# SIMULATED ANNEALING TECHNIQUE FOR NEXT GENERATION ACCESS NETWORK DESIGN

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**2012.12.28.**

MŰEGYETEM 1782

# ABSTRACT

Next Generation Access (NGA) networks offer enormous bandwidth and low latency, mainly due to the exploitation of optical transmission. Deploying optical fiber in the access network, however, requires a huge investment, therefore topology design and optimization plays an important role. In the recent years, algorithmic access network design became viable, mainly due to the existence of digital maps and GIS databases. In the previous work, we have proposed technology-dependent, scalable heuristics for Passive Optical Network (PON) and Active Ethernet (AETH) network design. In this paper, we present a novel technology-independent solution based on the Simulated Annealing (SA) metaheuristics for any point-to-multipoint optical access network technology. The newly proposed heuristics deliver competitive results, within 5-10% of the theoretical optimum, even for scenarios with up to thousands of demand points. The key for scalability is the concept Voronoi-diagrams applied for demand point clustering and evaluation within the Simulated Annealing scheme.

# 1. INTRODUCTION

After the successful of optical transmission in core networks, time has come for fiber communications in the access [1]. Optical access networks provide a future proof platform for a wide range of services at the expense of replacing the cable plant [2]. Unfortunately this "expense" is an enormous investment, which has to be justified by long term sustainability. Deployment costs have to be minimized, therefore optimal network planning plays crucial role regarding profitability.

Recent advances in the field of techno-economic evaluations have shown that making topology design and integral part of the process pays off [3]. Preliminary cost estimation based on the optimized network infrastructure has a significant accuracy gain over statistical approaches [4].

These altogether show the necessity of topology design algorithms, which are capable to deliver the optimal access network topology based on the digital map of the service area. These algorithms have to be scalable, in order to handle problems of practical interest, i.e. thousands of even tens of thousands of demand points.

In this paper, we present a scalable, technology-independent topology design algorithm based on the Simulated Annealing (SA) metaheuristics [20], addressing all point-to-multipoint access network technologies. The proposed solution offers remarkable scalability. The key is the concept of using Voronoi-diagrams for state evaluation, demand point clustering and network link selection, solving these altogether at the same time.

## 1.1. RELATED WORK

Theory of network design in itself has a long history and a massive research background [5]. It was motivated by its high economic and technical impact, while the appearance of digital maps and GIS databases made is viable.

The algorithmic PON network design literature started with optimization techniques for regular grid structures [6], and then in the following years various heuristic approaches were investigated, e.g. evolutionary algorithms, genetic algorithm or particle swarm optimization [7]-[11]. However, all of them suffer from scalability issues: service areas with thousands or even tens of thousands of demand points were beyond possibilities of these methods. Exact optimization, i.e. (Mixed) Integer Programming approaches were also demonstrated [12], with even stronger similar scalability issues. Clustering methods, and in particular the K-means algorithm was also applied in point-multipoint access networks [13]. Later, specialized heuristics were developed for PON network design [14]. In our recent publications, we have proposed a set of specialized, scalable heuristics for GPON, Active Ethernet and VDSL networks [15].

# 2. BACKGROUND

The term "Next Generation Access" refers to the fully or partly optical access networks that fulfill the challenging requirements of future (internet) services [2]. NGA network technologies include point-to-point fiber optics and various point-to-multipoint networks, e.g. Passive Optical Networks (GPON, WDM PON) or Active Ethernet (AETH).

Algorithmic topology design of NGA networks strongly relies on Geographic Information System (GIS) databases and digital maps, as the street system, and the map of the service area serves as the primary input of the topology design problem. This digital map defines a graph model, in which the edges represent the set of "potential" network links, which may be deployed, based on the decision of the network designer (or the optimization algorithm).

## 2.1. THE OPTIMIZATION PROBLEM

At the necessary level of abstraction, any point-to-multipoint access network may be described by a proper graph model (Fig. 1). The nodes have specific roles in this graph: the *demand points* have to be connected to *Distribution Units (DUs*, e.g. splitter, switch*)* which may be located at several available *DU locations*; the *DUs* will be then connected to the *Central Office (CO)*. Network deployment costs including cable plant (trenching & fiber), and also DU equipment costs have to be minimized, with respect to the length limitations and DU capacity constraints. This NGA Topology Design (NTD) problem combines three fundamental subproblems:

- clustering problem of the demand points, defining which demand points will be assigned to the same DU,
- location problem of the DUs, defining the optimal position of the DU for each demand point group,
- path/flow problem, defining the network links which connect the demand points to the respective DU, and the DUs to the CO.

Complexity and approximability of the NTD problem was investigated [16], and it was proven to be NP-hard, even with several simplifications and relaxations [15].

NP-hardness of the NTD problem, coupled with the fact that a typical access network scenario contains tens of thousands of nodes in the graph poses a though algorithmic challenge, calling for highly efficient and scalable approximation algorithms (i.e. heuristics).

In the recent years we have published specialized heuristics for GPON or AETH networks [15]. These *technology-dependent* algorithms offer notable scalability and high quality approximation, but heavily rely on specifications of these technologies, e.g. the weights of distinct cost factors or the typical length constraints and DU capacities. Hence, a more *technology-independent* approach supplements this "heuristics portfolio", addressing a wide set of NGA technologies.

The earlier published heuristics implement the designer's way of thinking, which is beneficial for scalability. However, a heuristic search over the solution could help to eliminate potential flaws of the implemented intelligence. These altogether has directed our attention to metaheuristics.

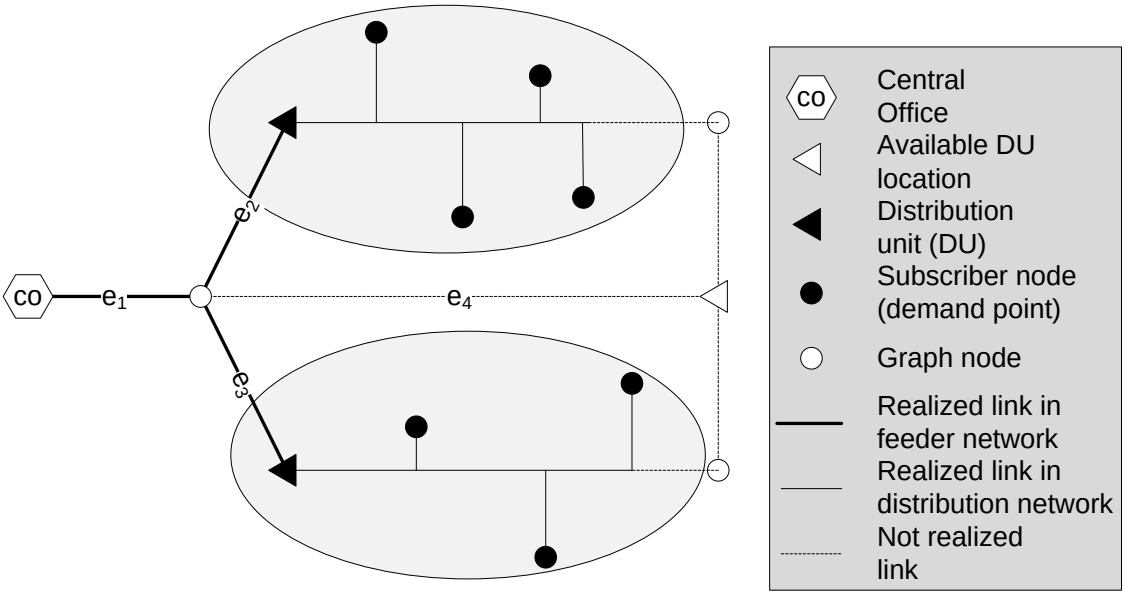


Fig. 1 Access network graph model [15]

## 2.2. Why Simulated Annealing?

Several *metaheuristic* approaches exist in the literature, and some of them were investigated regarding their applicability for the NTD problem [17], e.g. Random Optimization [18], Genetic Algorithm [19], Simulated Annealing [20], Tabu Search [21], and several nature inspired algorithms, such as Ant Colony Optimization [22] or the Firefly Algorithm [23].

*Simulated Annealing* [20] turned out to be the most promising alternative, among others due to its scalability, and its ability to avoid local optima. SA does not require numerous solutions, neither enormous amount of visited previous states to store. Every heuristics offers a trade-off between scalability and approximation quality. Simulated Annealing allows us to keep this trade-off under control, which was a strong argument for SA. SA could be effectively adapted to point-to-multipoint topology design, as the following sections will describe.

## 2.3. Voronoi-diagrams

Before we dive in the details of the proposed SA scheme, we have to spend a few words on Voronoi-diagrams, which lie in the heart of the state evaluation process.

In general, a Voronoi diagram (Fig. 2) is a decomposition of points in a given space, according to a set of “sites”. Every point in the space is assigned to the closest one of these sites, and the points assigned to the same site form a Voronoi cell. In our case, the Voronoi-diagram is interpreted on the network graph, with the distribution units (DUs) as its “sites”, while the cells are the demand point clusters (groups) around the DUs. In an optimal point-to-multipoint access network topology the demand points should be connected to their closest DU, i.e. the network topology forms a Voronoi-diagram.

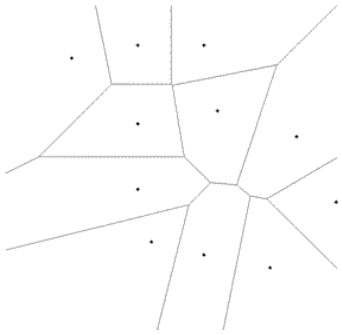

Fig. 2 Voronoi-diagram

# 3. SIMULATED ANNEALING FOR THE PON TOPOLOGY DESIGN PROBLEM

Applying a SA scheme is not a straightforward process: SA itself just gives a strategy, but it needs adaptation to the addressed problem. This adaptation itself makes the difference between highly efficient and totally useless applications of SA.

Namely, the following features have to be defined:

- state evaluation
- neighbor state selection
- initial state generation
- decision function
- cooling (temperature control) strategy
- termination

## 3.1. STATE EVALUATION

The key for the scalability of the algorithm was the idea to focus on the set of Distribution Units (DUs), by adding, removing or moving DUs during the state transitions. Using the concept of Voronoi-diagrams, the "state evaluation", i.e. the cost with the given set of DUs may be relatively quickly derived. Moreover, the following steps not only provide the state evaluation, but define the DU - demand point assignment, and give the network connections at the same time:

- **Step 1**: Following the concept of Voronoi diagrams, as an initial step, assign all demand points to their closest DU.
- **Step 2**: In order to meet DU capacity constraints ($K$), refine the assignment. Keep the closest $K$ demand points assigned to "overloaded" DUs, but disconnect the remainders. Obviously, if $n$ DUs are located at the same location, assign the closest $n \cdot K$ demand points to them.
- **Step 3**: Remove these fully loaded DUs along with their assigned $K$ demand points, and iterate from Step 1 again. Connect the remaining demand points to the remaining DUs, until any unconnected demand point exists.

Based on the assignment procedure a network topology is created with respect to the given DU allocation. The cable plant cost calculations follow the shortest paths between DUs and their assigned demand points, and minimal cost Steiner-tree connecting the DUs to the CO. Result of the state evaluation is the sum of the cable plant costs and the cost of the DU units.

## 3.2. Neighbor state selection

Convergence and scalability is also influenced by neighbor state selection, supporting walks across the solution space: the number of necessary steps between the two most distant states defines the "diameter" of the solution space.

We propose a neighbor state selection method based on the number and location of DUs. A neighbor state is a network topology, which is created via modification of the current topology by any of the following operations:

- **ADD:** Add a new DU to the current solution. Location of new DU will be chosen by a weighted random selection, based on the distribution of demand points, with higher probability in a region where overloaded DUs exist.
- **DELETE:** Remove a DU from the current solution. A weighted random selection will be applied again: a DU will be removed with higher probability from a region where underutilized DUs exist.
- **MOVE:** Move a (randomly chosen) DU to the nearest neighboring DU location.

The neighbor state is the result of a random choice among these operations. Combined with the demand point assignment process described at the state evaluation, adding a DU splits oversized groups, deleting a DU contracts undersized groups, while movement slightly rearranges the neighboring groups.

The number of necessary DUs in an optimal solution is typically close to *S/K*, i.e. the population per DU capacity ratio. Removing all DUs of solution $Y_1$, and adding all DUs of solution $Y_2$ therefore requires at most *2·S/K* steps, i.e. the diameter of the solution space is:

$$d \sim 2 \cdot \frac{S}{K}$$

In a typical PON network scenario, with a few thousand demand points (*S~3.000*), and 1:64 splitting ratio (*K=64*), diameter of the solution space is in the magnitude of 100 steps.

## 3.3. Initial state generation

Since the DUs play central role, the initial number and location of distribution units has to be determined, preferably not far away from the optimum. As a "naïve" approach, the initial DU allocation will follow the geographic distribution of demand points. Assuming DU capacity K, and *S* demand points (population), at least *S/K* DUs are necessary. The algorithm will start with 20% more, i.e. *1.2·S/K* DUs, in order to allow all state transitions, including DU deletions at the beginning. For the initial allocation of these DUs, a weighted random selection is applied among the potential DU locations, based on the demand point density distribution.

## 3.4. Decision function

The decision function gives the probability of a "backward" step during the minimization process, towards a higher cost solution. It lies in the heart of Simulated Annealing, prevents it from being stuck in a local minimum. In order to ensure convergence, this probability is decreasing in time, controlled by the decreasing temperature parameter (see below).

Suppose an actual state ($N_0$) and a neighbor state ($N_1$). If $N_1$ has lower cost, it will be accepted as a next state; otherwise a random number ($R$) is chosen, and compared to the decision function $d$, which is the function of the temperature ($T$) and the cost difference ($\Delta c$) between $N_0$ and $N_1$:

$$if\left[d\left(T,\Delta c\right)<R\right],\ accept\ N_1$$

Decision function $d$ is monotonic in $T$: with decreasing temperature, the probability of a "backward" step is also decreased. When applying SA for the topology design problem, the following decision function was used:

$$e^{\frac{-\Delta c}{T\cdot\mu_{price}}}>R$$

Here $\mu_{price}$ is a scaling factor for compensating the difference between the price and temperature absolute values, which can differ by several orders of magnitude, and depends on e.g. the unit used.

## 3.5. Temperature control

The schedule and rate of temperature reductions give a tradeoff between optimality and convergence speed. Typically faster processes stop earlier, with higher probability in a local minimum, while slower processes have a higher probability to reach a global optimum (or at least a "better" local minimum). Fig. 3 shows an example for this phenomenon.

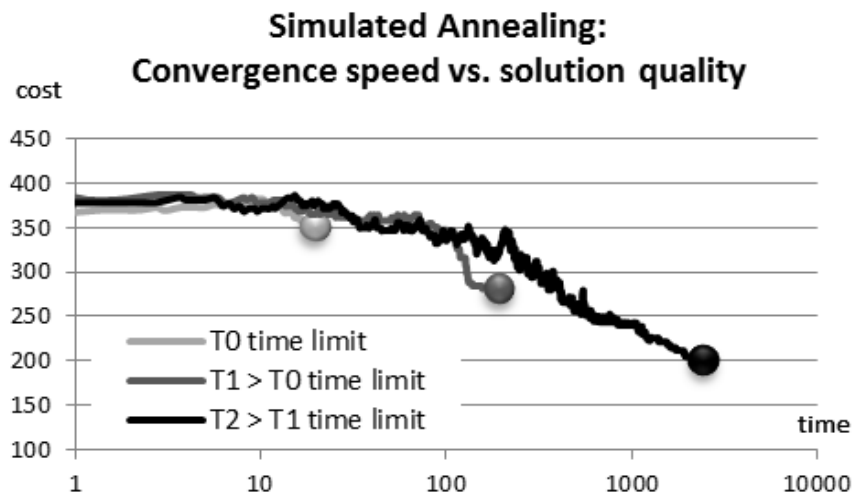


Fig. 3 Convergence speed vs. optimality

The temperature should obviously decrease monotonically. Here I have chosen an exponentially decreasing function from $T=1000$, multiplying it by an constant $\alpha<1.0$ in every loop, which leads to a fast decrease in the beginning, and slower process at the end, around the presumed optimum (Fig. 4).

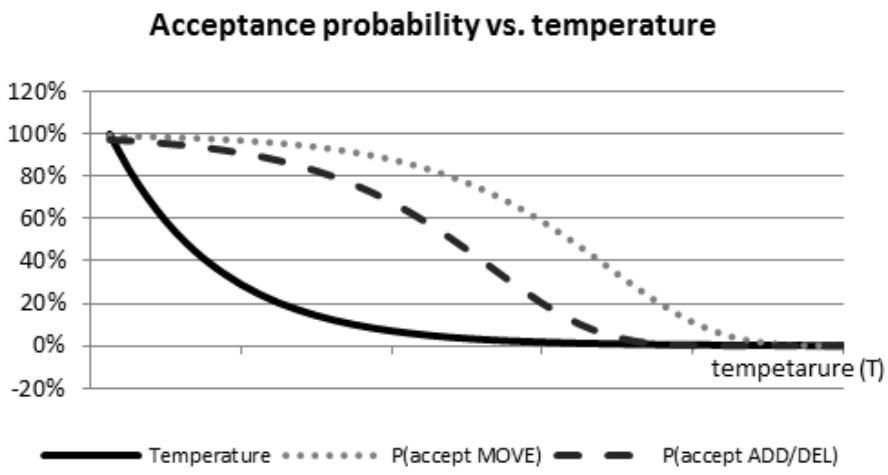


Fig. 4 Simulated annealing process dynamics

## 3.6. Termination

The Simulated Annealing process stops:

- when T falls below *1.0*
- if the cost remains unchanged for a sufficiently long period of time, i.e. no neighbor states are accepted for a number of iterations (in this case: 1000)

## 3.7. Parameter setup

The decision function and the temperature control strategy together controls the "dynamics" of the SA process. Fine tuning its parameters has key importance. The chosen parameters keep the decision "alive" throughout the cooling process. Fig. 4 shows the temperature decrease, the probability of an ADD/DELETE and a MOVE state transition together. These state transition probabilities have a "slow start" curve, which supports the initial steps across the solution space, in order to avoid local optima. The convergence speed is increased during the SA process. The ADD/DELETE transitions have a more pronounced effect on the topology, while a MOVE operation is responsible for the "fine tuning", therefore acceptance probability of the MOVE operation decreases slower. The curves are controlled via the above described parameters, namely $T$, $\alpha$ and $\mu_{price}$:

- with lower initial T values, the "slow start" phase gets shorter, while higher initial temperature makes it longer;
- $\mu_{price}$ scaling factor affects the height of the curves: higher values result in higher acceptance probabilities,
- α temperature decrease coefficient affects the number of iterations: the closer it is to 1.0, the more iterations we get.

# 4. EVALUATION

The evaluation of heuristics has two primary aspects: scalability and approximation quality. Therefore, we will use two "benchmarks". On the one hand, the SA-based heuristics will be compared to the theoretical optimum, calculated by a simplified Mixed Integer Programming (MIP) formulation [16]. On the other hand, SA will be compared to our specialized heuristics for GPON or AETH networks (namely the BCA and INCA heuristics [15]): to our knowledge, these belong to the fastest, scalable heuristics for GPON and AETH networks specifically.

## 4.1. SCENARIOS

We have chosen a set of reference areas for evaluation purposes, all of which use data from real-world scenarios: characteristics of them are concluded in Table 1. The last, large-scale urban scenario is a tough scalability challenge, with more than 26.000 demand points.

TABLE 1 CASE STUDIES

| **Parameter ↓** | **Scenario →** | Little town | Agglomeration | Dense urban | Large scale urban |
|---|---|---|---|---|---|
| Area ($km^2$) | | 7.21 | 4.42 | 1.18 | 13.56 |
| Buildings | | 3 071 | 2 134 | 1 080 | 4 151 |
| Demand points | | 3 165 | 2 714 | 6 994 | 26 489 |
| Buildings per $km^2$ | | 430 | 480 | 920 | 310 |
| Demand points per $km^2$ | | 440 | 610 | 5 930 | 1 950 |
| Graph nodes | | 6 712 | 4 783 | 2 265 | 19 331 |
| Graph edges | | 6 850 | 4 846 | 2 319 | 20 136 |
| Total street system length (km) | | 97,8 | 55,7 | 26,6 | 303 |

## 4.2. APPROXIMATION QUALITY

In this subsection we focus on the first three scenarios: for these, resource requirement of SA are satisfied, while the extremely large scale urban scenario will be investigated in the next section. We have compared the cost achieved by the MIP solver ("lower bound"); Simulated Annealing with various time limits; and the fast, specialized BCA/INCA heuristics. We have used four different time limits for SA: the shortest time limit ($T_0$) was equal to the running time of the respective fast specialized heuristics (BCA/INCA), then we used 10 and 100 times $T_0$. Finally SA was used without time limit, i.e. until one if terminating conditions (Section 3.6) are met.

The results for GPON networks are seen on Fig. 5. The cost values are normalized, so that the MIP theoretical optimum equals 100%. The total cost achieved by SA decreases monotonically as the computation times increase, as the grey bars between the black (MIP) and white (BCA) column show. As expected, SA provides lower quality results than the respective BCA/INCA fast heuristics with strict time limits, and comparable (or even better!) results when time limits does not affect it substantially. In these cases, the difference between the MIP lower bound and SA falls below 10%. Since it is just a lower bound on the minimum cost, SA could be even closer to the theoretical optimum, i.e. the approximation quality of SA is convincing. We also have to mention,

that in a few cases, SA results fall within the MIP solver's optimality gap (which explains how we can get below the lower bound).

As the problem size increases (from left to right on the horizontal axis), the impact of time limits for SA becomes visible: SA needs longer times to provide results comparable to the BCA algorithm. However, without these strict time limits, SA slightly outperforms BCA. For AETH networks (Fig. 6), similar trends are visible: SA suffers performance degradation with strict time limits, but it outperforms the INCA algorithm as it has "enough time". The running time difference between SA and INCA increases with problem size.

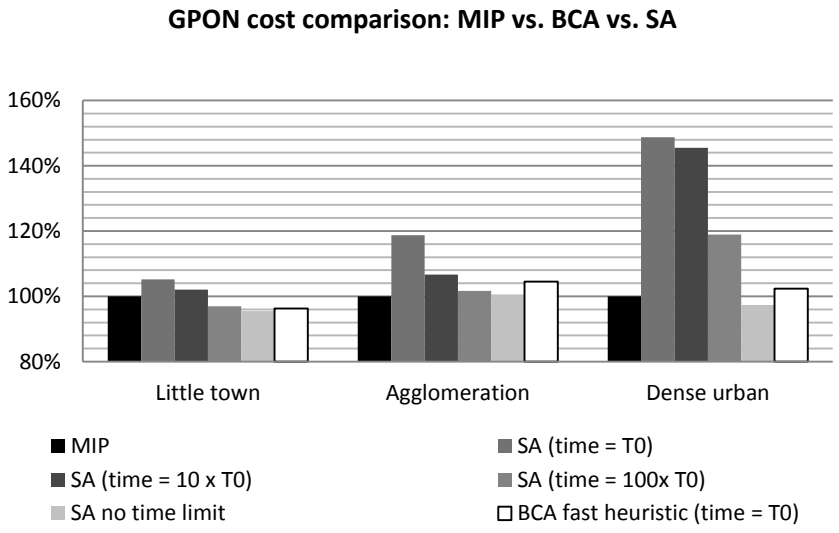


Fig. 5 Approximation quality I. (GPON)

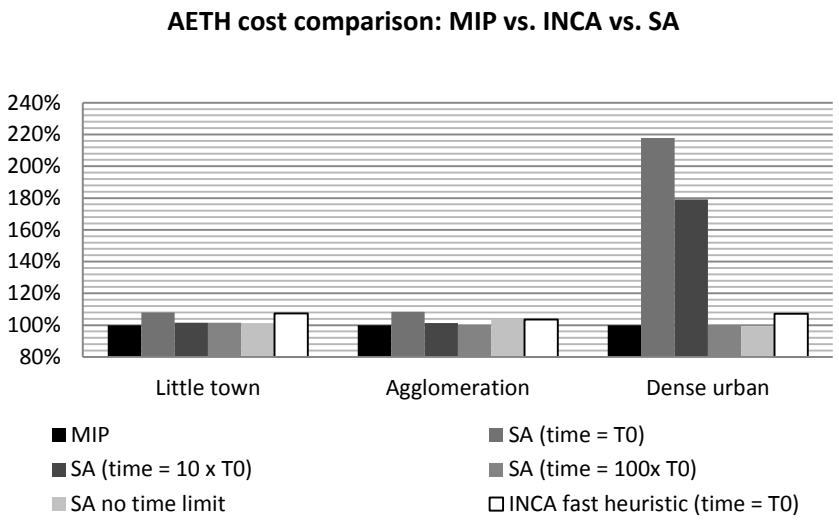


Fig. 6 Approximation quality II. (AETH)

## 4.3. SCALABILITY

We have seen that if SA has "enough time", it could outperform even the BCA/INCA heuristics. However, for this remarkable optimization performance, SA needs 2-3 orders of magnitude longer calculation times as the fast heuristics (Fig. 7). On the other hand, these longer times are still far acceptable: a couple of thousand seconds, i.e. a few hours for these scenarios with 3-7.000 demand points. Since network design is an "offline" problem, a couple of hours, or even a few days of computation is still acceptable. Obviously, BCA and INCA has its own application areas, where multiple, repetitive calculations are necessary.

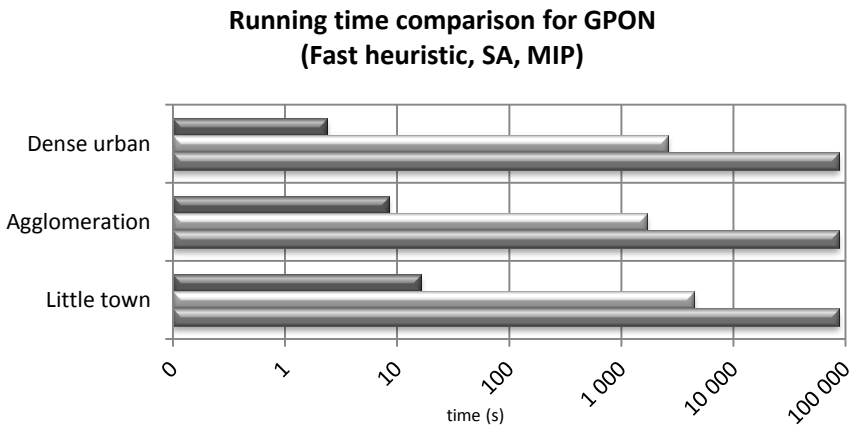


Fig. 7 Scalability comparison I.

The significant scalability difference between MIP, SA and the fast BCA/INCA heuristics needs explanation.

As one extremity, MIP virtually makes an exhaustive search all over the solution space (branch & bound): the most resource-intensive approach, but it offers exact optimization. In contrary, our earlier published technology-specific fast heuristics implement and follow the network designer's way of thinking in an almost deterministic manner. These do not analyze a wide set of candidate solutions, rather create a single topology, which makes them extremely fast.

The Simulated Annealing heuristics takes something from both worlds. On the one hand, it performs a random walk in the solution space, visiting a number of candidate solutions (potentially any of them). On the other hand, the idea of focusing on the Distribution Units and using the concept of Voronoi-diagrams for clustering and connecting demand points to DUs cuts a majority of the solution space. It leads to a significant scalability gain of SA over MIP, but SA still provides near-optimal results. Moreover, SA is technology-independent, i.e. it does not rely on specific cost weights or typical constraints of a given access network technology, which opens application possibilities towards any current and future point-multipoint optical access network technology.

## 4.4. Limits of Simulated Annealing

Difference between the specialized fast heuristics and SA (or even MIP) implies a more pronounced difference for really large-scale scenarios. Our last case study ("large-scale urban scenario") is a complete district of Budapest, Hungary, with more than 4.000 buildings and 25.000 demand points, which was beyond possibilities of MIP.

The total cost comparison of SA and the fast heuristics is depicted on Fig. 8, and the respective running time comparison is given on Fig. 9. The specialized, highly scalable BCA and INCA heuristics were extremely fast (75/45 seconds): these are capable to solve even larger problems. Simulated Annealing, even with an optimized implementation and carefully chosen parameters needed approximately 1-2 days for calculations.

For an offline network design problem, these computation times are just in the acceptable range, i.e. SA was capable to solve large-scale scenarios with 25.000 demand points, but SA has reached its limits there.

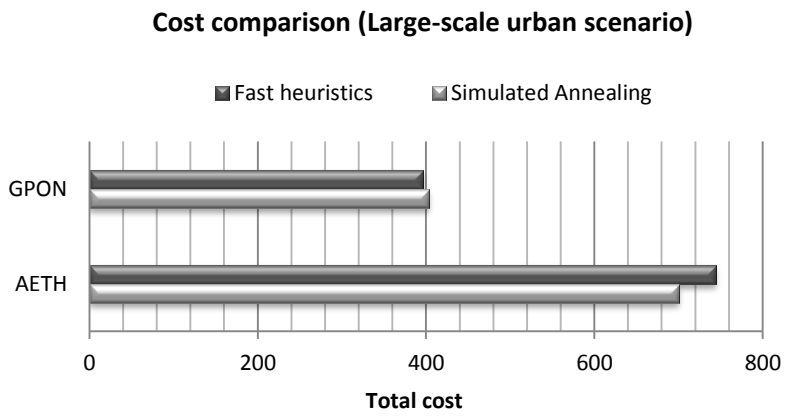


Fig. 8 Approximation quality III. (Large-scale urban scenario)

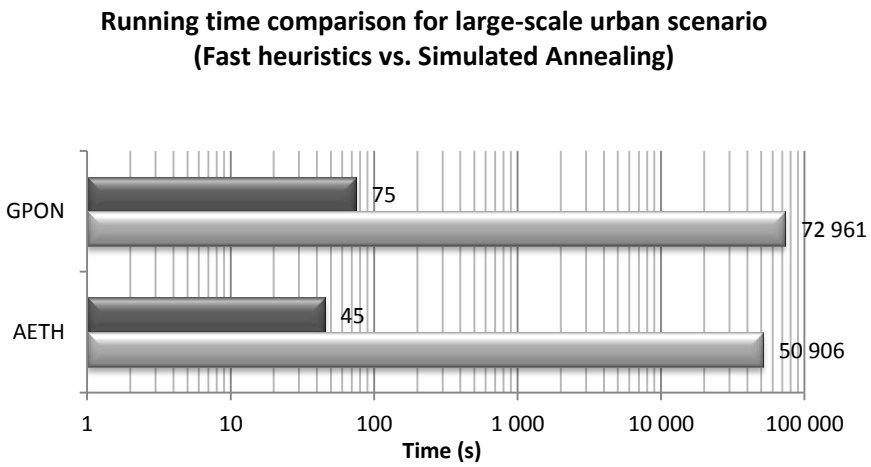


Fig. 9 Scalability comparison II.

# 5. CONCLUSION

In this paper, a technology-independent heuristic algorithm was proposed for the topology design of point-to-multipoint Next Generation Access networks. The proposed Simulated Annealing based heuristics approximates the theoretic lower bound within 10%, and scales for scenarios with up to ten or twenty thousand demand points. Beyond similar approximation performance, we could observe significant differences in scalability of the proposed Simulated Annealing solution, the earlier published technology-specific fast heuristics, and the Mixed Integer Programming.

As a conclusion, we can define the typical application areas. The earlier published, fast technology-dependent heuristics are the perfect choice for repetitive calculation, e.g. for analyzing sensibility of network deployment cost to various parameters or cost factors.

Simulated Annealing is a strong competitor in the final stage, when the network technology and its parameters are chosen, and we need a high quality optimized network topology. In this final step of the topology design process, time limits are not tight: with these conditions, SA outperforms its competitors.

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