# All-Optical Phase Regeneration

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## 1 Introduction

Phase-sensitive amplifiers (PSAs) have recently drawn significant attention due to their potential for phase regeneration through phase squeezing and noiseless amplification [1]. The most common technique of implementing a PSA is to use four-wave mixing (FWM) in a highly non-linear fiber (HNLF) based fiber-optic parametric amplifier (FOPA) to generate phase correlated signal, idler and pump waves that are combined at an input to a second FOPA operated as a PSA [1, 2, 3, 4]. However, the use of cascaded second-order non-linearity in periodically-poled lithium-niobate (PPLN) waveguides also offers the prospect of compact, low latency, broadband devices due to high non-linear coefficients, low spontaneous noise emission, low crosstalk and no intrinsic frequency chirp [2]. Additionally, PPLNs are immune to stimulated Brillouin scattering (SBS), which limits the pump power in FOPA-based devices and requires significant additional complexity, such as phase modulation, or specially strained fibers to overcome. Furthermore, the length of HNLF required increases the requirement of active phase correlation schemes compared to non-linear chips, particularly in interferometric set-ups.

## 2 Four-wave mixing for phase-sensitive amplification

Most phase-sensitive amplifiers exploit Four-Wave Mixing (FWM) for the phase-sensitive operation. The coupled nonlinear equations for the phenomenon are [5]:

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = \frac{in_2'\omega_1}{c}\left[(f_{11}|A_1|^2 + 2\sum_{k\neq 1} f_{1k}|A_k|^2)A_1 + 2f_{1234}A_2^*A_3A_4e^{i\Delta kz}\right] \tag{2.1}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}z} = \frac{in_2'\omega_2}{c}\left[(f_{22}|A_2|^2 + 2\sum_{k\neq 2} f_{2k}|A_k|^2)A_2 + 2f_{2134}A_1^*A_3A_4e^{i\Delta kz}\right] \tag{2.2}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = \frac{in_2'\omega_3}{c}\left[(f_{33}|A_3|^2 + 2\sum_{k\neq 3} f_{3k}|A_k|^2)A_3 + 2f_{3412}A_1A_2A_4^*e^{i\Delta kz}\right] \tag{2.3}$$

$$\frac{\mathrm{d}A_4}{\mathrm{d}z} = \frac{in_2'\omega_4}{c}\left[(f_{44}|A_4|^2 + 2\sum_{k\neq 4} f_{4k}|A_k|^2)A_4 + 2f_{4312}A_1A_2A_3^*e^{i\Delta kz}\right] \tag{2.4}$$

$$\Delta k = (n_3\omega_3 + n_4\omega_4 - n_1\omega_1 - n_2\omega_2)/c \tag{2.5}$$

where the first term is Self-Phase Modulation (SPM) the second is Cross-Phase Modulation (XPM) and the third one is FWM. As can be seen FWM can only occur permanently if the phase mismatch is zero. If the mismatch is not zero, the FWM-caused power transfer will periodically change sign, thus move around zero. It can be seen that XPM and SPM can be handled as the special cases of FWM, which inherently guarantees the phase matching condition. Thus, these effects appear at any dispersion in the fiber.

Let us now introduce the effective core area instead of the overlap integral and the nonlinear constant and assume they are the same for all waves:

$$A_{\text{eff}} = \frac{1}{f_{1234}} = \frac{1}{f_{2134}} = \frac{1}{f_{3412}} = \frac{1}{f_{4312}} \tag{2.6}$$

$$\gamma = \frac{n_2'\omega}{cA_{\text{eff}}} \tag{2.7}$$

In normal fiber phase matching is reached through operating in the near-zero dispersion domain. To be precise, $\Delta k$ should be exactly zero at high powers due to SPM and XPM effects shifting the signal and pump phase. This can be taken into account by solving the above equation for one strong wave (e.g. pump) neglecting fiber losses. We only need to take into accout SPM and get

$$\frac{\mathrm{d}A}{\mathrm{d}z} = i\gamma|A|^2 A. \tag{2.8}$$

From the undepleted pump approximation $|A|^2 = P$, where $P$ is the pump power. Then we get

$$A(z) = \sqrt{P}e^{i\gamma P z}. \tag{2.9}$$

From which the pump phase shift is obvious.

As the effect of SPM and XPM can be compensated by tuning $\Delta k$, from now on let's only take into accout FWM. If we choose equation 2.1 and assume $\Delta k = 0$ for FWM we get:

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = iA_2^* A_3 A_4 \cdot \text{const.} \tag{2.10}$$

If $A_2$, $A_3$ and $A_4$ are undepleted, they generate a fourth signal ($A_1$), with $pi/2$ phase shift. If there is already a signal present at this wavelength, the superposition of the original and the generated signal will be present. This phase shift will play an important role in determining the power flow direction. Next let us assume a degenerate setup with $A_1$ and $A_2$ coincident (figure 2.1)

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = i\gamma \cdot 2A_1^* A_3 A_4 e^{j\Delta k z} \tag{2.11}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = i\gamma \cdot 2A_1 A_1 A_4^* e^{-j\Delta k z} \tag{2.12}$$

$$\frac{\mathrm{d}A_4}{\mathrm{d}z} = i\gamma \cdot 2A_1 A_2 A_3^* e^{-j\Delta k z} \tag{2.13}$$

assuming perfect phase matching ($\Delta k = 0$) we get

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = i\gamma \cdot 2A_1^* A_3 A_4 \tag{2.14}$$

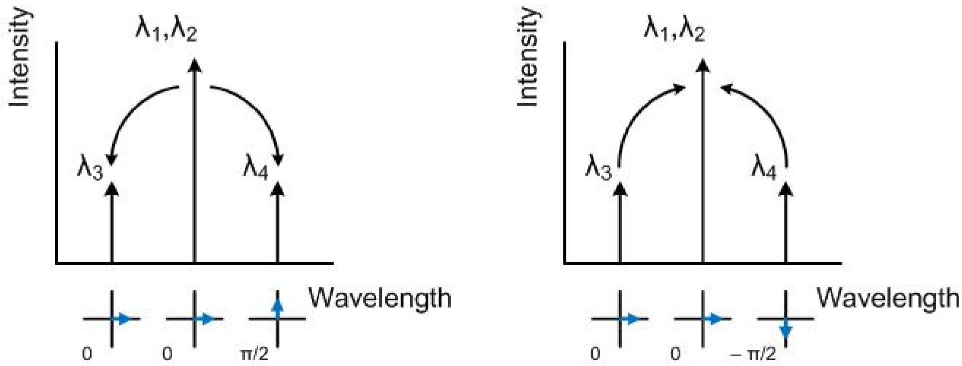


Figure 2.1: Phase changes and power flow through four-wave mixing controlling a non-degenerate wave

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = i\gamma \cdot 2A_1A_1A_4^* \tag{2.15}$$

$$\frac{\mathrm{d}A_4}{\mathrm{d}z} = i\gamma \cdot 2A_1A_2A_3^* \tag{2.16}$$

If there are only two waves present in the system on the input, a new wave will be generated. If we use $\phi_3$ and $\phi_4$ for the phases of $A_1(0)$ and $A_3(0)$, we get for $A_4(\mathrm{d}z)$

$$\phi_4(\mathrm{d}z) = 2\phi_1(0) - \phi_3(0) + \pi/2. \tag{2.17}$$

This results in stable phases for the signal and the idler with $\mathrm{d}A_s$ and $\mathrm{d}A_i$ always pointing in the same direction as their current value while gives decreasing pump power as $\mathrm{d}A_p$ will always point to the opposite complex direction as its current value. Let us now see how the other waves change using the notation $\mathrm{d}\phi_m$ for the phase of $\mathrm{d}A_m$

$$\mathrm{d}\phi_1 = \pi/2 - \phi_1 + \phi_3 + \phi_4 = \pi/2 - \phi_1 + \phi_3 + 2\phi_1 - \phi_3 + \pi/2 = \pi + \phi_1 = -\phi_1 \tag{2.18}$$

$$\mathrm{d}\phi_3 = \pi/2 + 2\phi_1 - \phi_4 = \pi/2 + 2\phi_1 - 2\phi_1 + \phi_3 - \pi/2 = \phi_3 \tag{2.19}$$

$$\mathrm{d}\phi_4 = \phi_4 \tag{2.20}$$

Which shows that the power from the degenerate wave will keep flowing from the other two waves. Now if the third wave is present with arbitrary phase:

$$\phi_4 = 2\phi_1 - \phi_3 + \pi/2 + \Delta\phi \tag{2.21}$$

$$\mathrm{d}\phi_1 = -\phi_1 + \Delta\phi \tag{2.22}$$

$$\mathrm{d}\phi_3 = \phi_3 - \Delta\phi \tag{2.23}$$

$$\mathrm{d}\phi_4 = \pi/2 + 2\phi_1 - \phi_3 = \phi_4 - \Delta\phi \tag{2.24}$$

In the general case this is difficult to evaluate. But let us now take the special cases of $\Delta\phi = 0$ and $\Delta\phi = \pi$. In the first case, obviously, $A_3$ and $A_4$ will keep being amplified and their phase will not be distorted. On the other hand, the changes on $A_1$ will have the opposite sign as the current amplitude, thus its phase will not be distorted, but its amplitude will decrease. Effectively, the process will transfer power from the $A_1$ to the $A_3$ and $A_4$. For $\Delta\phi = \pi$ the process will reverse, transferring power from $A_3$ and $A_4$ to $A_1$. Next, let us check what happens when we apply the phase shift to $A_1$.

$$\phi_1 = \phi_{10} + \Delta\phi \tag{2.25}$$

$$\mathrm{d}\phi_3 = \pi/2 + 2\phi_1 - \phi_4 = \pi/2 + 2\phi_1 + 2\Delta\phi - 2\phi_1 + \phi_3 - \pi/2 = \phi_3 + 2\Delta\phi \tag{2.26}$$

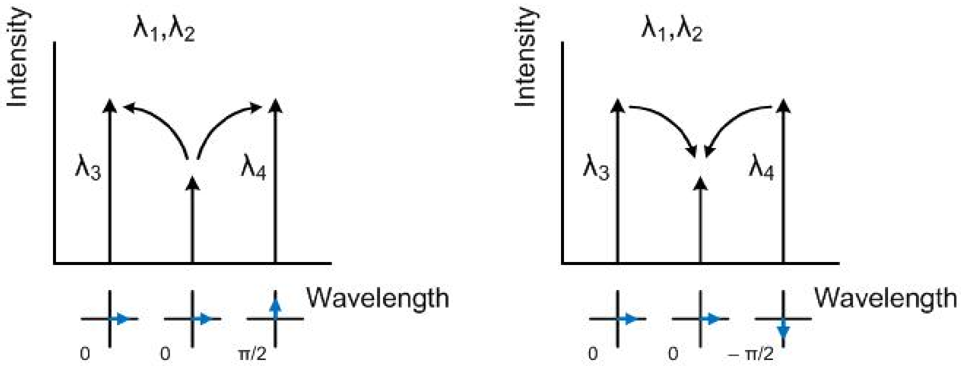


Figure 2.2: Phase changes and power flow through four-wave mixing controlling a degenerate wave

$$\mathrm{d}\phi_4 = 2\phi_1 - \phi_3 + \pi/2 = 2\phi_{10} + 2\Delta\phi - \phi_3 + \pi/2 = \phi_4 + 2\Delta\phi \tag{2.27}$$

Similiar to the the above discussion it can be seen, that power will be transferred from $A_1$ to $A_3$ and $A_4$ if the phase difference is $n \cdot \pi$, and from $A_3$ and $A_4$ to $A_1$ if the phase difference is $(2n-1) \cdot \pi/2$.

As a consequence we can deduct that we need to apply the phase shift for the degenerate wave (in a degenerate configuration) if we want to regenerate a BPSK signal. In this paper we will focus on the latter, which is usually referred to as a double pump or degenerate signal/idler configuration. From now on we will call the waves pump1, signal and pump2. Let us solve the equation for the signal in the undepleted pump approximation.

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = i\gamma \cdot 2A_s^* A_{p1} A_{p2} \tag{2.28}$$

Let us select the phases of pump1 and pump2 so that the coefficient of $A_s^*$ be real.

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = C \cdot A_s^* \tag{2.29}$$

where $C = i\gamma \cdot 2A_{p1}A_{p2}$. Let us use $z = x + i \cdot y$, and so we get for the real and imaginary components

$$\frac{\partial A_s}{\partial x} = CA_s^R \tag{2.30}$$

$$\frac{\partial A_s}{\partial y} = -CA_s^I \tag{2.31}$$

where $A_s^R$ and $A_s^I$ denote the real and imaginary component of $A_s$. The solutions are

$$A_s^R = C^R \exp(Cx) \tag{2.32}$$

and

$$A_s^I = C^I \exp(-Cy) \tag{2.33}$$

resulting in

$$A_s = C_R \exp(Cz) + iC_I \exp(-Cz). \tag{2.34}$$

which clearly shows that the imaginary component of the signal is suppressed while the real component is amplified.

Despite being unsuitable for BPSK regeneration, the undepleted pump configuraion is often important too. Let us now try a similar derivation for the signal and idler using the undepleted pump approximation.

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = i\gamma \cdot 2A_p^2 A_i^* \tag{2.35}$$

$$\frac{\mathrm{d}A_i}{\mathrm{d}z} = i\gamma \cdot 2A_p^2 A_s^* \tag{2.36}$$

Let us use the constant $C = \gamma 2A_p^2$ and assume the phase of the pump is zero without the loss of generality.

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = iCA_i^* \tag{2.37}$$

$$\frac{\mathrm{d}A_i}{\mathrm{d}z} = iCA_s^* \tag{2.38}$$

Then we get

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = iCA_i^R + CA_i^I \tag{2.39}$$

$$\frac{\mathrm{d}A_i}{\mathrm{d}z} = iCA_s^R + CA_s^I \tag{2.40}$$

$$\begin{matrix} \frac{\mathrm{d}A_s^R}{\mathrm{d}x} = CA_i^I & \frac{\mathrm{d}A_s^I}{\mathrm{d}y} = CA_i^R \\ \frac{\mathrm{d}A_i^R}{\mathrm{d}x} = CA_s^I & \frac{\mathrm{d}A_i^I}{\mathrm{d}y} = CA_s^R \end{matrix} \tag{2.41}$$

Let us differentiate the first equation again

$$\frac{\mathrm{d}^2 A_s^R}{\mathrm{d}x^2} = C\frac{\mathrm{d}A_i^I}{\mathrm{d}x} = C^2 A_s^R \tag{2.42}$$

Similarly

$$\frac{\mathrm{d}^2 A_s^I}{\mathrm{d}y^2} = C\frac{\mathrm{d}A_i^R}{\mathrm{d}y} = C^2 A_s^I \tag{2.43}$$

which gives for the signal

$$A_s(z) = C_1 e^{Cz} + C_2 e^{-Cz} + i\left(C_3 e^{Cz} + C_4 e^{-Cz}\right) \tag{2.44}$$

for the idler we get

$$A_i(z) = i\left(C_1 e^{Cz} - C_2 e^{-Cz}\right) - C_3 e^{Cz} + C_4 e^{-Cz} \tag{2.45}$$

Let us choose 0 for the input signal phase and $\pi/2$ for the idler, which gives the highest phase-sensitive amplification as seen before. In this case $A_s(0) = A$ and $A_i(0) = iA$. This gives $C_1 + C_2 = C_1 - C_2 = A$, thus, $C_1 = A$, $C_2 = 0$.

$$A_s(z) = Ae^{Cz} \tag{2.46}$$

$$A_i(z) = iAe^{Cz} \tag{2.47}$$

Next, let us see what happens to the imaginary component of some small imaginary input signal (e.g. noise) keeping the idler stable. In this case $A = C_3 + C_4$ and $C_1 + C_2 = 0$. This gives $C_1 = -C_2 = A/2$ and $C_3 = C_4 = A/2$.

$$A_s(z) = A/2e^{Cz} - A/2e^{-Cz} + i(A/2e^{Cz} + A/2e^{-Cz}) \tag{2.48}$$

$$A_i(z) = i\left(A/2e^{Cz} + A/2e^{-Cz}\right) - A/2e^{Cz} + A/2e^{-Cz} \tag{2.49}$$

If we neglect the decaying components we get

$$A_s(z) = A/2e^{Cz} + iA/2e^{Cz} \tag{2.50}$$

$$A_i(z) = iA/2e^{Cz} - A/2e^{Cz} \tag{2.51}$$

Finally, let us check the powers if we apply an input signal with the opposite sign than the maximum gain phase. This means $C_1 + C_2 = -A$, $C_3 + C_4 = 0$, $C_1 - C_2 = A$, and $C_3 = C_4$. This gives $C_3 = C_4 = 0$ and $C_2 = -A$, $C_1 = 0$.

$$A_s(z) = -Ae^{-Cz} \tag{2.52}$$

$$A_i(z) = iAe^{-Cz} \tag{2.53}$$

Which gives purely decaying components. Finally, what happens if we have no idler present: $C_1 = C_2$, $C_3 = C_4$, $C_1 = C_2 = A/2$, $C_3 = C_4 = 0$. In this case we get

$$A_s(z) = A/2e^{Cz} + A/2e^{-Cz} \tag{2.54}$$

$$A_i(z) = i\left(A/2e^{Cz} - A/2e^{-Cz}\right) \tag{2.55}$$

Which means that in the phase insensitive mode the parametric amplifier gives 6dB lower gain the the phase-sensitive amplifier.

# 3 Cascaded second-order nonlienarities for phase-sensitive amplification

Despite the many differences between the two, second-order nonlinearities can be exploited for phase-sensitive operation much like third-order nonlienarities. Second-order are not present in optical fibers due to the molecular symmetry of the $SiO_2$ molecule. For this purpose nonlinear optical crystals can be used.

The problem with second order nonlinearities compared with third-order nonlinearities is that the interacting waves fall into different wavelength domains of the material. This gives fundamentally different refractive indices for the interacting waves, which destroys phase matching ($\Delta k$). One solution is to exploit crystal birefringence by using nonparallel polarization states for the input waves. Although this process can theoretically provide perfect phase matching, in practice it is very sensitive and complicated to perform. Another solution is to periodically invert the crystal structure of the material, thus, effectively inverting the sign of the phase mismatch. This process – Quasi Phase Matching – provide somewhat lower nonlinear conversion efficiency than traditional phase matching, still, it is more feasible due to its simpler alignment, and the fact, that input signals can be paralelly polarized in this case. The coupled mode equations describing three-wave mixing are [southampton]

$$\frac{\mathrm{d}E_p(z)}{\mathrm{d}z} = -\frac{-\alpha_p}{2}E_p(z) + i\kappa_{pp}\omega_p E_{\mathrm{SH}}(z)E_p^*(z)e^{i\Delta k_{pp}}z \tag{3.1}$$

$$\frac{\mathrm{d}E_{\mathrm{SH}}(z)}{\mathrm{d}z} = -\frac{\alpha_{SH}}{2}E_{\mathrm{SH}}(z) + i\kappa_{pp}\omega_p E_p^2 e^{-i\Delta k_{pp}z} + 2i\kappa_{si}\omega_p E_s(z)E_i(z)e^{i\Delta k_{si}z} \tag{3.2}$$

$$\frac{\mathrm{d}E_s(z)}{\mathrm{d}z} = -\frac{\alpha_s}{2}E_s(z) + i\kappa_{si}\omega_s E_{SH}(z)E_i^*(z)e^{-i\Delta k_{si}z} \tag{3.3}$$

$$\frac{\mathrm{d}E_i(z)}{\mathrm{d}z} = -\frac{\alpha_i}{2}E_i(z) + i\kappa_{si}\omega_i E_{SH}(z)E_s^*(z)e^{-i\Delta k_{si}z} \tag{3.4}$$

The phase mismatches for the pump and the signal+idler are

$$\Delta k_{pp} = k_{SH} - 2k_p - \frac{2\pi}{\Lambda} \tag{3.5}$$

$$\Delta k_{si} = k_s + k_i - k_{SH} - \frac{2\pi}{\Lambda} \tag{3.6}$$

The coupling coefficients are

$$\kappa_{pp} = d_{\mathit{eff}}\sqrt{\frac{2\mu_0}{cn_p^2 n_{\mathrm{SH}} A_{\mathit{eff}}}} \tag{3.7}$$

$$\kappa_{si} = d_{\mathit{eff}}\sqrt{\frac{2\mu_0}{cn_s n_i n_{\mathrm{SH}} A_{\mathit{eff}}}} \tag{3.8}$$

Neglecting propagation losses and phase mismatch we get

$$\frac{\mathrm{d}E_p(z)}{\mathrm{d}z} = i\kappa_{pp}\omega_p E_{\mathrm{SH}}(z)E_p^*(z) \tag{3.9}$$

$$\frac{\mathrm{d}E_{\mathrm{SH}}(z)}{\mathrm{d}z} = i\kappa_{pp}\omega_p E_p^2(z) + 2i\kappa_{si}\omega_p E_s(z)E_i(z) \tag{3.10}$$

$$\frac{\mathrm{d}E_s(z)}{\mathrm{d}z} = i\kappa_{si}\omega_s E_{SH}(z)E_i^*(z) \tag{3.11}$$

$$\frac{\mathrm{d}E_i(z)}{\mathrm{d}z} = i\kappa_{si}\omega_i E_{SH}(z)E_s^*(z) \tag{3.12}$$

As can be seen, the phases relations of the signal, idler and pump are almost exactly the same as for the undepleted pump four-wave mixing. The only difference here is the phase relations' absolute value as here all phases should be zero for amplification whereas in the FWM case one of the was at $\pi/2$.

# 4 Experimental results

We have demonstrated record phase-sensitive amplification in a PPLN waveguide with the idler wave generated in HNLF. The experimental set-up and the phase-sensitive gain function can be seen in figure 4.2 and 4.1. For the next step, we demonstrated regeneration of BPSK signals based on phase-sensitive amplfication. The resulting constellation diagrams can be seen in figure 4.3. The input signal heavily degraded by phase noise is clearly better on the phase-sensitive amplifier output.

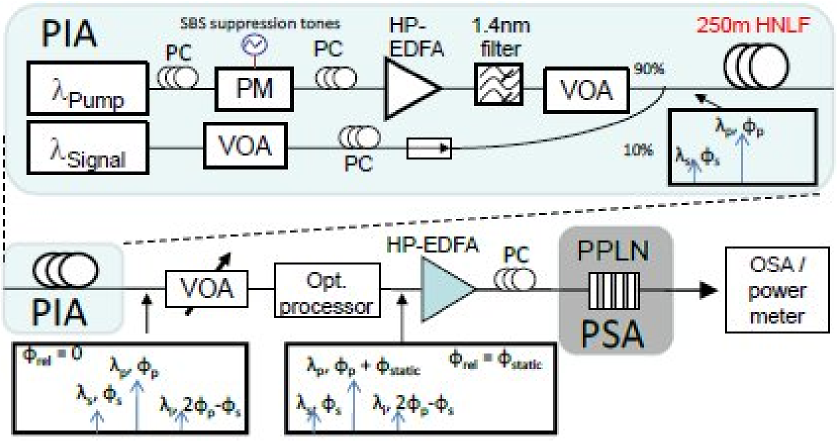


Figure 4.1: Experimental setup for measuring phase-sensitive operation in PPLNs

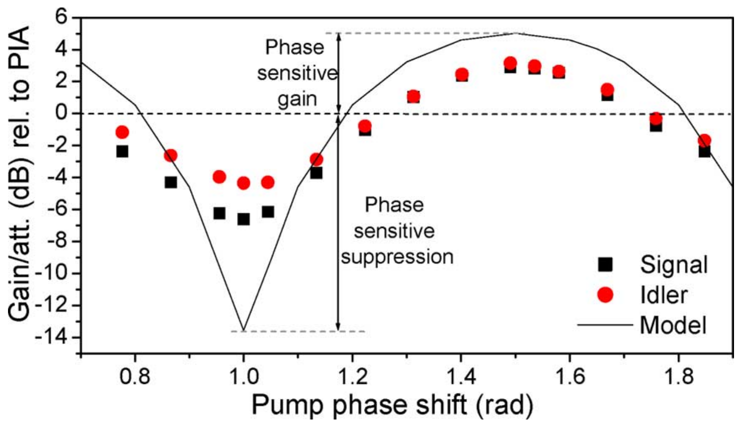


Figure 4.2: PPLN phase-sensitive characteristics

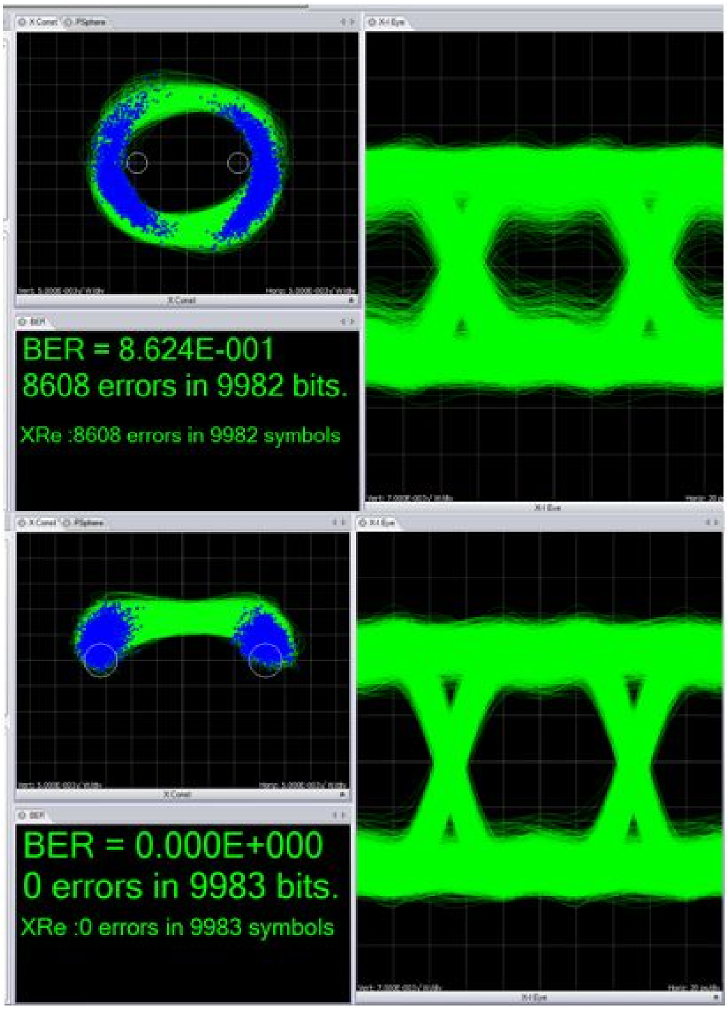


Figure 4.3: BPSK phase regeneration using PPLN

# References

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