{\infty} n\frac{\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1}\frac{1}{p1}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1}\frac{1}{p1}\right)}}{\lambda 1}$$

$$\frac{p1\mu 1}{(\lambda 1-p1\mu 1)^{2}\left(1-\frac{\lambda 1}{\lambda 1-p1\mu 1}\right)}$$

most megnezzuk, hogy hovatart a fayolle fele w1, ha g2 tart vegtelenhez*)

w1[λ1, λ2, μ1, μ2, g1, g2]

$$\frac{1+\frac{(-g1+g2)\lambda 2\mu 1}{\mu 2\left(g1\left(1-\frac{\lambda 1}{\mu 1}\right)\mu 1+g2\left(1-\frac{\lambda 2}{\mu 2}\right)\mu 2\right)}}{\mu 1\left(1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}\right)}$$

Limit[w1[λ1, λ2, μ1, μ2, g1, g2], g2 → ∞]

$$\frac{\lambda 2(\mu 1-\mu 2)+\mu 2^{2}}{(\lambda 2-\mu 2)(\lambda 2\mu 1+(\lambda 1-\mu 1)\mu 2)}$$

Limit[w2[λ1, λ2, μ1, μ2, g1, g2], g2 → ∞]

$$\frac{1}{-\lambda 2+\mu 2}$$

$$\mathrm{Limit}\left[-\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)},g2\to\infty\right]$$

$\frac{\lambda 1-\mu 1}{\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2}$

$\sum_{n=0}^{\infty} n\rho^n(1-\rho)$

$\frac{(1-\rho)\rho}{(-1+\rho)^2}$

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, \mathrm{g1} = 1, \mathrm{g2} = 10000000, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{\mathrm{w1}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}], \mathrm{w2}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]\}$

$\{0.1, 0.2, 0.5, 0.4, 1, 10000000, 0.7\}$

$\{15., 5.\}$

$\left\{\left\{\mathrm{w1} \to -\frac{-\mathrm{g1}\lambda 2+\mathrm{g2}\lambda 2+\mathrm{g1}\mu 2}{\mathrm{g1}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, \mathrm{w2} \to -\frac{\mathrm{g1}\lambda 1-\mathrm{g2}\lambda 1+\mathrm{g2}\mu 1}{\mathrm{g2}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$

$\{\{\mathrm{w1} \to 3.33333 \times 10^7, \mathrm{w2} \to 6.66667\}\}$

$\left\{\left\{\mathrm{w1} \to -\frac{-\mathrm{g1}\lambda 2+\mathrm{g2}\lambda 2+\mathrm{g1}\mu 2}{\mathrm{g1}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, \mathrm{w2} \to -\frac{\mathrm{g1}\lambda 1-\mathrm{g2}\lambda 1+\mathrm{g2}\mu 1}{\mathrm{g2}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$

$\{\{\mathrm{w1} \to 3.33333 \times 10^7, \mathrm{w2} \to 6.66667\}\}$

$\mathrm{Clear}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]$

$\mathrm{Solve}\left[\left\{\frac{\mathrm{g1}+\mathrm{g2}\lambda 2\mathrm{w2}}{-\mathrm{g1}\lambda 1+\mathrm{g1}\mu 1} == \mathrm{w1}, \frac{\mathrm{g2}+\mathrm{g1}\lambda 1\mathrm{w1}}{-\mathrm{g2}\lambda 2+\mathrm{g2}\mu 2} == \mathrm{w2}\right\}, \{\mathrm{w1}, \mathrm{w2}\}\right]$

$\left\{\left\{\mathrm{w1} \to -\frac{-\mathrm{g1}\lambda 2+\mathrm{g2}\lambda 2+\mathrm{g1}\mu 2}{\mathrm{g1}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, \mathrm{w2} \to -\frac{\mathrm{g1}\lambda 1-\mathrm{g2}\lambda 1+\mathrm{g2}\mu 1}{\mathrm{g2}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .5, \mathrm{g1} = 1, \mathrm{g2} = 1, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{\lambda 1\mathrm{w1}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}] + \lambda 2\mathrm{w2}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]\}$

$\{0.1, 0.2, 0.5, 0.5, 1, 1, 0.6\}$

$\{1.5\}$

$-\lambda 1\frac{-\mathrm{g1}\lambda 2+\mathrm{g2}\lambda 2+\mathrm{g1}\mu 2}{\mathrm{g1}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)} - \lambda 2\frac{\mathrm{g1}\lambda 1-\mathrm{g2}\lambda 1+\mathrm{g2}\mu 1}{\mathrm{g2}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}$

1.5

azonos atlagos meret es sulyok eseten a nagy atlag mindketonel ugyanaz*)

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .5, \mathrm{g1} = 1, \mathrm{g2} = 1, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{\lambda 1\mathrm{w1}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}] + \lambda 2\mathrm{w2}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]\}$

$\{0.1, 0.2, 0.5, 0.5, 1, 1, 0.6\}$

$\{1.5\}$

$-\lambda 1\frac{-\mathrm{g1}\lambda 2+\mathrm{g2}\lambda 2+\mathrm{g1}\mu 2}{\mathrm{g1}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)} - \lambda 2\frac{\mathrm{g1}\lambda 1-\mathrm{g2}\lambda 1+\mathrm{g2}\mu 1}{\mathrm{g2}(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}$

1.5

ha g2 nagyon nagy akkor is ugyanaz lesz a nagy atlaga a varakozasi idonek a fayolle esetben de az enkepletemben nagyon nagy lesz a varhato ertek, emiatt lehet, hogy nem jo az en kepletem*)

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .5, \mathrm{g1} = 1, \mathrm{g2} = 1000000000000000, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{\lambda 1\mathrm{w1}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}] + \lambda 2\mathrm{w2}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]\}$

$\{0.1, 0.2, 0.5, 0.5, 1, 1000000000000000, 0.6\}$

$\{1.5\}$

$-\lambda 1\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}-\lambda 2\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}$

$2.\times 10^{14}$

$\left\{\lambda 1=.1,\lambda 2=.2,\mu 1=.5,\mu 2=.5,g1=1,g2=2,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$

$\left\{\frac{\lambda 1}{\lambda 1+\lambda 2}w1[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]+\frac{\lambda 2}{\lambda 1+\lambda 2}\ w2[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]\right\}$

$\{0.1,0.2,0.5,0.5,1,2,0.6\}$

$\{5.\}$

$-\frac{\lambda 1}{\lambda 1+\lambda 2}\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}-\frac{\lambda 2}{\lambda 1+\lambda 2}\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}$

5.33333

$\text{Limit}\left[-\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)},\{g2\to\infty,\lambda 1\to 0\}\right]$

$\left\{\frac{\lambda 1-\mu 1}{\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2},\frac{1}{-\lambda 2+\mu 2}\right\}$

$\text{Limit}[w2[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2],\lambda 1\to 0]$

$\frac{1}{-\lambda 2+\mu 2}$

$\left\{\lambda 1=.1,\lambda 2=.3,\mu 1=.5,\mu 2=.5,g1=1,g2=2,\frac{\lambda 1}{\mu 1}+\frac{\lambda 2}{\mu 2}\right\}$

$\{w1[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2],w2[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2]\}$

$\{0.1,0.3,0.5,0.5,1,2,0.8\}$

$\{13.75,8.75\}$

$\left\{-\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)},-\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}$

$\{16.,9.\}$

——————————————-———*)

$\text{Clear}[\lambda 1,\lambda 2,\mu 1,\mu 2,g1,g2,p1,n2]$

$\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda 1}{\mu 1}\frac{1}{p1}$

$\left(\frac{\lambda 1}{p1\mu 1}\right)^{\frac{g2n2p1}{g1-g1p1}}$

$\text{Solve}\left[\frac{1}{p1}==\frac{g1 i+g2n2}{g1 i},i\right]$

$\left\{\left\{i\to -\frac{g2n2p1}{g1(-1+p1)}\right\}\right\}$

a peak rate-et figyelmbe vevo szummaban ez az elso tag...*)

$\sum_{n=1}^{\frac{g2n2p1}{g1(1-p1)}}n\prod_{i=1}^{n}\frac{\lambda 1}{\mu 1 i p1}/\left(1+\sum_{m=1}^{\frac{g2n2p1}{g1(1-p1)}}\prod_{i=1}^{m}\frac{\lambda 1}{\mu 1 i p1}+\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda 1}{\mu 1 i p1}\sum_{m=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty}\prod_{i=1}^{m-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda 1}{\mu 1}\frac{g1 i+g2n2}{g1 i}\right)$

$\text{FullSimplify}[\%327]$

$\text{FullSimplify}[\%292,\{\lambda 1>0,\lambda 2>0,\mu 1>0,\mu 2>0,g1>0,g2>0,p1>0\}]$

$-\left(p1\lambda 1\left(-g1(-1+p1)\mu 1+\left(\frac{\lambda 1}{p1\mu 1}\right)^{\frac{g2n2p1}{g1-g1p1}}(g1(-1+p1)\mu 1+g2n2(\lambda 1-p1\mu 1))\right)\right)/\left(g1(-1+p1)(\lambda 1-p1\mu 1)^2\left(1+ \right.\right.$

most vegyuk a peak ratet figyelembe vevo szummaban a masodik reszt*)

$\sum_{n=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty}n\frac{\left(\frac{\lambda 1}{\mu 1p1}\right)^{\frac{g2n2p1}{g1(1-p1)}}\prod_{i=1}^{n-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda 1}{\mu 1}\frac{g1 i+g2n2}{g1 i}}{1+\sum_{m=1}^{\frac{g2n2p1}{g1(1-p1)}}\left(\frac{\lambda 1}{\mu 1p1}\right)^{m}+\left(\frac{\lambda 1}{\mu 1p1}\right)^{\frac{g2n2p1}{g1(1-p1)}}\sum_{m=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty}\prod_{i=1}^{m-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda 1}{\mu 1}\frac{g1 i+g2n2}{g1 i}}$

$$\left(\left(\frac{1}{g1}\right)^{\frac{g2n2p1}{-g1+g1p1}} g1^{-2+\frac{g2n2p1}{-g1+g1p1}}(g1+g2n2)\lambda1^{1+\frac{g2n2p1}{-g1+g1p1}}\left(\frac{1}{\mu1}\right)^{\frac{g2n2p1}{-g1+g1p1}}\left(\frac{\lambda1}{\mu1}\right)^{\frac{g2n2p1}{g1(1-p1)}}\left(\frac{\lambda1}{p1\mu1}\right)^{\frac{g2n2p1}{g1(1-p1)}}\left(-2g1\mu1\text{HypergeometricF}\right.\right.$$

FullSimplify[%294, {λ1 > 0, λ2 > 0, μ1 > 0, μ2 > 0, g1 > 0, g2 > 0, p1 > 0}]

$$\left(\left(\frac{\lambda1^2}{p1\mu1^2}\right)^{\frac{g2n2p1}{g1-g1p1}}\left(\frac{\lambda1}{\mu1}\right)^{\frac{g2n2p1}{-g1+g1p1}}(-\lambda1+\mu1)^{-2-\frac{g2n2}{g1}}(-\lambda1+p1\mu1)\left(g2n2p1(-\lambda1+\mu1)^{2+\frac{g2n2}{g1}}-\mu1^{1+\frac{g2n2}{g1}}(g1(\lambda1-p1\lambda1)+g\right.\right.$$

$$\textbf{FullSimplify}\left[\%294+\%295,\left\{\lambda1>0,\lambda2>0,\mu1>0,\mu2>0,g1>0,g2>0,p1>0,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}<1,p1<1\right\}\right]$$

$Aborted

Limit[%296, p1 → 1]

$$\left\{\lambda1=.1,\lambda2=.2,\mu1=.5,\mu2=.4,g1=1,g2=2,p1=0.01,n2=50,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}\right\}$$

{0.1, 0.2, 0.5, 0.4, 1, 2, 0.01, 50, 0.7}

$$\frac{g1+g2n2}{-g1\lambda1+g1\mu1}$$

252.5

$$\sum_{n=1}^{\frac{g2n2p1}{g1(1-p1)}} n\left(\prod_{i=1}^{n}\frac{\lambda1}{\mu1ip1}\right)\Big/\left(1+\sum_{m=1}^{\frac{g2n2p1}{g1(1-p1)}}\prod_{i=1}^{m}\frac{\lambda1}{\mu1ip1}+\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda1}{\mu1ip1}\sum_{m=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty}\prod_{i=1}^{m-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda1}{\mu1}\frac{g1i+g2n2}{g1i}\right)$$

Sum::div : Sum does not converge. More...

Sum::div : Sum does not converge. More...

$$\frac{20.}{21.+20.\sum_{m=2.0101}^{\infty}0.048024520.2^m}$$

$$\sum_{n=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty} n\left(\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\left(\frac{\lambda1}{\mu1ip1}\right)\prod_{i=1}^{n-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda1}{\mu1}\frac{g1i+g2n2}{g1i}\right)\Big/$$
$$\left(1+\sum_{m=1}^{\frac{g2n2p1}{g1(1-p1)}}\prod_{i=1}^{m}\frac{\lambda1}{\mu1ip1}+\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda1}{\mu1ip1}\sum_{m=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty}\prod_{i=1}^{m-\frac{g2n2p1}{g1(1-p1)}}\frac{\lambda1}{\mu1}\frac{g1i+g2n2}{g1i}\right)$$

Sum::div : Sum does not converge. More...

Sum::div : Sum does not converge. More...

$$\sum_{n=2.0101}^{\infty}\frac{0.96049120.2^n n}{21.+20.\sum_{m=2.0101}^{\infty}0.048024520.2^m}$$

$$\prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}}\left(\frac{\lambda1}{\mu1ip1}\right)$$

$$\frac{\left(\frac{1}{p1}\right)^{\frac{g2n2p1}{g1-g1p1}}\lambda1^{\frac{g2n2p1}{g1-g1p1}}\left(\frac{1}{\mu1}\right)^{\frac{g2n2p1}{g1-g1p1}}}{\frac{g2n2p1}{g1-g1p1}!}$$

$$\textbf{FullSimplify}\left[\%14,\textbf{Assumptions->}\left\{\lambda1>0,\lambda2>0,\mu1>0,\mu2>0,g1>0,g2>0,p1>0,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}<1,p1<1\right\}\right]$$

$$\frac{\left(\frac{\lambda1}{p1\mu1}\right)^{\frac{g2n2p1}{g1-g1p1}}}{\frac{g2n2p1}{g1-g1p1}!}$$

$$\sum_{\square=\square}^{\square}\frac{\left(\frac{\lambda1}{p1\mu1}\right)^{\frac{g2n2p1}{g1-g1p1}}}{\frac{g2n2p1}{g1-g1p1}!}$$

Assuming

Options[Simplify]

{Assumptions :→ $Assumptions, ComplexityFunction → Automatic, TimeConstraint → 300, TransformationFunctions → Automatic, Trig → True}

$$\textbf{Assuming}\left[\left\{\lambda1>0,\lambda2>0,\mu1>0,\mu2>0,g1>0,g2>0,p1>0,\frac{\lambda1}{\mu1}+\frac{\lambda2}{\mu2}<1,p1<1\right\},\right.$$

$$\sum_{n=1}^{\frac{g2n2p1}{g1(1-p1)}} n \prod_{i=1}^{n} \frac{\lambda 1}{\mu 1 i p1} / \left(1 + \sum_{m=1}^{\frac{g2n2p1}{g1(1-p1)}} \prod_{i=1}^{m} \frac{\lambda 1}{\mu 1 i p1} + \prod_{i=1}^{\frac{g2n2p1}{g1(1-p1)}} \frac{\lambda 1}{\mu 1 i p1} \sum_{m=\frac{g2n2p1}{g1(1-p1)}+1}^{\infty} \prod_{i=1}^{m-\frac{g2n2p1}{g1(1-p1)}} \frac{\lambda 1}{\mu 1} \frac{g1 i + g2n2}{g1 i}\right)\Big]$$

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, g1 = 1, g2 = 2, p1 = 0.01, n2 = 50, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{0.1, 0.2, 0.5, 0.4, 1, 2, 0.01, 50, 0.7\}$

%26

1.2645284540662655*^-10 − 1.7285328835108723*^-17i

RealValues

Clear[$\lambda 1, \lambda 2, \mu 1, \mu 2$, g1, g2, p1, n2]

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, g1 = 1, g2 = 2, p1 = 0.01, n2 = 50, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{0.1, 0.2, 0.5, 0.4, 1, 2, 0.01, 50, 0.7\}$

%12

$Aborted

sajnos an umerikus kiertekelest valami miatt nem tudta rovid idon belul megcsinalni*)

Clear[$\lambda 1, \lambda 2, \mu 1, \mu 2$, g1, g2, p1, n2]

$$\frac{\frac{\sum_{n=1}^{\infty} n \prod_{i=1}^{n} \frac{\lambda 1}{\mu 1} \frac{g1 i + g2\lambda 2n2}{g1 i}}{1 + \sum_{m=1}^{\infty}\left(\prod_{i=1}^{m} \frac{\lambda 1}{\mu 1} \frac{g1 i + g2\lambda 2n2}{g1 i}\right)}}{\lambda 1}$$

$$\frac{(g1+g2n2\lambda 2)\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2\lambda 2}{g1}}}{g1(\lambda 1-\mu 1)^2\left(1-\frac{\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{-\frac{g2n2\lambda 2}{g1}}\left(\mu 1+\lambda 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2\lambda 2}{g1}}-\mu 1\left(\frac{-\lambda 1+\mu 1}{\mu 1}\right)^{\frac{g2n2\lambda 2}{g1}}\right)}{\lambda 1-\mu 1}\right)}$$

FullSimplify[%16]

$$\frac{g1+g2n2\lambda 2}{g1(-\lambda 1+\mu 1)}$$

sajnos az egyuttes kiertekelesben benn maradtak gamma fuggvenyek szummai. Meg kellene probalni kulon kiertekelni a masodik tagot is*)

ez kicsit maskeppen nez ki mint az egyuttes kiertekelesben*)

$\left\{\lambda 1 = .1, \lambda 2 = .2, \mu 1 = .5, \mu 2 = .4, g1 = 1, g2 = 2, p1 = 0.01, n2 = 50, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{0.1, 0.2, 0.5, 0.4, 1, 2, 0.01, 50, 0.7\}$

%20

$Aborted

sajnos ez sem megy numerikusan*)

——————————*)

$\left\{\lambda 1 = .2, \lambda 2 = 10, \mu 1 = .5, \mu 2 = 50, g1 = 1, g2 = .1, \frac{\lambda 1}{\mu 1} + \frac{\lambda 2}{\mu 2}\right\}$

$\{0.2, 10, 0.5, 50, 1, 0.1, 0.6\}$

$\left\{\left\{w1 \to -\frac{-g1\lambda 2+g2\lambda 2+g1\mu 2}{g1(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}, w2 \to -\frac{g1\lambda 1-g2\lambda 1+g2\mu 1}{g2(\lambda 2\mu 1+\lambda 1\mu 2-\mu 1\mu 2)}\right\}\right\}$

$\{\{w1 \to 4.1, w2 \to 0.23\}\}$

$$\{\mathrm{w1}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}], \mathrm{w2}[\lambda 1, \lambda 2, \mu 1, \mu 2, \mathrm{g1}, \mathrm{g2}]\}$$

$\{4.89535, 0.259302\}$

$$\frac{1}{\lambda 2} \frac{\lambda 2/\mu 2}{1-\lambda 2/\mu 2} \frac{\mathrm{g2}+\mathrm{g1}\frac{\frac{\lambda 1}{\mu 1}}{1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}}}{\mathrm{g2}}$$

0.275

$$\frac{\frac{1}{\mu 1}}{1-\frac{\lambda 1}{\mu 1}-\frac{\lambda 2}{\mu 2}}$$

$5.$

$$\frac{\prod_{i=1}^{n} \frac{\lambda 2}{\mu 2} \frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}}{1+\sum_{m=1}^{\infty}\left(\prod_{i=1}^{m} \frac{\lambda 2}{\mu 2} \frac{\mathrm{g2}i+\mathrm{g1n1}}{\mathrm{g2}i}\right)}$$

$$\frac{\left(\frac{1}{\mathrm{g2}}\right)^{n} \mathrm{g2}^{n} \lambda 2^{n} \left(\frac{1}{\mu 2}\right)^{n} \mathrm{Pochhammer}\left[1+\frac{\mathrm{g1n1}}{\mathrm{g2}}, n\right]}{\left(1-\frac{\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{-\frac{\mathrm{g1n1}}{\mathrm{g2}}}\left(\mu 2+\lambda 2\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}}-\mu 2\left(\frac{-\lambda 2+\mu 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}}\right)}{\lambda 2-\mu 2}\right) n!}$$

Assuming[{i>=0, n1>=0, n2>=0, $n \geq 0$, n_Integer}, FullSimplify[%1]]

$$\frac{\left(\frac{1}{\mathrm{g2}}\right)^{n} \mathrm{g2}^{n} \lambda 2^{n} \left(1-\frac{\lambda 2}{\mu 2}\right)^{\frac{\mathrm{g1n1}}{\mathrm{g2}}} \left(\frac{1}{\mu 2}\right)^{1+n} (-\lambda 2+\mu 2) \mathrm{Pochhammer}\left[1+\frac{\mathrm{g1n1}}{\mathrm{g2}}, n\right]}{n!}$$

6 Conclusion

In this paper we have analyzed a bandwidth economical discriminatory processor sharing system with access rate limitations, as a possible and realistic model for bandwidth sharing of (elastic) network traffic flows subject to flow control and access rate limits. We have characterized the state-space and determined the unique state-dependent bandwidth shares of such a capacity conserving system, in which the unused capacity of users due to the effect of their access rate limits is fully re-distributed among other users. We have also presented two asymptotic regimes of the system and the detailed performance evaluation of the system by using Wolfram Mathematica.

Appendix

6.1 Random time changes driven by Poisson process

Let $\beta_{\underline{l}} : \mathbb{R}^K \to [0, \infty)$ be given a measurable function for each $\underline{l} \in \mathbb{Z}^K$ such that $\sum_{\underline{l}} \beta_{\underline{l}}(\underline{x}) < \infty$ for any $\underline{x} \in \mathbb{R}^K$. For any bounded function $f : \mathbb{R}^K \to \mathbb{R}$ with finite support set

$$Af(\underline{x}) = \sum_{\underline{l}} \beta_{\underline{l}}(\underline{x}) \left(f\left(\underline{x} + \underline{l}\right) - f\left(\underline{x}\right)\right) \quad \underline{x} \in \mathbb{Z}^K.$$

Let $Y_{\underline{l}}$, $\underline{l} \in \mathbb{Z}^K$ be independent Poisson processes, let $\underline{X}(0)$ be nonrandom, and suppose $\underline{X}$ satisfies

$$\underline{X}(t) = \underline{X}(0) + \sum_{\underline{l}} \underline{l}\ Y_{\underline{l}} \left(\int_0^t \beta_{\underline{l}}(\underline{X}(s))\, \mathrm{d}s\right) \tag{34}$$

Given $\underline{X}(0)$ the solution of (34) is unique. If Af is bounded for any bounded function f with finite support then $\underline{X}(t), t \geq 0$ is a Markov process with generator A. Remark that $\beta_{\underline{l}}(\underline{x})$ is the rate of jumping from $\underline{x}$ to $\underline{x} + \underline{l}$.

The Markov process $\underline{\widehat{X}}_n$ with intensities $q^{(n)}_{\underline{x},\underline{x}+\underline{l}} = n\beta_{\underline{l}}(\underline{x}/n)$ satisfies

$$\underline{\widehat{X}}_n(t) = \underline{\widehat{X}}_n(0) + \sum_{\underline{l}} \underline{l}\, Y_l \left(\int_0^t \beta_{\underline{l}} \left(\frac{\underline{\widehat{X}}_n(s)}{n} \right) \mathrm{d}s \right).$$

Let $\underline{X}_n(t) = \dfrac{\underline{\widehat{X}}_n}{n}$ we have

$$\underline{X}_n(t) = \underline{X}_n(0) + \sum_{\underline{l}} \underline{l} \frac{1}{n} Y_l \left(\int_0^t \beta_{\underline{l}} \left(\underline{X}_n(s) \right) \mathrm{d}s \right). \tag{35}$$

Let

$$F(\underline{x}) = \sum_{\underline{l}} \underline{l} \beta_{\underline{l}}(\underline{x}).$$

The following theorem holds.

Theorem 5 *Suppose that for each compact set $B \subset \mathbb{R}^K$,*

$$\sum_{\underline{l}} |\underline{l}| \sup_{\underline{x} \in B} \beta_{\underline{l}}(\underline{x}) < \infty$$

and there exists $M_B > 0$ such that

$$\left| F(\underline{x}) - F(\underline{y}) \right| \leq M_B \left| \underline{x} - \underline{y} \right|, \quad \underline{x}, \underline{y} \in B.$$

Suppose $\underline{X}_n$ satisfies (35), $\lim_{n\to\infty} \underline{X}_n(0) = \underline{x}_0$, and $\underline{X}$ satisfies

$$\underline{X}(t) = \underline{x}_0 + \int_0^t F(\underline{X}(s))\, \mathrm{d}s, \quad t \geq 0.$$

Then for every $t \geq 0$,

$$\lim_{n\to\infty} \sup_{s \leq t} \left| \underline{X}_n(s) - \underline{X}(s) \right| = 0 \quad a.s.$$

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